Scientific Research Publishing

Energy and Power Engineering, 2024, 16, 1-20
https://www.scirp.org/journal/epe
ISSN Online: 1947-3818
ISSN Print: 1949-243X

Estimation of Daily Global Solar Radiation with Different Sunshine-Based Models for Some Burundian Stations

Mathias Bashahu[1]*, Gratien Ndacayisaba[2]

[1]Department of Physics and Technology, University of Burundi, Bujumbura, Burundi
[2]Department of Physics, Ecole Normale Supérieure, Bujumbura, Burundi
Email: *bashahuma@yahoo.fr

How to cite this paper: Bashahu, M. and Ndacayisaba, G. (2024) Estimation of Daily Global Solar Radiation with Different Sunshine-Based Models for Some Burundian Stations. *Energy and Power Engineering*, **16**, 1-20.
https://doi.org/10.4236/epe.2024.161001

Received: November 10, 2023
Accepted: January 13, 2024
Published: January 16, 2024

Abstract

Sunshine duration (*S*) based empirical equations have been employed in this study to estimate the daily global solar radiation on a horizontal surface (*G*) for six meteorological stations in Burundi. Those equations include the Ångström-Prescott linear model and four amongst its derivatives, *i.e.* logarithmic, exponential, power and quadratic functions. Monthly mean values of daily global solar radiation and sunshine duration data for a period of 20 to 23 years, from the Geographical Institute of Burundi (IGEBU), have been used. For any of the six stations, ten single or double linear regressions have been developed from the above-said five functions, to relate in terms of monthly mean values, the daily clearness index ($y=\bar{k}_t=\frac{\bar{G}}{\bar{G}_0}$) to each of the next two kinds of relative sunshine duration (RSD): $x=\bar{s}_r=\frac{\bar{S}}{\bar{S}_0}$ and $x'=\bar{s}'_r=\frac{\bar{S}}{\bar{S}'_0}$. In those ratios, G_0, S_0 and S'_0 stand for the extraterrestrial daily solar radiation on a horizontal surface, the day length and the modified day length taking into account the natural site's horizon, respectively. According to the calculated mean values of the clearness index and the RSD, each station experiences a high number of fairly clear (or partially cloudy) days. Estimated values of the dependent variable (*y*) in each developed linear regression, have been compared to measured values in terms of the coefficients of correlation (*R*) and of determination (R^2), the mean bias error (*MBE*), the root mean square error (*RMSE*) and the *t*-statistics. Mean values of these statistical indicators have been used to rank, according to decreasing performance level, firstly the ten developed equations per station on account of the overall six stations, secondly the six stations on account of the overall ten equations. Nevertheless, the obtained values of those indicators lay in the next ranges for all the developed sixty equations: $R \in [0.8751;0.9494]$; $R^2 \in [0.7657;0.9013]$; $MBE \in [-0.000817;+0.001700]$; $RMSE \in [0.011872;0.113109]$; $t \in [0.000000;0.130589]$, with $t_c = t(n-1=11;\gamma=99.5\%) = 3.106$. These results lead to assert that any of the sixty developed linear regressions (and thus equations $\overline{k}_t$ in terms of $\overline{s}_r$ and $\overline{s}'_r$), fits very adequately measured data, and should be used to estimate monthly average daily global solar radiation with sunshine duration for the relevant station. It is also found that using $\overline{s}'_r$ as RSD, is slightly more advantageous than using $\overline{s}_r$ for estimating the monthly average daily clearness index, $\overline{k}_t$. Moreover, values of statistical indicators of this study match adequately data from other works on the same kinds of empirical equations.

Keywords

Clearness Index, Two Kinds of Relative Sunshine Duration, Ångström-Prescott Linear Model and Four Derivatives, Statistical Tests, Six Burundian Stations

1. Introduction

In many countries, electricity production is inadequate considering the increasing energy demand due to industrialization effort and to high rate of population growth. As an example, that production in Burundi originates for 98.2% from hydroelectric plants and for 1.8% from imported fossil fuels [1]. As a result, power outages are frequently observed daily in different urban areas. In order to increase electricity production and alleviate the constantly scaling cost and environmental concern of fossil fuels, attempts have been (and are currently still) made to supplement traditional energy sources with free, clean and inexhaustible energy sources. Solar energy is one of these last sources. However, the design and the performance assessment of solar energy conversion systems (SECS) at a specific site, require reliable data of the solar radiation components, especially of the global solar radiation (GSR) on a horizontal surface. These data are generally important for science and engineering applications in various sectors, e.g. in meteorology, energy, agriculture, transportation, communications, health, architecture, tourism and leisure. In many geographical locations, GSR is not measured since the required apparatuses are too expensive to purchase and to maintain. Even if measured data are available, they are not always complete due particularly to equipment failures. Accordingly, researchers have developed empirical models which are able to estimate (or to predict) GSR for a specific location. Many of these models require the input of meteorological variables for which data are more accessible than GSR data. Any of such models is classified according to the input variable (or set of variables) which it requires. The most common category includes relative sunshine duration (RSD) based models. Other categories require the next sets of variables as examples: relative sky cloud cover [2] [3] [4]; RSD and atmospheric water vapor content [5]; air temperature [6] [7] [8]; RSD and air temperature [9] [10] [11] [12]; RSD, normalized air temperature and relative humidity [13] [14]; RSD, air temperature and precipitation [15]; RSD, air temperature, precipitation and relative sky cloud cover [16].

