



Investigation of the Corona Discharge Problem Based on Different Computational Approaches of Dimensional Analysis

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Authors' contributions

This work was carried out in collaboration between the two authors. Author JOO performed the analysis, solved the detailed example, prepared the tables, wrote the first draft of the manuscript and initiated the literature search. Author AMAR envisioned and designed the study, contributed to the symbolic and numerical analysis, checked the solution of the detailed example, enhanced and clarified the tables, managed and finalized the literature search and substantially edited and improved the entire manuscript. Both authors read and approved the final manuscript.

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ABSTRACT

Although corona discharge is notorious for its detrimental effects, it is also used in many beneficial practical applications. Despite the existence of a variety of sophisticated theoretical and experimental methods for investigating corona discharge, we explore yet a much simpler method that relies on the use of Dimensional Analysis (DA). The DA method does not demand profound knowledge of the underlying phenomenon or its governing equations, as it only needs the correct identification of the variables influencing the phenomenon, and the specification of their physical dimensions. The classical and well-known Gauss-Jordan elimination method is compared with other matrix-oriented computational approaches in analyzing the pertinent dimensional system. This method relies upon solution-preserving elementary row operations, i.e., operations that one can use on a matrix without spoiling the solution set for an associated matrix equation. A distinct

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advantage of this method is that it does not pre-suppose a particular value of the matrix rank but tackles the task of determining this rank while proceeding towards its ultimate result. Moreover, this method can also be used to find the inverse of a regular (invertible) matrix or to determine the solution (if any) of a system of linear equations. As a bonus, novel results of numerical investigations of bases, regimes, and dimensionless products are extensively presented in this paper. A remarkable observation made herein is that the set of variables pertinent for a particular phenomenon cannot be arbitrarily partitioned into basis (input) variables and regime (output or isolated) variables. The paper running example deals with expressing a specific variable, viz. ozone generation rate per unit length of wire (r^0) through dimensionless products in terms of a set of determining or influencing variables.

Keywords: Corona discharge; dimensionless products; dimensional analysis; Gauss-Jordan method.

1. INTRODUCTION

A corona discharge might be viewed as a stable electric discharge that emerges in a non-uniform geometry involving two (or more) electrodes when a sharp electrode is connected to a high voltage source, while the other, much flatter, electrode is (typically) connected to the ground. A region of high electric field is formed near the sharp electrode, as a result of the disparity in the lengths of the radii of curvature of the two electrodes. A free electron that happens to be in this region is accelerated and might eventually acquire kinetic energy that is sufficiently high to enable it to collide with a neutral atom or molecule and to detach another electron from that molecule. Therefore, the number of free electrons within the region (those detached from neutral atoms) grows steadily, and gas ionization takes place [1]. Corona discharge has many noticeable effects (typically detrimental). It usually coincides with a sharp hissing or crackling audible noise, visual violet glow (associated with invisible ultra-violet radiation), harmful ozone gas generation, power loss, and interference with radio communications.

Corona discharges occur in the gas between two electrodes, which have radii of curvature that are significantly different. Specifically, they occur on the electrode with a smaller radius of curvature, which might be a wire, a needle tip, or a blade edge, wherein the electric field strength increases significantly [2]. Both positive ions and electrons are generated under such conditions, and they move in opposite directions along and against the direction of the electric field (from the positive electrode to the negative one). When an electron leaves the region of the high field, it ceases to possess enough energy to further ionize more neutral particles, and hence the spread of the discharge is spatially limited. Electrons might attach to neutral molecules to

form negative ions, a phenomenon usually referred to as one of the electronegative gases. Corona discharges are classified as positive or negative coronas, according to the polarity of the voltage on the sharper electrode. If the sharper electrode is positive with respect to the flatter one, the corona is positive, and otherwise, it is negative.

Two regions can be identified in the air gap between the two electrodes: a lower field drifts region and a high field ionization region [3]. The corona voltage ranges from 6 to 19 kV, depending on the inter-electrode gaps [4]. The atmosphere, conductor size, and the spacing between the conductors are among the factors that affect corona discharge. The gas motion driven by the corona discharge recently drew much attention in scientific research circles. Besides, the use of bundled conductors and corona rings can effectively mitigate corona discharge, thereby reducing its inadvertent effects. Dimensional Analysis (DA) and other methods of partial analysis are techniques used to obtain partial solutions to problems that are too complex to be solved by complete mathematical analysis. For instance, DA was used to identify fundamental masses in terms of four fundamental constants that describe, respectively, the relativistic, quantum, gravitational, and cosmological aspects of the Universe in [5]. Typically, these methods have been found useful in many engineering applications, in general, and in power systems, in particular [6]. We strive herein to utilize classical DA, augmented by matrix-theoretical background and techniques, in the notable area of power systems concerning corona discharges. We observe that mathematicians, who are naturally knowledgeable about matrix theory, are not always knowledgeable regarding the use of this theory in DA. In the meanwhile, experimental researchers, who are expected to use DA, are

often neither deeply knowledgeable about nor highly experienced in applications of matrix theory. Therefore, we stress matrix theory application to DA in our presentation in this paper. The prominent problem of corona discharge will serve as a vehicle for demonstrating our matrix-enhanced DA paradigm.

Furthermore, the early major published works on corona discharge have been surveyed in [7]. Though corona discharge was known before the end of the third decade of the twentieth century, it took almost two decades for pioneering work on it as a mature subject to appear. This work may be credited to Loeb [7], who was the first to study corona in air subject to standard atmospheric pressure. He analyzed the mechanism of the pulses of negative corona and investigated the differences between positive and negative coronas. Robinson [8] was the first to report the existence of a phenomenon named ionic wind (coronal wind or ionic wind), which is the airflow caused by electric forces linked to corona discharge. However, Chattok [9] was the first to analyze this phenomenon quantitatively. Corona discharges from the surface of a conducting liquid, which might comprise liquid jets from electrospray nozzles or charged droplets, were also explored a variety of perspectives, including those of current-voltage characteristics, recording of current waveforms, steady or streak photography, mass spectrometry, light intensity measurements, and optical emission spectroscopy [10].

Investigations of various corona discharge phenomena are still of contemporary paramount importance due to two reasons: (a) that our understanding of the physical processes present in corona discharges is still far from being complete, and (b) the growing number of important practical applications of this phenomenon both in industry and research. These applications include the charging of thin insulating films, electrophotography, and electrostatic precipitation. On the negative side, corona discharges are generally detrimental and deleterious in electrical power systems, and they should be avoided or mitigated since they result in power loss, generate audible noise, and cause radio interferences [11].