Within the RSD-based models, the oldest is the Ångström linear model [17]. Ångström type models, e.g. in [2] [4] [18], relate the ratio $\frac{G}{G_c}$ to the next kind of RSD: $s_r = \frac{S}{S_0}$, where the quantities G, G_c, S, S_0 are the daily actual global solar radiation, the global solar radiation in very clear sky conditions, the actual sunshine duration and the day length, respectively on a horizontal surface at a specific location. Later on, Prescott [19] has replaced in the Ångström model the ratio $\frac{G}{G_c}$ by the daily clearness index $k_t = \frac{G}{G_0}$, where the daily extraterrestrial solar radiation on a horizontal surface, G_0 is easier to handle than G_c. Ångström-Prescott type empirical equations have been developed for a huge number of locations all around the world, e.g. in [20]-[25]. Both Ångström and Ångström-Prescott linear models are also commonly used, together with their derivatives such as logarithmic, power, exponential, quadratic, cubic functions and higher degree polynomials (or some of their combinations), e.g. in [26]-[39]. Some extensive review papers on such sub-categories of models are proposed in literature, e.g. in [35] [38].

Records of several meteorological variables have been performed for long periods at different stations in Burundi. Data of those variables are accessible within the Geographical Institute of Burundi (IGEBU) in terms of monthly means of daily values, and it is noticed that sunshine duration (SD) data are more available than GSR ones. As a continuation of previous studies on solar radiation estimation for Burundian locations, e.g. in [5] [40] [41] [42] [43], the present work aims mainly to develop, in terms of monthly mean values, RSD-based Ångström-Prescott linear regressions and modified functions, *i.e.* logarithmic, exponential, power and quadratic empirical equations, to predict GSR on a horizontal surface (through the clearness index, $\bar{k}_t$) for selected stations in Burundi. Two sets of empirical equations are implemented for each station: in the first one, $\bar{k}_t$ depends on $\bar{s}_r = \frac{\bar{S}}{\bar{S}_0}$ as a first kind of RSD; in the second set, $\bar{k}_t$ is a function of $\bar{s}'_r = \frac{\bar{S}}{\bar{S}'_0}$ as a second kind of RSD. The performance of the developed equations is assessed and compared using common statistical test methods, and the results are discussed.

2. Materials and Methodology

2.1. Selected Stations and Basic Data

Monthly mean values of daily SD ($\bar{S}$) and GSR on a horizontal surface ($\bar{G}$)

used as basic data in this study have been collected within the Geographical Institute of Burundi. They refer to the six stations and the periods shown in Table 1, where the elevation is the station's height above sea level. Figure 1 indicates the position of those stations on a map of Burundi.

Two criteria have guided the selection of those stations and periods: firstly to match long-period for simultaneous continuous records of the two variables (SD and GSR), secondly to represent different country's climatic regions. Monthly variations of those variables are shown in Table 2 and Table 3 for the six stations and in Figure 2 for one station. For any of the six stations, higher SD mean values are observed during the dry period from June to August. Moreover, according to decreasing annual mean values of the SD and GSR, the six stations rank as following: Musasa, Bujumbura-Airport, Gitega-Zege, Imbo-SEMS, Ruvyironza, Rwegura.

Table 1. Geographical locations and observation periods of $\overline{S}$ and $\overline{G}$.

Station name	Latitude (˚*S*)	Longitude (˚*E*)	Elevation (*m*)	Observation period	Number of years
Bujumbura-Airport	3˚19'	29˚19'	783	1968-1988	21
Gitega-Zege	3˚24'	29˚55'	1663	1968-1989	22
Imbo-SEMS	3˚11'	29˚21'	820	1968-1987	20
Musasa	4˚00'	30˚06'	1260	1968-1987	20
Ruvyironza	3˚49'	29˚46'	1822	1968-1988	21
Rwegura	**2˚55'**	**29˚31'**	2120	1968-1990	23

Table 2. Monthly mean values of sunshine duration ($\overline{S}$ in hours) for the six stations.

Month↓ Station→	Bujumbura-Airport	Gitega-Zege	Imbo-SEMS	Musasa	Ruvyironza	Rwegura
January	5.323	5.010	4.806	5.310	4.458	4.429
February	6.150	5.164	5.354	6.257	4.700	5.514
March	5.848	5.103	5.471	5.942	4.923	4.671
April	6.193	4.983	5.580	5.953	4.840	3.740
May	7.355	6.277	6.474	6.932	6.061	4.300
June	8.960	8.053	7.817	9.000	7.717	6.733
July	8.897	9.242	7.877	9.465	8.016	7.816
August	8.523	8.306	7.684	9.174	7.294	7.065
September	7.204	6.233	6.633	7.940	6.240	6.043
October	6.603	6.016	6.139	6.274	5.677	5.590
November	5.533	4.817	5.010	5.950	4.283	4.220
December	5.910	5.235	4.977	5.887	4.216	4.977
Annual mean	**6.881**	**6.214**	**6.154**	**7.009**	**5.711**	**5.425**

Table 3. Monthly mean values of global solar radiation ($\overline{G}$ in J·cm^{-2}·day^{-1}) for the six stations.