This paper aims to investigate quantitatively the effect and theory of corona discharge in various mechanisms in addition to solving corona discharge related problems using three

computational approaches for dimensional analysis. To this goal, various articles focusing on corona discharge have been reviewed and the different areas where corona discharge is produced have been extensively evaluated. Our DA results cannot be considered as conclusive or complete ones as they need other methods (outside DA scope) to supplement the knowledge obtained about dimensionless products and establish a form of mathematical inter-dependence among these products. However, our DA results certainly provide a clear view of how the pertinent variables are inter-related. They also allow useful qualitative reasoning about the explored phenomenon without demanding any deep theorization or sophisticated computation.

The remainder of this paper is organized as follows. Section 2 presents a detailed literature review of corona discharge. Section 3 formulates and analyzes in detail a typical problem of corona discharge. Finally, conclusions are given in section 4.

2. LITERATURE REVIEW

In this section, a detailed literature review is presented regarding the theory and numerical simulation of corona discharge. This review is deemed useful for a full appreciation of the DA results obtained later. It also guides our selection of the physical variables to be included in any particular DA problem.

2.1 Early Discovery of Corona Discharge

Peek is believed to be the first scholar to mention the existence of corona in his 1929 book [12]. He noted that this phenomenon is initiated when the voltage between two smooth conductors exceeds a certain critical threshold and that it is manifested as an audible hissing noise, a violet light that is observable in a dark medium, and a noticeable reading on a wattmeter. Also, he noted that when the air is electrically overstressed, the air two main constituents, oxygen O_2 and nitrogen N_2 react chemically to form various nitrogen oxides, which are notorious for being undesirable pollutants. Consequently, the corona discharge is accompanied by power loss, which can be attributed to one of several reasons: chemical reactions, noise, light, and heat. Peek [12] also reported that the power loss recorded by a wattmeter increases significantly with the increase of the voltage level. He also observed the existence of several differences

between AC and DC corona discharges. The appearance of the AC corona for the positive/negative half-cycle of the supplied voltage is the same as that of the positive/negative DC corona. Moreover, Peek utilized a stroboscope (an instrument through which a rotating object appears stationary), to explore the difference in the discharge patterns for the positive and negative coronas. He observed that some reddish beads are formed on the wire in the case of a negative applied voltage, while a smoother bluish-white glow is visible in the case of a positive applied voltage.

In addition, Peek derived an analytical formula for the electric field intensity E on the active corona electrode at the point of corona onset. In his analysis, he considered both influences of the mass volumetric density of air and the corona electrode radius of curvature. It should be noted that any configuration that has a non-uniform gap can be used for the generation of the corona discharge current. However, in practical situations, geometries that possess electrodes differing significantly in sharpness (as measured inversely by the radii of curvature) are more effective for such a generation. The most prominent systems that have been explored in the open literature are the point-to-plane system (where the point can be a tip of a sharp needle), and the coaxial wire-cylinder system. In 1965, Loeb [13] explained the difference in the corona patterns arising in these two systems. While the point-plane geometry has a confined discharge region, the discharge in the coaxial geometry can be initiated at different points on the corona wire.

2.2 Simulation of Corona Discharge

Diverse electrical, mechanical, and chemical processes are related to the phenomenon of corona discharge. Significant simplifications are usually needed to analyze these processes. Notwithstanding such simplifications, desirable analytical solutions are not obtainable for geometries other than the one-dimensional one. Consequently, numerical simulation of corona discharges has become the main tool of scholars to enhance their understanding of these discharges [14]. The two major parameters involved in such numerical simulation are the electric field intensity and the space charge density. These two quantities enable the calculation of the electric current, which, in turn, is the main parameter required for the computation of the power and energy loss. The electric field intensity is affected by the space

charge magnitude and distribution and the space charge depends on the distribution of the electric-field intensity. Therefore, both quantities are mutually inter-related.

Furthermore, the heat transfer resulting from corona discharge was examined in [4] using a thin plate as the corona electrode, while the collecting electrode is formed by grounding the heated plate. The experiments conducted in [4] reveal that the heat transfer coefficient at the center of the heated plate is increased by a factor in the range 2.6 - 4.8 times as compared to that of natural convection. In comparison, forced convection heat transfer enhancement using a coaxial wire-tube corona system was numerically analyzed in [15], where the proposed corona system could change the flow pattern pushing the central high-velocity flow towards the hot surface. Mehalaine et al. [16] evaluated the effects of corona discharges on a wing and flap system via numerical simulation. They indicated that the numerical outcomes of the high lift system show a remarkable increase of the lift coefficients between the angles of attack of 1.8° and 6° and a delay of the boundary layer separation. Grosu et al. [17] simulated corona discharge by the similarity theory methods where a general system of equations for a corona discharge was derived and reduced to a dimensionless form. Their outcome reveals that the quadratic pattern of the current-voltage characteristics of the corona discharge results from the linear dependence of the electric field intensity and the linear threshold dependence of the density of free space carriers on the voltage.

2.3 Investigation of Corona Discharge

Morrow [18] reported some pioneering work in 1985 on simulating corona discharge in several gases including oxygen, wherein he considered a few chemical reactions. He determined the electric field intensity by using a hybrid technique, which combined (a) the Flux-Corrected Transport method that solves the three drift-diffusion equations for ionic species, and (b) the finite difference method (FDM) that solves Poisson's equation.

However, the first published work, devoted entirely to using the finite element method (FEM) to simulate corona discharge was co-authored by Janischewskyj and Gela [19], who simulated corona in a wire-cylinder configuration assuming a one-dimensional unipolar corona model. A useful (albeit controversial) assumption called

the Deutsch Hypothesis (DH) is extensively adopted for simplifying the calculation of the ionized electric fields surrounding electrodes in corona [20–23]. Jones and Davies [24] asserted that the Deutsch assumption was satisfied in the simulation in [19] since the electric field was always radial in the used configuration. Abdel-Salam et al. [25] used a hybrid FEM-charge simulation method (CSM) technique, for simulating corona discharge in a wire-to-ground configuration. Chen and Davidson simulated positive and negative corona discharges in a wire-cylinder configuration [26].

Also, Yanallah and Pontiga [27] introduced a semi-analytical stationary discharge model in a point plane configuration for both positive and negative corona discharges in oxygen. The approximate analytical expressions for the electric field and the ionic densities were found by solving the Gauss and the continuity equations [27]. A semi-empirical equation was derived in [28] to provide a correlation between the ozone generation rate of a negative wire-to-plate corona discharge in both dry and humid air. Adamiak et al. [29] innovated an approach based on a direct ionization criterion and applied it for a two-dimensional hyperbolic needle-ground configuration. This new approach was compared with two others: one based on Kaptzov's hypothesis [30–33] and another based on the analytical Peek formula with an equivalent electrode radius [25]. The discharge current was practically the same for the three approaches utilized. However, the electric field distributions on the corona electrode surface were slightly different. The current-voltage and light emission characteristics of electro-spraying of various liquids in standard atmospheric air were investigated in [10]. The spectroscopic measurements reveal that the onset of corona discharge coincides with the beginning of electro-spraying where the dependence of the amplitude of selected spectral lines on capillary-nozzle voltage was determined from the measured light emission spectra of discharges from the capillary nozzle and liquid jet. A novel dimensionless approach to analyzing the capability of a solar-based power supply system with seasonal hydrogen storage to supply a constant load demand all year round was studied in [34].

The dimensionless model is applied to a set of 78 cities with varying latitudes, which are selected to be lying across the five major continents. For a round-trip storage efficiency of

around 45% and an assumption of the base-case unit costs of components, solar-hydrogen systems were found to be economic in a majority (55%) of the cities considered. Carsimamovic et al. [35] examined the AC corona discharge parameters of atmospheric air using a calculation model. They concluded that the analyses of AC corona discharge parameters of atmospheric air over a long period (measured in hours, days, or even weeks) allow for the determination of the ionization and attachment coefficients as functions of the electric field intensity and inter-electrode spacing.

2.4 Corona Discharge in Various Applications/Mechanisms/Techniques

Needle-mesh and needle-fin electrodes were utilized in the enhancement of heat transfer via corona discharge and a direct comparison of the ionic wind devices with the two electrodes is carried out experimentally [36]. The investigation of the heat transfer in [36] reveals that the needle-fin configuration has a superior performance making a fall in temperature from 54.5°C to 39.1°C with a low power consumption of 0.85 W. The possibility and advantages of utilizing fins as collector electrodes were also confirmed in [36]. Lee et al. [37] analyzed the intermittent corona discharge plasma jet (ICDPJ) for improving the quality of tomato where the tomatoes were treated with corona plasmas produced by using 8 kV DC electricity at 2.0-4.0 ampere currents. It was indicated that the ICDPJ treatment decreased the contaminants loads by 0.68-1.02 log CFU/g at 2.0 A, by 1.42-1.71 log CFU/g at 3.0 A, and by 2.00 log CFU/g to a non-detectable level at 4.0 A. Here, the symbol CFU stands for a colony-forming unit, which is a unit used in microbiology to estimate the number of viable bacteria or fungal cells in a sample, where the term viable is defined as the ability to multiply via binary fission under specific controlled conditions. Usually, the unit CFU/g (colony-forming unit per gram) is given in association with logarithm (to base 10) notation.

Also, the generalizations from the experimental current-voltage properties of a corona discharge for helium and synthetic air at the positive and negative polarities of a star-shaped discharge electrode at different gas pressures were studied by Grosu et al. [38], who confirmed Townsend's structure of the characteristics and stated that this fact can serve as a basis for wide application.

3. FORMULATION AND ANALYSIS OF A CORONA DISCHARGE PROBLEM VIA DA

3.1 Determination of Dimensionless Products Involving Seven Physical Variables

The dimensional system to be studied herein is formulated using seven physical parameters, namely, the ozone generation rate per unit length of wire in dry air (r^0 , mg/(s·m)), the wire radius (b , mm), the inter-electrode gap (d , mm), the applied voltage (V , Volt), the excess voltage (V_e , Volt) which is referred to as the difference between the applied voltage (V , Volt) and the corona inception voltage (V_i , Volt), the permittivity for the inter-electrode drift region (ϵ , F/m), and the ion mobility (μ , $m^2/(V \cdot s)$). The analysis is based on the four fundamental dimensions for electromagnetic entities (mass (M), length (L), time (T), and current (I)) in the International System (SI) of Units [28].

In this section, DA will be used to obtain the dimensionless products of the seven physical variables mentioned above as well as to show how these variables are interrelated. The above variables are a mixture of fundamental and derived quantities, and as such it is possible to mathematically derive/express their dimensions using the afore-mentioned four main fundamental dimensions, i.e., mass (M), length (L), time (T), and current (I). It is, however, worthy to note that the expressions to be derived are for general dimensionless products that relate the seven physical variables on equal footing without giving weight or priority to any of them.

Also, three different computational techniques/methods/approaches will be utilized to broaden the scope of the DA approach and to know the benefit and complexity involved in each of the selected techniques. The list of variables involved is illustrated in Table 1. The units and dimensions of each variable are derived using basic knowledge of physics and mathematics.

Table 1. Seven physical variables involved in the calculation of the dimensionless products for ozone generation using DA

Variable	Symbol	Unit	Dimensions
Ozone generation rate per unit length of wire in dry air	r^0	mg/(s·m)	$M L^{-1} T^{-1}$
Wire radius	a	mm	L
The inter-electrode gap	d	mm	L
Applied voltage	V	V or $m^2kg/(s^3.A)$	$M L^2 T^{-3} I^{-1}$
Excess voltage	V_e	V or $m^2kg/(s^3.A)$	$M L^2 T^{-3} I^{-1}$
permittivity for the inter-electrode drift region	ϵ	F/m or $A^2s^4/(m^3.kg)$	$M^{-1} L^{-3} T^4 I^2$
Ion mobility	μ	$m^2/(V \cdot s)$ or s^2A/kg	$M^{-1} T^2 I$

The derivation of the applied and excess voltage (V and Ve) units/dimensions:

We start our derivation by the well-known result that

Voltage or electric potential = electric field strength (E) × distance (d)

i.e.

$$V = E \times d \tag{1}$$

$$E = \frac{\text{Force}, F}{\text{Charge}, Q} \tag{2}$$

$$\text{Charge}, Q = \text{current}, I \times \text{time}, t \tag{3}$$

$$\text{Force}, F = \text{mass}, m \times \text{acceleration}, a \tag{4}$$

$$\text{Acceleration}, a = \frac{\text{velocity}}{\text{time}} = \text{displacement}/(\text{time})(\text{time}) \tag{5}$$

Substituting equations (3), (4), and (5) into equation (2) gives

$$E = \frac{m \times d}{t^3 \times I} \text{ (kgm/s}^3\text{A)} \quad (6)$$

Substituting equation (6) in equation (1) results in:

$$V = \frac{m \times d}{t^3 \times I} \times d = \frac{m \times d^2}{t^3 \times I} \text{ (} \frac{\text{kgm}^2}{\text{s}^3\text{A}} \text{)} \quad (7)$$

The derivation of the permittivity for the inter-electrode drift region (ϵ) units/dimensions

$$\text{permittivity, } \epsilon = \frac{\text{capacitance,C}}{\text{distance,d}} \quad (8)$$

$$C = \frac{Q}{V} = \frac{It}{\left(\frac{m \times d^2}{t^3 \times I}\right)} = \frac{I^2 t^4}{d^2 m} \text{ (} \frac{\text{A}^2 \text{s}^4}{\text{m}^2 \text{kg}} \text{)} \quad (9)$$