Month ↓ Station→	Bujumbura-Airport	Gitega-Zege	Imbo-SEMS	Musasa	Ruvyironza	Rwegura
January	1709.62	1504.80	1560.93	1738.88	1345.96	1467.18
February	1759.78	1592.58	1588.40	1751.42	1463.00	1442.10
March	1876.64	1651.10	1731.12	1918.62	1592.58	1421.20
April	1830.84	1550.78	1677.97	1826.66	1500.62	1262.36
May	1835.02	1554.96	1726.94	1885.18	1580.04	1270.72
June	1818.30	1663.64	1707.83	1943.70	1642.74	1471.36
July	1818.30	1717.98	1695.69	1897.72	1630.20	1559.14
August	1860.10	1617.66	1700.07	1943.70	1634.38	1609.30
September	1872.64	1634.38	1672.00	1931.16	1613.48	1546.60
October	1851.74	1538.24	1681.55	1830.84	1513.16	1492.26
November	1784.86	1500.62	1585.41	1726.34	1383.58	1358.50
December	1793.22	1588.40	1634.38	1776.50	1429.56	1429.56
Annual mean	**1793.43**	**1549.39**	**1663.53**	1847.56	**1512.81**	1444.19

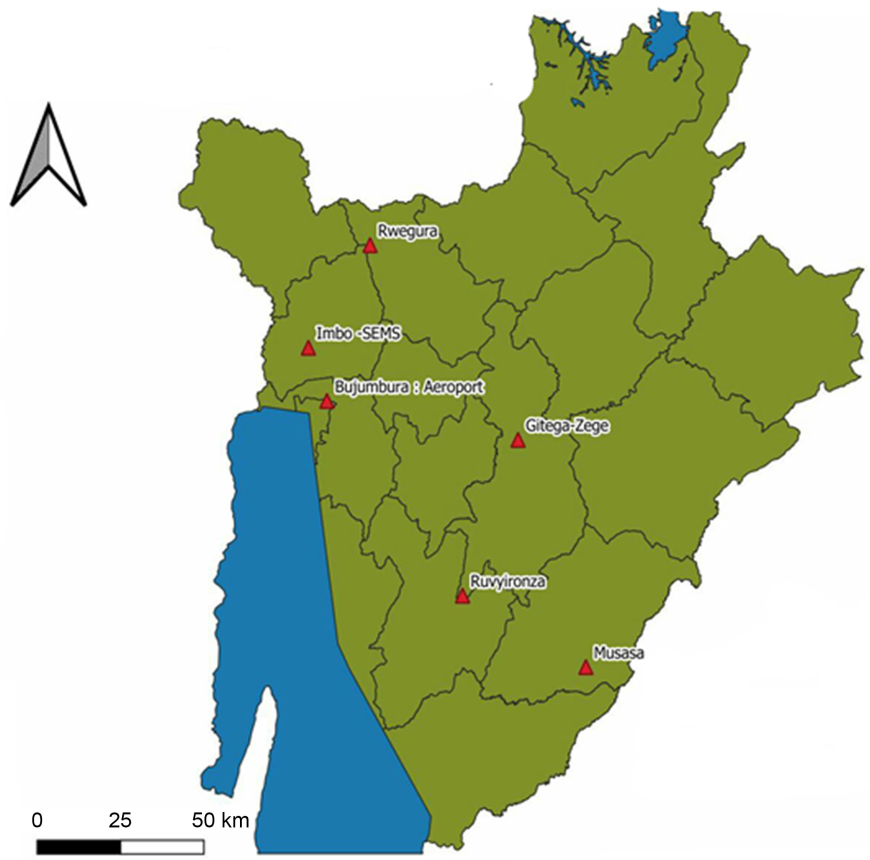

Figure 1. Geographical position of the six stations of this study.

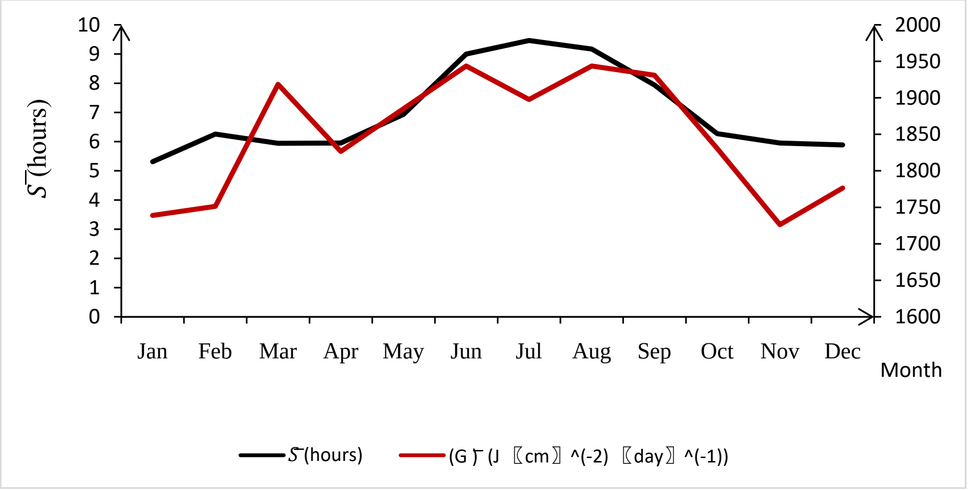

Figure 2. Monthly variation of average daily SD and GSR for one station, e.g.: Musasa.

2.2. Empirical Correlations Development and Evaluation Methods

The ten Ångström-Prescott type linear empirical model and modified functions to be developed for any of the six locations of this study are shown in **Table 4**, where the independent variable is one of the next two kinds of monthly mean daily RSD: $x = \bar{s}_r = \frac{\bar{S}}{\bar{S}_0}$ and $x' = \bar{s}'_r = \frac{\bar{S}}{\bar{S}'_0}$, while the dependent variable is the monthly mean daily clearness index, $y = \bar{k}_t = \frac{\bar{G}}{\bar{G}_0}$.

For each station and day of the year, the day length (S_0), the modified day length (S'_0) taking into account the natural horizon of the site, and the extraterrestrial daily solar radiation on a horizontal surface (G_0), have been computed by using relationships from the next theoretical background. With the convention of the solar hour angle w_s equal to zero at noon, negative in the morning (AM) and positive in the afternoon (PM), w_s (in degrees) is related to the solar time t (in hours) as follows:

$$w_s = 15(t - 12) \tag{6.1}$$

At sunrise and sunset (subscripts "*sr*" and "*ss*", respectively), the expressions of w_s are:

$$w_{sr/ss} = \mp \cos^{-1}\left[-\text{tg}\,\varphi\,\text{tg}\,\delta\right] \tag{6.2}$$

In Equation (6.2), φ is the latitude of the site (negative in the southern hemisphere) and δ is the solar declination, for which expressions and table of values in terms of the order number (J = 1 to 365) of the day of the year are given in the literature [3] [44]. Therefore, as difference between sunset and sunrise solar times, the day length (on a horizontal surface) is expressed as:

$$S_0 = t_{ss} - t_{sr} = \frac{2}{15} w_{ss} \tag{6.3}$$

At its turn, the above mentioned modified day length is given by [2] [4]:

Table 4. Sets of empirical correlations to be developed in this paper.

Model n°	Model type	Correlation equation			
		Set 1		Set 2	
1	Linear	$y = ax + b$	(1.1)	$y = a'x' + b'$	(1.2)
2	Logarithmic	$y = a\ln x + b$	(2.1)	$y = a'\ln x' + b'$	(2.2)
3	Exponential	$y = be^{ax}$	(3.1)	$y = b'e^{a'x'}$	(3.2)
4	Power	$y = b(x)^{a}$	(4.1)	$y = b'(x')^{a'}$	(4.2)
5	Quadratic	$y = a + bx + cx^2$	**(5.1)**	$y = a' + b'x' + c'x'^2$	(5.2)

$$S_0' = \frac{2}{15}\cos^{-1}\left[\frac{\cos 85^\circ - \sin\varphi\sin\delta}{\cos\varphi\cos\delta}\right] \tag{6.4}$$

The extraterrestrial daily solar radiation on a horizontal surface is expressed (in $J \cdot m^{-2} \cdot day^{-1}$) as follows:

$$G_0 = \frac{24}{\pi}E_0\left[\dot{I}_{sc}\right](3600)\left[\cos\delta\cos\varphi\sin w_{ss} + \frac{\pi}{180}w_{ss}\sin\delta\sin\varphi\right] \tag{6.5}$$

where the solar constant $\dot{I}_{sc}$ is equal to 1367 $W \cdot m^{-2}$, w_{ss} is expressed in degrees, and E_0 is the eccentricity correction factor of the earth's orbit, for which expressions and table of values in terms of *J* are also available in the literature [3] [44]. As example of results of the previous computation procedure, variations of S_0, S_0' and G_0 in terms of *J* are shown in **Figures 3(a)-(c)** for one of the six stations.

From the computed values of the day length, modified day length and extraterrestrial daily solar radiation on a horizontal surface, monthly mean values $\bar{S}_0, \bar{S}_0'$ and $\bar{G}_0$ have been gathered for each station. Then, a set of twelve monthly values of each variable within the next couples: ($x = \bar{s}_r = \frac{\bar{S}}{\bar{S}_0}$; $y = \bar{k}_t = \frac{\bar{G}}{\bar{G}_0}$); ($x' = \bar{s}_r' = \frac{\bar{S}}{\bar{S}_0'}$; *y*), has been made up for each station. Values of the three variables *x*, *x*' and *y* for the six stations are summarized in **Tables 5-7**. Their monthly variations are shown in **Figure 4** for one station as an example.

Data from the last three tables indicate firstly that higher mean values of RSD and clearness index for the six stations are observed during the dry period, *i.e.* either from June to August, or from May to September. Secondly, as already stated in section 2.1 for $\bar{S}$ and $\bar{G}$, the six stations rank as follows according to decreasing annual mean values of $\bar{s}_r, \bar{s}_r'$ and $\bar{k}_t$: Musasa, Bujumbura-Airport, Gitega-Zege, Imbo-SEMS, Ruvyironza, Rwegura. Thirdly, according to some classifications of sky categories based on clearness index and RSD values, e.g. in [24] [35] [45], the observed annual means of RSD and clearness index, *i.e.* $\bar{s}_r \in [0.453; 0.586]$; $\bar{s}_r' \in [0.481; 0.622]$; $\bar{k}_t \in [0.40; 0.52]$, indicate that for most of the days of the year, sky over any of the six stations is partially cloudy (or fairly clear) with predominance of diffuse radiation.