Putting equation (9) into equation (8) gives

$$\epsilon = \frac{\left(\frac{I^2 t^4}{d^2 m}\right)}{d} = \frac{I^2 t^4}{d^3 m} \text{ (} \frac{\text{A}^2 \text{s}^4}{\text{m}^3 \text{kg}} \text{)} \quad (10)$$

The derivation of the ion mobility (μ) units/dimensions

$$\mu = \frac{d^2}{V \times t} \quad (11)$$

since

$$V = \frac{m \times d^2}{t^3 \times I}$$

$$\mu = \frac{d^2}{\left(\frac{m \times d^2}{t^3 \times I}\right)} = \frac{t^2 \times I}{m} \text{ (s}^2\text{A/kg)} \quad (12)$$

According to the theorem presented in [5,39], each dimensionless product of the set of the seven physical variables in Table 1 above will be of the form:

$$\pi = k r^{0b} a^e d^f V^g V_e^h \epsilon^k \mu^m \quad (13)$$

Where k is a dimensionless constant, while b, e, f, g, h, k, and m are exponents yet to be (partially) determined or inter-related.

If we denote by [y] the dimension of the parameter y we note that $[\pi] = 1$, where $[\pi] = (M L^{-1} T^{-1})^b (L)^e (L)^f (M L^2 T^{-3} I^{-1})^g (M L^2 T^{-3} I^{-1})^h (M^{-1} L^{-3} T^4 I^2)^k (M^{-1} T^2 I)^m$ (14a)

or equivalently:

$$M^0 L^0 T^0 I^0 = M^{b+g+h-k-m} L^{-b+e+f+2g+2h-3k} T^{-b-3g-3h+4k+2m} I^{-g-h+2k+m} \quad (14b)$$

The product π is dimensionless if

$$b + g + h - k - m = 0 \quad (15a)$$

$$-b + e + f + 2g + 2h - 3k = 0 \quad (15b)$$

$$-b - 3g - 3h + 4k + 2m = 0 \quad (15c)$$

$$-g - h + 2k + m = 0 \tag{15d}$$

There are seven unknowns (i.e., the b, e, f, g, h, k, and m exponents) in equations (15) with just four conditions (equations). It will be seen shortly that these equations are linearly independent i.e., the dimensional matrix derived from them has a full rank of four. Thus, we can determine only four of the seven unknowns in terms of the remaining three. This implies that only four of the seven unknown variables (to be termed basis variables) can be found in terms of the remaining three (to be termed regime variables). In the meantime, from our knowledge of combinatorial analysis, there are 35 ways for choosing 4 objects out of 7 (with neither order nor repetition). However, it might turn out that some of these 35 ways are not realizable. The dimensional-matrix form of the system is shown in Table 2.

Table 2. The dimensional matrix

	r^0	a	V	d	Ve	ϵ	μ
M	1	0	1	0	1	-1	-1
L	-1	1	2	1	2	-3	0
T	-1	0	-3	0	-3	4	2
I	0	0	-1	0	-1	2	1

3.1.1 Fundamental solutions method

In this method, the dimensional matrix presented in Table 2 is utilized to obtain the dimensionless products. The original homogeneous matrix equation is replaced by a number of inhomogeneous equations equal to the number of regime variables (three in the present case). For each of these equations, we assign a value of a unit vector to the vector of regime indices. For the first equation of our present problem, we assign the unit-vector value $[1 \ 0 \ 0]^T$ to a selected regime vector $[b \ e \ g]^T$, thereby obtaining the inhomogeneous equation below for the corresponding vector of basis indices $[f \ h \ k \ m]^T$. Note that the matrix columns under the zero-valued indices e and g, are irrelevant or don't-cares, and hence are not specified, so as to emphasize that they do not pertain to the new equation. The column beneath the index b has each of its entries negated, since it is assumed to have been moved to the other side of the equation, and it is the reason why the equation is now inhomogeneous.

	r^0	a	V	d	Ve	ϵ	μ
	$b=1$	$e=0$	$g=0$	f	h	k	m
M	-1	-	-	0	1	-1	-1
L	1	-	-	1	2	-3	0
T	1	-	-	0	-3	4	2
I	0	-	-	0	-1	2	1

The matrix equation above is a compact form of four scalar equations, namely

$$h - k - m = -1; \quad f + 2h - k = 1; \quad -3h + 4k + 2m = 1; \quad -h + 2k + m = 0$$

We now solve the above matrix equation (or its equivalent set of four scalar equations) using Cramer's rule to obtain:

$$A = \begin{bmatrix} 0 & 1 & -1 & -1 \\ 1 & 2 & -3 & 0 \\ 0 & -3 & 4 & 2 \\ 0 & -1 & 2 & 1 \end{bmatrix} \quad b_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad \text{and } x = \begin{bmatrix} f \\ h \\ k \\ m \end{bmatrix}$$

$$\Delta \text{ (determinant of } A) = -1$$

$$A_f = \begin{bmatrix} -1 & 1 & -1 & -1 \\ 1 & 2 & -3 & 0 \\ 1 & -3 & 4 & 2 \\ 0 & -1 & 2 & 1 \end{bmatrix}$$

$$\Delta_f \text{ (determinant of matrix } A_f) = 0, \quad f = \frac{\Delta_f}{\Delta} = \frac{0}{(-1)} = 0$$

$$A_h = \begin{bmatrix} 0 & -1 & -1 & -1 \\ 1 & 1 & -3 & 0 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

$$\Delta_h \text{ (determinant of matrix } A_h) = 1$$

$$h = \frac{\Delta_h}{\Delta} = \frac{1}{(-1)} = -1$$

$$A_k = \begin{bmatrix} 0 & 1 & -1 & -1 \\ 1 & 2 & 1 & 0 \\ 0 & -3 & 1 & 2 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$$\Delta_k \text{ (determinant of matrix } A_k) = 1$$

$$k = \frac{\Delta_k}{\Delta} = \frac{1}{(-1)} = -1$$

$$A_m = \begin{bmatrix} 0 & 1 & -1 & -1 \\ 1 & 2 & -3 & 1 \\ 0 & -3 & 4 & 1 \\ 0 & -1 & 2 & 0 \end{bmatrix}$$

Δ_m (determinant of matrix A_m) = -1

$$m = \frac{\Delta_m}{\Delta} = \frac{(-1)}{(-1)} = 1$$

The first fundamental solution is now completed as:

	r^0	a	V	d	Ve	ε	μ
	b	e	g	f	h	k	m
π_1	1	0	0	0	-1	-1	1

Therefore, the first regime or dimensionless product is

$$\pi_1 = ((r^0)^1(\mu)^1(V_e)^{-1}(\varepsilon)^{-1}) = \left(\frac{r^0\mu}{V_e\varepsilon}\right)$$