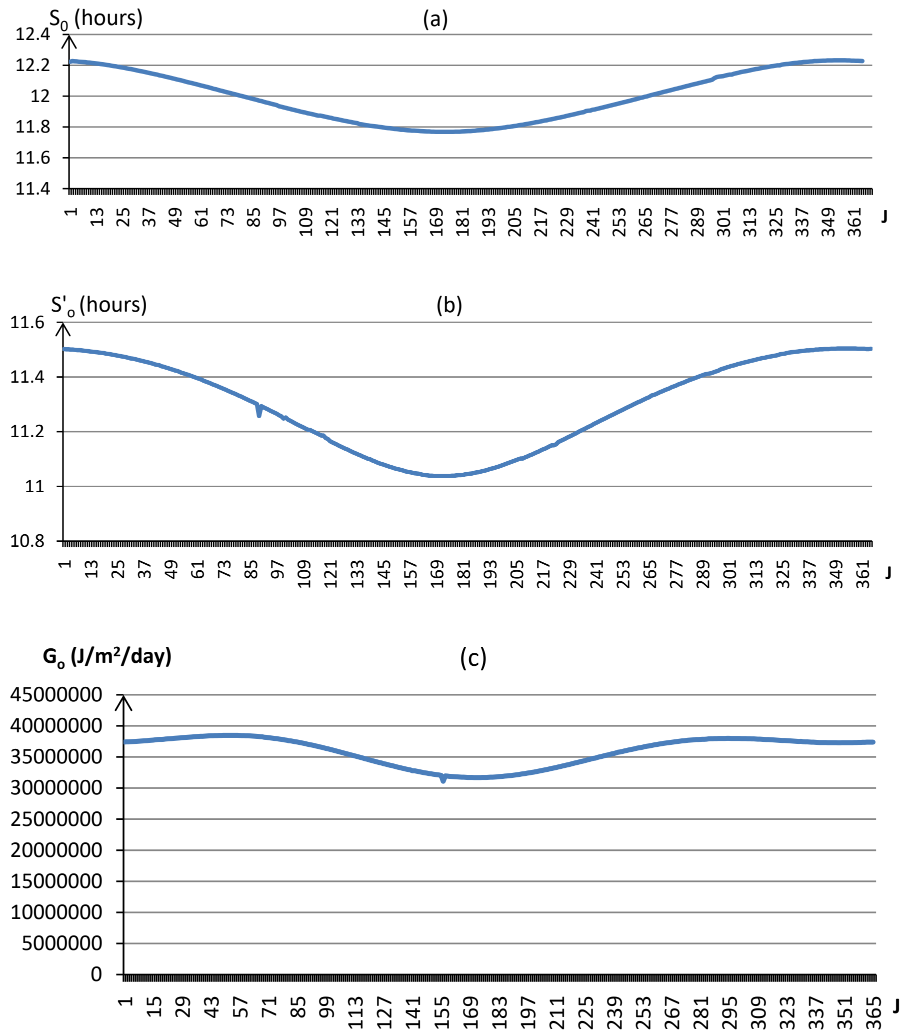

Figure 3. Variations of S_0, S_0' and G_0 with the day of the year for one station, e.g.: Musasa; (a) day length, S_0 vs J; (b) modified day length, S_0' vs J; (c) extraterrestrial daily solar radiation on a horizontal surface, G_0 vs J.

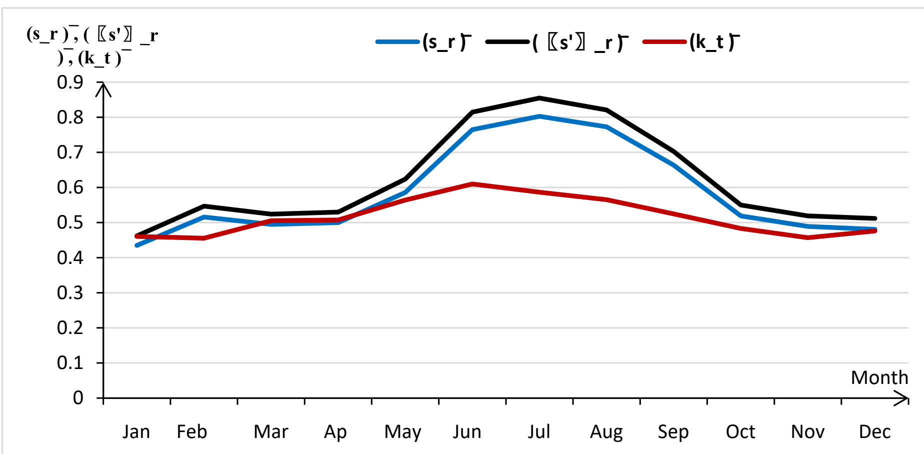

Figure 4. Monthly variation of average measured daily RSD and clearness index for one station, e.g.: Musasa.

Table 5. Input monthly mean relative sunshine duration ($\overline{s}_r$) data for the six stations.

Month ↓ Station →	Bujumbura-Airport	Gitega-Zege	Imbo-SEMS	Musasa	Ruvyironza	Rwegura
January	0.437	0.412	0.395	0.435	0.366	0.365
February	0.508	0.427	0.442	0.516	0.388	0.456
March	0.487	0.425	0.455	0.495	0.410	0.389
April	0.519	0.418	0.468	0.500	0.406	0.313
May	0.621	0.530	0.546	0.586	0.513	0.362
June	0.759	0.682	0.661	0.765	0.655	0.569
July	0.752	0.782	0.666	0.803	0.679	0.660
August	0.711	0.699	0.647	0.773	0.614	0.593
September	0.602	0.520	0.554	0.664	0.521	0.504
October	0.547	0.498	0.509	0.519	0.470	0.464
November	0.456	0.387	0.399	0.489	0.352	0.348
December	0.485	0.429	0.409	0.481	0.345	0.409
Annual mean	**0.573**	**0.518**	**0.513**	0.586	**0.476**	0.453

Table 6. Input monthly mean relative sunshine duration ($\overline{s}'_r$) data for the six stations.