The matrix and scalar forms for the second fundamental solution ($[b \ e \ g]^T = [0 \ 1 \ 0]^T$) are given by

	r^0	a	V	d	Ve	ε	μ
	b=0	e=1	g=0	f	h	k	m
M	-	0	-	0	1	-1	-1
L	-	-1	-	1	2	-3	0
T	-	0	-	0	-3	4	2
I	-	0	-	0	-1	2	1

$$h - k - m = 0; \quad f + 2h - 3k = -1; \quad -3h + 4k + 2m = 0; \quad -h + 2k + m = 0$$

Solution of the above matrix equation (or of its equivalent set of four scalar equations) via Cramer's rule gives:

$$A = \begin{bmatrix} 0 & 1 & -1 & -1 \\ 1 & 2 & -3 & 0 \\ 0 & -3 & 4 & 2 \\ 0 & -1 & 2 & 1 \end{bmatrix} \quad b_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \text{and } x = \begin{bmatrix} f \\ h \\ k \\ m \end{bmatrix}$$

$$\Delta (\text{determinant of } A) = -1$$

$$\Delta_f (\text{determinant of matrix } A_f) = 1$$

$$f = \frac{\Delta_f}{\Delta} = \frac{1}{(-1)} = -1$$

$$\Delta_h (\text{determinant of matrix } A_h) = 0$$

$$h = \frac{\Delta_h}{\Delta} = \frac{0}{(-1)} = 0$$

$$\Delta_k (\text{determinant of matrix } A_k) = 0$$

$$k = \frac{\Delta_k}{\Delta} = \frac{0}{(-1)} = 0$$

$$\Delta_m (\text{determinant of matrix } A_m) = 0$$

$$m = \frac{\Delta_m}{\Delta} = \frac{0}{(-1)} = 0$$

The second fundamental solution is now completed as:

	r^0	a	V	d	Ve	ϵ	μ
	b	e	g	f	h	k	m
π_2	0	1	0	-1	0	0	0

Therefore, the second regime or dimensionless product is

$$\pi_2 = ((a)^1(d)^{-1}) = \left(\frac{a}{d}\right)$$

The matrix and scalar forms for the third fundamental solution ($[b \ e \ g]^T = [0 \ 0 \ 1]^T$) are given by

	r^0	a	V	d	Ve	ϵ	μ
	b=0	e=0	g=1	f	h	k	m
M	-	-	-1	0	1	-1	-1
L	-	-	-2	1	2	-3	0
T	-	-	3	0	-3	4	2
I	-	-	1	0	-1	2	1

$$h - k - m = -1; \quad f + 2h - 3k = -2; \quad -3h + 4k + 2m = 3; \quad -h + 2k + m = 1$$

Solving the above matrix equation (or its equivalent set of four scalar equations) using Cramer's rule, we obtain:

$$A = \begin{bmatrix} 0 & 1 & -1 & -1 \\ 1 & 2 & -3 & 0 \\ 0 & -3 & 4 & 2 \\ 0 & -1 & 2 & 1 \end{bmatrix} \quad b_3 = \begin{bmatrix} -1 \\ -2 \\ 3 \\ 1 \end{bmatrix} \quad \text{and} \quad x = \begin{bmatrix} f \\ h \\ k \\ m \end{bmatrix}$$

$$\Delta (\text{determinant of } A) = -1$$

$$\Delta_f (\text{determinant of matrix } A_f) = 0$$

$$f = \frac{\Delta_f}{\Delta} = \frac{0}{(-1)} = 0$$

$$\Delta_h (\text{determinant of matrix } A_h) = 1$$

$$h = \frac{\Delta_h}{\Delta} = \frac{1}{(-1)} = -1$$

$$\Delta_k (\text{determinant of matrix } A_k) = 0$$

$$k = \frac{\Delta_k}{\Delta} = \frac{0}{(-1)} = 0$$

$$\Delta_m (\text{determinant of matrix } A_m) = 0$$

$$m = \frac{\Delta_m}{\Delta} = \frac{0}{(-1)} = 0$$

The third fundamental solution is now stated as:

	r^0	a	V	d	Ve	ϵ	μ
	b	e	g	f	h	k	m
π_3	0	0	1	0	-1	0	0

Therefore, the d regime or dimensionless product is

$$\pi_3 = ((V)^1(V_e)^{-1}) = \left(\frac{V}{V_e}\right)$$

Hence, the three dimensionless products obtained are:

$$\pi_1 = \left(\frac{r^0 \mu}{V_e \epsilon}\right); \quad \pi_2 = \left(\frac{a}{d}\right); \quad \pi_3 = \left(\frac{V}{V_e}\right)$$

and they constitute regimes for the three variables r^0 , a , and V . In fact, each of these variables appears only in one and only one of the three dimensionless products.

3.1.2 Matrix analysis method

$$\pi = k r^{0b} a^e d^f V^g V_e^h \epsilon^k \mu^m$$

The dimensional matrix given in Table 2 is assumed to be column-wise partitioned into a unit matrix A sharing the same rows with another matrix B . The partitioned matrix is used to obtain the dimensionless products as follows. The operations applied to convert matrix A to unit matrix amount to left multiplying it by its inverse. Therefore, these operations are also applied B , thereby left multiplying it by the inverse of A

$$A = \begin{bmatrix} 0 & 1 & -1 & -1 \\ 1 & 2 & -3 & 0 \\ 0 & -3 & 4 & 2 \\ 0 & -1 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 2 \\ -1 & 0 & -3 \\ 0 & 0 & -1 \end{bmatrix}$$

Therefore

$$A^{-1} = \begin{bmatrix} 3 & 1 & 2 & -1 \\ 0 & 0 & -1 & 2 \\ 1 & 0 & 0 & 1 \\ -2 & 0 & -1 & 1 \end{bmatrix}$$

Hence

$$A^{-1}B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

Now, the matrix defined as $C = -[A^{-1}B]^T$ is computed and placed to the left of an appropriate unit matrix I of the same row dimension, thereby resulting in a matrix of the dimensional products, which is conveniently called the Szirtes matrix [40].

$$C = \begin{bmatrix} 0 & -1 & -1 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

	r^0	a	V	d	Ve	ε	μ
	b	e	g	f	h	k	m
π_1	1	0	0	0	-1	-1	1
π_2	0	1	0	-1	0	0	0
π_3	0	0	1	0	-1	0	0

According to the Szirtes matrix above, the problem has three regimes or dimensional products, which might be written in explicit algebraic form as

$$\pi_1 = ((r^0)^1(\mu)^1(V_e)^{-1}(\varepsilon)^{-1}) = \left(\frac{r^0\mu}{V_e\varepsilon}\right) \tag{16}$$

$$\pi_2 = ((a)^1(d)^{-1}) = \left(\frac{a}{d}\right) \tag{17}$$

$$\pi_3 = ((V)^1(V_e)^{-1}) = \left(\frac{V}{V_e}\right) \tag{18}$$

3.1.3 Gauss-Jordan elimination method

The procedure of Gauss-Jordan elimination is applied to the matrix equation (15) in a step-by-step fashion to obtain the dimensionless products of our corona discharge problem. We do not write the vector of indices as a column vector to the right of the dimensional matrix but write it as a row vector on top of it. This is a well-known trick used frequently [5,41-45] to enhance the readability of matrix multiplication. We also omit the equality sign in the matrix equation and add the zero vector in the R.H.S. of this equation as an extra vector for the dimensional matrix resulting in what is called an *augmented matrix* [5]. Now, elementary row operations [5,46] are applied to the whole rows of the augmented matrix and are depicted by the labels to the left of the matrix. We follow the following steps to obtain the dimensionless products.