Month ↓ Station →	Bujumbura-Airport	Gitega-Zege	Imbo-SEMS	Musasa	Ruvyironza	Rwegura
January	0.465	0.437	0.420	0.462	0.388	0.387
February	0.539	0.452	0.469	0.547	0.411	0.484
March	0.515	0.450	0.482	0.524	0.434	0.412
April	0.551	0.443	0.496	0.530	0.431	0.332
May	0.660	0.564	0.581	0.624	0.545	0.385
June	0.809	0.727	0.705	0.815	0.698	0.606
July	0.801	0.832	0.709	0.855	0.723	0.702
August	0.761	0.742	0.686	0.821	0.652	0.630
September	0.637	0.551	0.585	0.703	0.552	0.534
October	0.580	0.528	0.539	0.550	0.498	0.491
November	0.484	0.421	0.438	0.519	0.374	0.369
December	0.516	0.456	0.434	0.512	0.367	0.435
Annual Mean	**0.610**	**0.550**	**0.545**	0.622	**0.506**	0.481

The next step has been to write any of the ten equations of Table 4 in the form of a (simple or double) linear regression. Then, the use of data from Tables 5-7, together with least square method-based common relationships from linear regression analysis, has allowed the computation of relevant regression coefficients, correlation coefficient (R) and coefficient of determination (R^2) for any of the ten regressions per station. Besides these two last coefficients, the performance of each obtained empirical regression has been assessed through the computation of three other complementary statistical indicators expressed as follows [2] [5] [46] [47].

Table 7. Input monthly mean daily clearness index ($\bar{k}_t$) data for the six stations.

Month ↓ Station →	Bujumbura-Airport	Gitega-Zege	Imbo-SEMS	Musasa	Ruvyironza	Rwegura
January	0.4561	0.4008	0.4166	0.4606	0.3571	0.3912
February	0.4600	0.4161	0.4155	0.4556	0.3814	0.3774
March	0.4924	0.4350	0.4560	0.5055	0.4192	0.3744
April	0.5068	0.4293	0.4643	0.5077	0.4174	0.3485
May	0.5453	0.4625	0.5120	0.5643	0.4720	0.3760
June	0.5671	0.5180	0.5316	0.6101	0.5141	0.4553
July	0.5434	0.5268	0.5205	0.5866	0.5021	0.4613
August	0.5509	0.4645	0.4902	0.5652	0.4728	0.4620
September	0.5572	0.4439	0.4539	0.5248	0.4385	0.4196
October	0.4891	0.4068	0.4443	0.4833	0.3997	0.3942
November	0.4758	0.3998	0.4231	0.4571	0.3671	0.3632
December	0.4837	0.4281	0.4421	0.4762	0.3836	0.3878
Annual mean	**0.5107**	**0.4443**	**0.4642**	**0.5264**	**0.4271**	**0.4009**

1) the mean bias error:

$$MBE = \frac{1}{n}\sum_{i=1}^{n} d_i \quad (7.1)$$

$$d_i = y_{i,calc} - y_{i,meas} \quad (7.2)$$

2) the root mean square error:

$$RMSE = \left[\frac{1}{n}\sum_{i=1}^{n} d_i^2\right]^{1/2} \quad (7.3)$$

3) the t-statistics:

$$t = \left[\frac{(n-1)(MBE)^2}{(RMSE)^2 - (MBE)^2}\right]^{1/2} \quad (7.4)$$

where n is the total number of values of the independent and the dependent variables in a given set (x, y) or (x', y), *i.e.* $n = 12$ here, while d_i is the difference between the i-th calculated value ($y_{i,calc}$) of the dependent variable y and its measured (or experimental) counterpart ($y_{i,meas}$).

R ranges from 0 to 1 for a positive correlation and from −1 to 0 for a negative one. R^2 is the proportion (from 0 to 1) of the variance in the dependent variable that is predictable from the independent variable. High values of these two coefficients are desired.

The MBE test provides information on the long-term performance of a correlation. A low MBE is desired. A positive value stands for average amount of overestimation in the calculated value, while a negative value stands for average amount of underestimation in the calculated value. Nevertheless, overestimation of an individual observation can cancel underestimation in a separate observation.

The *RMSE* test provides information on the short-term performance of a correlation. It allows a term by term comparison of the actual deviation between the calculated and the measured value. Small values of *RMSE* are also desired. However, a few large error in the sum can produce a significant increase in *RMSE*.

The simultaneous juncture of a large *RMSE* and a small *MBE* stands for a large scatter about the line of perfect estimation. At its turn, the simultaneous occurrence of a small *RMSE* and a large *MBE* means a consistently over or underestimation. Thus, although these two tests provide a reasonable procedure to compare models, they don't objectively indicate whether model's estimates are significantly different from their measured counterparts. That is why an additional indicator, the t-statistics (or the Student's statistical variable), is generally required. This variable indicates whether or not the model's estimates are statistically significant at a particular confidence level γ (or level of significance $\alpha = 1-\gamma$). For the model's estimates to be judged statistically significant at a confidence level γ, one has simply to read from standard statistical tables, e.g. in [48] [49], the critical *t* value $\left(t_c = t\left(n-1,\alpha\right)\right)$ at α level of significance (or confidence level γ) and *n*-1 degrees of freedom. The null hypothesis is accepted when the next relation is satisfied:

$$t < t_c \tag{7.5}$$

Smaller values of t indicate better model's performance. In the present study, $\alpha = 0.005$, $\gamma = 99.5\%$, $n-1 = 11$, and thus $t_c = 3.106$.

3. Results

For the stations and periods of this study, **Tables 8(a)-(f)** exhibit the obtained empirical correlations and values of the considered statistical indicators.

4. Discussion

Results from **Tables 8(a)-(f)** show that for any of the ten developed correlations per station, the coefficients of correlation and of determination are high ($R \in \left[0.8751; 0.9494\right]$; $R^2 \in \left[0.7657; 0.9013\right]$), the mean bias error is either close or equal to zero ($MBE \in \left[-0.000558; +0.001700\right]$), the root mean square error is small compared to $y_{i,meas}$ ($RMSE \in \left[0.011872; 0.113109\right]$) and the t-statistics is far lower than its critical value ($t \in \left[0.000000; 0.130589\right]$). These features lead to assert that, in terms of the Ångström-Prescott type linear model and its derivatives relating monthly average daily clearness index to RSD, all the proposed sixty equations fit adequately relevant measured data. They should be therefore used to estimate the monthly average daily GSR on a horizontal surface in the corresponding sites.

Table 8. Summary of resulting empirical correlations $y = y(x)$ and $y = y(x')$ (where $x = \overline{s}_r$; $x' = \overline{s}'_r$; $y = \overline{k}_t$) and values of computed statistical indicators for the stations and periods of this study.

(a) Bujumbura-Airport

Eq. n˚ and correlation type	Equation	*R*	*R*²	*MBE*	*RMSE*	*t*
1.1. Linear	$y = 0.3086x + 0.3336$	0.8841	0.7816	−0.000008	0.017873	0.001546
1.2. "	$y = 0.2862x' + 0.3361$	0.8812	0.7765	−0.000017	0.018076	0.003058
2.1. Logarithmic	$y = 0.1855\ln x + 0.6170$	0.9008	0.8114	0.000008	0.016884	0.001637
2.2. "	$y = 0.1831\ln x' + 0.6045$	0.8948	0.8007	0.000033	0.017069	0.006470
3.1. Exponential	$y = 0.3603e^{0.6020x}$	0.8805	0.7752	−0.000192	0.035512	0.017901
3.2. "	$y = 0.3623e^{0.5582x'}$	0.8775	0.7700	−0.000108	0.035915	0.010004
4.1. Power	$y = 0.6267x^{0.3623}$	0.8981	0.8066	0.000083	0.033112	0.008347
4.2. "	$y = 0.6116x'^{0.3575}$	0.8919	0.7956	0.000008	0.033864	0.000816
5.1. Quadratic	$y = -0.0061 + 1.4729x - 0.9634x^2$	0.9133	0.8342	−0.000425	0.015610	0.090334
5.2. "	$y = -0.0135 + 1.4130x' - 0.8755x'^2$	0.9122	0.8321	0.000183	0.015685	0.038769
Mean value of statistical indicator		**0.8934**	**0.7984**	**−0.000044**	**0.023960**	**0.017889**

(b) Gitega-Zege

Eq. n˚ and correlation type	Equation	*R*	*R*²	*MBE*	*RMSE*	*t*
1.1. Linear	$y = 0.2881x + 0.2951$	0.8975	0.8055	0.000083	0.017850	0.015484
1.2. "	$y = 0.2701x' + 0.2957$	0.8993	0.8088	−0.000008	0.017679	0.001563
2.1. Logarithmic	$y = 0.1600\ln x + 0.5537$	0.8921	0.7959	−0.000025	0.018385	0.004510
2.2. "	$y = 0.1563\ln x' + 0.5419$	0.8766	0.7684	−0.000017	0.018166	0.003043
3.1. Exponential	$y = 0.3203e^{0.6239x}$	0.8939	0.7990	0.000092	0.039428	0.007711
3.2. "	$y = 0.3208e^{0.5850x'}$	0.8956	0.8021	−0.000250	0.038616	0.021472
4.1. Power	$y = 0.5113x^{0.3474}$	0.8904	0.7928	−0.000075	0.040043	0.006212
4.2. "	$y = 0.5492x'^{0.3458}$	0.8919	0.7955	−0.000025	0.039764	0.002085
5.1. Quadratic	$y = 0.3306 + 0.1574x + 0.1131x^2$	0.8978	0.8061	−0.000017	0.017778	0.003109
5.2. "	$y = 0.3342 + 0.1367x' + 0.1089x'^2$	0.9002	0.8103	0.000033	0.017631	0.006270
Mean value of statistical indicator		**0.8935**	**0.7984**	**−0.000021**	**0.026534**	**0.008553**