Step 1: Finding a non-zero pivot in the first column in the first row, and making sure that this pivot is unity by keeping the first row (E1⁽¹⁾) intact:

	r^0	a	V	d	Ve	ε	μ	
E1 ⁽¹⁾	1	0	1	0	1	-1	-1	0
E2 ⁽¹⁾	-1	1	2	1	2	-3	0	0
E3 ⁽¹⁾	-1	0	-3	0	-3	4	2	0
E4 ⁽¹⁾	0	0	-1	0	-1	2	1	0

Step 2: Adding the first row to each of the second row and the third row and locating the pivot in the second column in the second row and making sure that this pivot is assigned a value of unity:

	r^0	a	V	d	Ve	ε	μ	
E1 ⁽²⁾ ← E1 ⁽¹⁾	1	0	1	0	1	-1	-1	0
E2 ⁽²⁾ ← E2 ⁽¹⁾ +E1 ⁽¹⁾	0	1	3	1	3	-4	-1	0
E3 ⁽²⁾ ← E3 ⁽¹⁾ +E1 ⁽¹⁾	0	0	-2	0	-2	3	1	0
E4 ⁽²⁾ ← E4 ⁽¹⁾	0	0	-1	0	-1	2	1	0

Step 3: Jumping to the third column (since the second column is already in the desired unit-vector form), and then locating a unit pivot in the third row and third column and swapping the third row and the fourth row, and then multiplying the new third row by -1:

	r^0	a	V	d	Ve	ε	μ	
E1 ⁽³⁾ ← E1 ⁽²⁾	1	0	1	0	1	-1	-1	0
E2 ⁽³⁾ ← E2 ⁽²⁾	0	1	3	1	3	-4	-1	0
E3 ⁽³⁾ ← -E4 ⁽²⁾	0	0	1	0	1	-2	-1	0
E4 ⁽³⁾ ← E3 ⁽²⁾	0	0	-2	0	-2	3	1	0

Step 4: Adding multiples of (-1), (-3), and (+2) of the third row to the first row, second row, and fourth row, respectively, and locating the next non-zero pivot in the fourth row(which is impossible for the fourth and fifth columns but possible for the sixth column, thereby changing the set of basis variables from r^0 , a , V , and d to r^0 , a , V , and ε , and changing the set of regime variables from Ve , ε , and μ to d , Ve , and μ).

	r^0	a	V	d	Ve	ε	μ	
$E1^{(4)} \leftarrow E1^{(3)} - E3^{(3)}$	1	0	0	0	0	1	0	0
$E2^{(4)} \leftarrow E2^{(3)} - 3 E3^{(3)}$	0	1	0	1	0	2	2	0
$E3^{(4)} \leftarrow E3^{(3)}$	0	0	1	0	1	-2	-1	0
$E4^{(4)} \leftarrow E4^{(3)} + 2 E3^{(3)}$	0	0	0	0	0	1	1	0

Step 5: Adding multiples of (-1), (-2), and (+2) of the third row to the first row, second row and fourth row, respectively

	r^0	a	V	d	Ve	ε	μ	
	b	e	g	f	h	k	m	
$E1^{(5)} \leftarrow E1^{(4)} - E4^{(4)}$	1	0	0	0	0	0	-1	0
$E2^{(5)} \leftarrow E2^{(4)} - 2 E4^{(4)}$	0	1	0	1	0	0	0	0
$E3^{(5)} \leftarrow E3^{(4)} + 2 E4^{(4)}$	0	0	1	0	1	0	1	0
$E4^{(5)} \leftarrow E4^{(4)}$	0	0	0	0	0	1	1	0

Therefore, we reproduce Equation (13) again herein for convenience

$$\pi = k r^{0b} a^e d^f V^g V_e^h \varepsilon^k \mu^m \tag{13}$$

$$b = m \tag{19a}$$

$$e = -f \tag{19b}$$

$$g = -h - m \tag{19c}$$

$$k = -m \tag{19d}$$

We can now substitute (19) in (13) to obtain regimes for d, Ve, and μ , namely

$$\pi = k r^{0m} a^{-f} d^f V^{-h-m} V_e^h \varepsilon^{-m} \mu^m \tag{13a}$$

$$= k (d/a)^f (V_e/V)^h (r^0 \mu / V \varepsilon)^m \tag{13b}$$

We obtain three independent regimes (which appear in line 3 of the forthcoming Table 3)

$$\pi_{a3} = (d/a), \quad \pi_{b3} = (V_e/V), \quad \pi_{c3} = (r^0 \mu / V \varepsilon). \tag{20}$$

The regimes in (14) differ from those obtained earlier in the previous subsections. If we insist on obtaining those earlier regimes, we could proceed as follows. From equation (19d), substitute $m = -k$ in equations (19a) and (19c) to obtain:

$$b + k = 0 \text{ or } k = -b; \quad e + f = 0 \text{ or } f = -e; \quad g + h - k = 0 \text{ or } h = k - g$$

Substituting the above equations into equation (13) gives:

$$\pi = k r^{0b} a^e d^{-e} V^g V_e^{(k-g)} \varepsilon^{-b} \mu^{-k}$$

$$\pi = k \left(\frac{r^0 \mu}{V_e \varepsilon} \right)^b \left(\frac{a}{d} \right)^e \left(\frac{V}{V_e} \right)^g$$

The three dimensionless products obtained are now those obtained earlier (and constitute line 1 in the forthcoming Table 3):

$$\pi_{a1} = \left(\frac{r^0\mu}{V_e\varepsilon}\right), \quad \pi_{b1} = \left(\frac{a}{d}\right), \quad \pi_{c1} = \left(\frac{V}{V_e}\right). \quad (21)$$

Each of the two sets of independent regimes in (20) and (21) can be directly obtained from the other. In fact

$$\pi_{a3} = 1/\pi_{b1}, \quad \pi_{b3} = 1/\pi_{c1}, \quad \pi_{c3} = \pi_{a1}/\pi_{c1}. \quad (22)$$

$$\pi_{a1} = \pi_{c3}/\pi_{b3}, \quad \pi_{b1} = 1/\pi_{a3}, \quad \pi_{c1} = 1/\pi_{b3}. \quad (23)$$

The Gauss-Jordan elimination method has been utilized to obtain the dimensionless products of the system while the process involved in the determination of dimensionless product using other techniques have been extensively shown in subsection 3.1.1 and 3.1.2 respectively.

π is given by

$$\pi = k \left(\frac{r^0\mu}{V_e\varepsilon}\right)^b \left(\frac{a}{d}\right)^e \left(\frac{V}{V_e}\right)^g$$

Rushdi and Rushdi [5] list several prominent advantages of the Gaussian or Gauss-Jordan elimination procedures. They point out that, in particular, these two variants of essentially the same procedure check for linear independence at no extra computational cost. In our present analysis, the Gauss-Jordan procedure does not produce an all-0 row, and hence it reveals that the dimensional matrix has a complete rank of 4, which implies that the equations in (15) are linearly independent. If the dimensional equations in (15) were linearly dependent, then an all-zero row would have been produced in the augmented matrix [46].

Each of the three independent dimensionless products can form an independent regime, which depends on the set of variables selected to form a basis (or equivalently, on the set of regimes (isolated) parameters. For instance, with our choice above of the four variables (d , V_e , ε , and μ) as input or basic variables, we are effectively making our first dimensionless product be identified by a single 'variable' r^0 , our second product is distinguished by a single 'variable' a with the third product being characterized by a single 'variable' V .

It is possible to deduce certain qualitative facts within each of the three regimes obtained [5,47]. The dependence of each of the regime variables r^0 , a , and V on the input variables belonging to the same regime is such that.

$$\frac{\partial r^0}{\partial \mu} < 0, \quad \frac{\partial r^0}{\partial V_e} > 0, \quad \frac{\partial r^0}{\partial \varepsilon} > 0 \quad (24)$$

$$\frac{\partial a}{\partial d} > 0 \quad (25)$$

$$\frac{\partial V}{\partial V_e} > 0 \quad (26)$$

The seven physical variables involved in this analysis are not necessarily 'variables,' and some of them (such as ε and μ) might be assumed to be almost constant. The partial differential relations in (24)-(26) in this case help in the assessment of uncertainty rather than feasible actual changes.

3.2 Regimes and Dimensionless Products

There are at most $\binom{7}{3} = \binom{7}{4} = 35$ selections for a set of four basis or input variables, associated with a complementary set of three isolated or output variables. We have already seen that not every such a selection corresponds to an actual triad of dimensional products. Table 3 considers all 35 candidate triads of dimensionless products, attempting to compute each of them from Equation (21), i.e., by using the products $\pi_{a1} = \left(\frac{r^0\mu}{V_e\varepsilon}\right)$, $\pi_{a2} = \left(\frac{a}{d}\right)$, and $\pi_{a3} = \left(\frac{V}{V_e}\right)$ as a starting point. According to Table 3, the attempt to compute triads of dimensionless products is successful in only 14 cases, whereas such triads do not exist in 21 cases of the 35 combinations explored in Table 3.

Out of the 42 dimensionless products obtained and presented in Table 3, there are just 8 distinct products. In retrospect, we note that with our knowledge of the three initial products $\pi_{a1} = \left(\frac{r^0\mu}{V_e\varepsilon}\right)$, $\pi_{a2} = \left(\frac{a}{d}\right)$, and $\pi_{a3} = \left(\frac{V}{V_e}\right)$, we can reduce the original problem to a problem involving five variables only, two lengths a and d , and three voltages V , V_e , and $V_2 = \frac{r^0\mu}{\varepsilon}$. The eight distinct

Table 3. Pairs of dimensionless products that form independent regimes

Case j	Set of basis (input) variables	Set of isolated (output) variables	π_{aj}	π_{bj}	π_{cj}
1	$\{d, V_e, \varepsilon, \mu\}$	$\{r^0, a, V\}$	$\pi_{a1} = \left(\frac{r^0 \mu}{V_e \varepsilon}\right)$	$\pi_{b1} = \left(\frac{a}{d}\right)$	$\pi_{c1} = \left(\frac{V}{V_e}\right)$
2	$\{\mu, r^0, a, V\}$	$\{d, V_e, \varepsilon\}$	$\pi_{a2} = \pi_{b1}^{-1}$ $= \left(\frac{d}{a}\right)$	$\pi_{b2} = \pi_{c1}^{-1}$ $= \left(\frac{V_e}{V}\right)$	$\pi_{c2} = \left(\frac{V \varepsilon}{r^0 \mu}\right)$
3	$\{\varepsilon, r^0, a, V\}$	$\{d, V_e, \mu\}$	$\pi_{a3} = \pi_{a2}$	$\pi_{b3} = \pi_{b2}$	$\pi_{c3} = \pi_{c2}^{-1}$ $= \left(\frac{r^0 \mu}{V \varepsilon}\right)$
4	$\{\varepsilon, \mu, a, V\}$	$\{r^0, d, V_e\}$	$\pi_{a4} = \pi_{c3}$	$\pi_{b4} = \pi_{a2}$	$\pi_{c4} = \pi_{b2}$
5	$\{\varepsilon, \mu, r^0, a\}$	$\{d, V, V_e\}$	$\pi_{a5} = \pi_{a2}$	$\pi_{b5} = \pi_{c2}$	$\pi_{c5} = \pi_{a1}^{-1}$ $= \left(\frac{V_e \varepsilon}{r^0 \mu}\right)$
6	$\{\varepsilon, \mu, r^0, V\}$	$\{a, d, V_e\}$	—	—	—
7	$\{V, \mu, d, a\}$	$\{r^0, V_e, \varepsilon\}$	—	—	—
8	$\{V, \mu, d, r^0\}$	$\{a, V_e, \varepsilon\}$	$\pi_{a8} = \pi_{b1}$	$\pi_{b8} = \pi_{b2}$	$\pi_{c8} = \pi_{c2}$
9	$\{d, \mu, a, r^0\}$	$\{V, V_e, \varepsilon\}$	—	—	—
10	$\{d, V, a, r^0\}$	$\{V_e, \varepsilon, \mu\}$	—	—	—
11	$\{a, d, V, V_e\}$	$\{r^0, \varepsilon, \mu\}$	—	—	—
12	$\{r^0, d, V, V_e\}$	$\{a, \varepsilon, \mu\}$	—	—	—
13	$\{r^0, a, V, V_e\}$	$\{d, \varepsilon, \mu\}$	—	—	—
14	$\{r^0, a, d, V_e\}$	$\{V, \varepsilon, \mu\}$	—	—	—
15	$\{r^0, \varepsilon, d, V_e\}$	$\{V, a, \mu\}$	$\pi_{a15} = \pi_{c1}$	$\pi_{b15} = \pi_{b1}$	$\pi_{c15} = \pi_{a1}$
16	$\{a, V, V_e, \mu\}$	$\{r^0, d, \varepsilon\}$	—	—	—
17	$\{r^0, V, V_e, \mu\}$	$\{a, d, \varepsilon\}$	—	—	—
18	$\{r^0, a, V_e, \mu\}$	$\{d, V, \varepsilon\}$	$\pi_{a18} = \pi_{a2}$	$\pi_{b18} = \pi_{c1}$	$\pi_{c18} = \pi_{c5}$
19	$\{a, V, V_e, \varepsilon\}$	$\{r^0, d, \mu\}$	—	—	—
20	$\{r^0, V, V_e, \varepsilon\}$	$\{a, d, \mu\}$	—	—	—
21	$\{r^0, a, V_e, \varepsilon\}$	$\{d, V, \mu\}$	$\pi_{a21} = \pi_{a2}$	$\pi_{b21} = \pi_{c1}$	$\pi_{c21} = \pi_{a1}$
22	$\{a, d, V, \varepsilon\}$	$\{r^0, V_e, \mu\}$	—	—	—
23	$\{r^0, d, V, \varepsilon\}$	$\{a, V_e, \mu\}$	$\pi_{a23} = \pi_{b1}$	$\pi_{b23} = \pi_{b2}$	$\pi_{c23} = \pi_{c3}$
24	$\{r^0, a, d, \varepsilon\}$	$\{V, V_e, \mu\}$	—	—	—
25	$\{V, V_e, \varepsilon, \mu\}$	$\{r^0, a, d\}$	—	—	—
26	$\{V, d, \varepsilon, \mu\}$	$\{r^0, a, V_e\}$	$\pi_{a26} = \pi_{c3}$	$\pi_{b26} = \pi_{b1}$	$\pi_{c26} = \pi_{b2}$
27	$\{d, V, V_e, \mu\}$	$\{r^0, a, \varepsilon\}$	—	—	—
28	$\{d, V, V_e, \varepsilon\}$	$\{r^0, a, \mu\}$	—	—	—
29	$\{a, V_e, \varepsilon, \mu\}$	$\{r^0, d, V\}$	$\pi_{a29} = \pi_{a1}$	$\pi_{b29} = \pi_{a2}$	$\pi_{c29} = \pi_{c1}$
30	$\{a, d, \varepsilon, \mu\}$	$\{r^0, V, V_e\}$	—	—	—
31	$\{a, d, V_e, \mu\}$	$\{r^0, V, \varepsilon\}$	—	—	—
32	$\{a, d, V_e, \varepsilon\}$	$\{r^0, V, \mu\}$	—	—	—
33	$\{r^0, V_e, \varepsilon, \mu\}$	$\{a, d, V\}$	—	—	—
34	$\{r^0, d, \varepsilon, \mu\}$	$\{a, V, V_e\}$	$\pi_{a34} = \pi_{b1}$	$\pi_{b34} = \pi_{c2}$	$\pi_{c34} = \pi_{c5}$
35	$\{r^0, d, V_e, \mu\}$	$\{a, V, \varepsilon\}$	$\pi_{a35} = \pi_{b1}$	$\pi_{b35} = \pi_{c1}$	$\pi_{c35} = \pi_{c5}$

products are the four ratios (a/d) , (V/V_e) , (V/V_2) , (V_2/V_e) , and their reciprocals. Therefore, four out of these eight distinct products involve five variables and have three missing variables each, while the remaining four products involve two variables only and have five missing variables

each. Moreover, each of the 8 distinct products is seen to appear in different places in Table 3. Also, there are 4 lines (1, 4, 26, and 29) in Table 3, in which the ozone generation rate per unit length of wire in dry air (r^0) appears as a regime or output variable. Based on this, the next logical

step is to derive four values of r^0 i.e. r^0_1, r^0_2, r^0_3 and r^0_4 , which might be obtained by writing the first regime in each of the aforementioned 4 lines as an arbitrary function of the other two regimes in the same line, namely

$$\pi_{a1} = f_1(\pi_{b1}, \pi_{c1}) \quad (27a)$$

$$\pi_{a4} = f_4(\pi_{b4}, \pi_{c4}) \quad (27b)$$

$$\pi_{a26} = f_{26}(\pi_{b26}, \pi_{c26}) \quad (27c)$$

$$\pi_{a29} = f_{29}(\pi_{b29}, \pi_{c29}) \quad (27d)$$

These relations can yield the following four expressions for r^0_1, r^0_4, r^0_{26} , and r^0_{29}

$$r^0_1 = \frac{V_e \epsilon}{\mu} f_1\left(\left(\frac{a}{d}\right), \left(\frac{V}{V_e}\right)\right) \quad (28a)$$

$$r^0_4 = \frac{V \epsilon}{\mu} f_4\left(\left(\frac{a}{d}\right), \left(\frac{V_e}{V}\right)\right) \quad (28b)$$

$$r^0_{26} = \frac{V \epsilon}{\mu} f_{26}\left(\left(\frac{a}{d}\right), \left(\frac{V_e}{V}\right)\right) \quad (28c)$$

$$r^0_{29} = \frac{V_e \epsilon}{\mu} f_{29}\left(\left(\frac{a}{d}\right), \left(\frac{V}{V_e}\right)\right) \quad (28d)$$

The values obtained for r^0 are just two rather than four. It is simply a product of $\left(\frac{V_e \epsilon}{\mu}\right)$ or $\left(\frac{V \epsilon}{\mu}\right)$ by an arbitrary function of the length ratio $\left(\frac{a}{d}\right)$ and voltage ratio $\left(\frac{V}{V_e}\right)$.

4. CONCLUSIONS

Simplified calculation models to analyze corona discharge problems via DA are presented in this paper. The application of three different computational approaches via DA generated an infinite number of dimensionless products that relate the ozone generation rate per unit length of wire in dry air (r^0) and six other parameters. It is possible to construct any of these dimensionless products using only any three independent products among them. Specifically, four dimensionless products only act as regimes for the ozone generation rate per unit length of wire in dry air (r^0), i.e., as a way for explicitly expressing r^0 in terms of its influencing variables. Eventually, the DA formula derived for r^0 become two rather than four. The correlation equation as a tool for predicting the ozone generation by negative wire-to-plate corona discharges in both dry and humid air has been derived mathematically by other researchers and validated with previously developed numerical

framework as well as experimental observations. Therefore, DA is only utilized in this article to find the dimensionless products that form independent regimes as well as variables derivable from those regimes. As a sequel of this work, we hope to explore DA utilization in the techno-economic assessment of power systems [48,49].

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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