(c) Imbo-SEMS

Eq. n˚ and correlation type	Equation	*R*	*R*²	*MBE*	*RMSE*	*t*
1.1. Linear	$y = 0.3494x + 0.2850$	0.8776	0.7702	−0.000108	0.018582	0.019337
1.2. "	$y = 0.3321x' + 0.2832$	0.8802	0.7747	0.000083	0.018399	0.015022
2.1. Logarithmic	$y = 0.1817\ln x + 0.5889$	0.8751	0.7657	0.000083	0.018774	0.014722
2.2. "	$y = 0.1847\ln x' + 0.5794$	0.8782	0.7713	0.000042	0.018559	0.007446
3.1. Exponential	$y = 0.3156e^{0.7452x}$	0.8806	0.7754	−0.000350	0.039593	0.029320
3.2. "	$y = 0.3150e^{0.7051x'}$	0.8790	0.7727	0.000233	0.039305	0.019690
4.1. Power	$y = 0.6036x^{0.3879}$	0.8789	0.7725	−0.000017	0.039778	0.001390
4.2. "	$y = 0.5910x'^{0.3927}$	0.8785	0.7718	0.000025	0.039393	0.002105
5.1. Quadratic	$y = 0.2987 + 0.2959x + 0.0504x^2$	0.8779	0.7707	−0.000367	0.018346	0.066301
5.2. "	$y = 0.2848 + 0.3263x' + 0.0052x'^2$	0.8803	0.7750	0.000175	0.018430	0.031494
Mean value of statistical indicator		**0.8786**	**0.7720**	**−0.000020**	**0.022985**	**0.020544**

(d) Musasa

Eq. n° and correlation type	Equation	*R*	*R*²	*MBE*	*RMSE*	*t*
1.1. Linear	$y = 0.3629x + 0.3037$	0.8851	0.7833	−0.000225	0.023922	0.031196
1.2. "	$y = 0.3406x' + 0.3045$	0.8871	0.7869	−0.000125	0.023723	0.017476
2.1. Logarithmic	$y = 0.2231\ln x + 0.6406$	0.8877	0.7881	−0.000125	0.023734	0.017468
2.2. "	$y = 0.2226\ln x' + 0.6270$	0.8897	0.7916	0.000017	0.023458	0.002356
3.1. Exponential	$y = 0.3608e^{0.6889x}$	0.8804	0.7751	−0.000292	0.046559	0.020777
3.2. "	$y = 0.3438e^{0.6465x'}$	0.8822	0.7783	−0.000092	0.113109	0.002688
4.1. Power	$y = 0.6508x^{0.4241}$	0.8842	0.7819	0.000042	0.045779	0.003019
4.2. "	$y = 0.6341x'^{0.4230}$	0.8860	0.7851	0.000017	0.045466	0.001216
5.1. Quadratic	$y = 0.1615 + 0.8366x - 0.3776x^2$	0.8881	0.7887	−0.000425	0.023686	0.059521
5.2. "	$y = 0.1610 + 0.7890x' - 0.3362x'^2$	0.8901	0.7923	−0.000817	0.023429	0.115676
Mean value of statistical indicator		**0.8862**	**0.7851**	**−0.000203**	**0.039287**	**0.027139**

(e) Ruvyironza

Eq. n° and correlation type	Equation	*R*	*R*²	*MBE*	*RMSE*	*t*
1.1. Linear	$y = 0.4163x + 0.2285$	0.9407	0.8848	−0.000175	0.017199	0.033745
1.2. "	$y = 0.3901x' + 0.2297$	0.9414	0.8862	0.000033	0.017097	0.006466
2.1. Logarithmic	$y = 0.2047\ln x + 0.5845$	0.9411	0.8857	0.000058	0.017133	0.011292
2.2. "	$y = 0.2041\ln x' + 0.5718$	0.9424	0.8880	−0.000300	0.017286	0.057570
3.1. Exponential	$y = 0.2686e^{0.9578x}$	0.9398	0.8720	−0.000408	0.042013	0.032236
3.2. "	$y = 0.2694e^{0.8973x'}$	0.9343	0.8729	0.001700	0.043209	0.130589
4.1. Power	$y = 0.6100x^{0.4726}$	0.9375	0.8789	0.001575	0.041801	0.125056
4.2. "	$y = 0.5924x'^{0.4712}$	0.9385	0.8808	0.001625	0.041493	0.129988
5.1. Quadratic	$y = 0.1742 + 0.6501x - 0.2381x^2$	0.9328	0.8700	−0.000558	0.017045	0.108699
5.2. "	$y = 0.1669 + 0.6364x' - 0.2279x'^2$	0.9428	0.8889	0.000125	0.016898	0.024535
Mean value of statistical indicator		**0.9385**	**0.8808**	**0.000368**	**0.027117**	**0.065925**

(f) Rwegura

Eq. n° and correlation type	Equation	*R*	*R*²	*MBE*	*RMSE*	*t*
1.1. Linear	$y = 0.3416x + 0.2462$	0.9483	0.8993	0.000108	0.012113	0.029664
1.2. "	$y = 0.31207x + 0.2466$	0.9491	0.9008	−0.000192	0.011874	0.053544
2.1. Logarithmic	$y = 0.1677\ln x + 0.5299$	0.9486	0.8999	−0.000050	0.012786	0.012970
2.2. "	$y = 0.1573\ln x' + 0.5202$	0.9417	0.8868	−0.000042	0.012668	0.010908
3.1. Exponential	$y = 0.2815e^{0.7708x}$	0.8761	0.7675	−0.000267	0.030279	0.029211
3.2. "	$y = 0.2740e^{0.7821x'}$	0.9477	0.8981	−0.000325	0.029365	0.036710
4.1. Power	$y = 0.5474x^{0.3858}$	0.9425	0.8883	0.000000	0.030746	0.000000
4.2. "	$y = 0.5346x'^{0.3849}$	0.9434	0.8901	0.000017	0.030521	0.001811
5.1. Quadratic	$y = 0.2624 + 0.2702x + 0.0735x^2$	0.9485	0.8996	0.000383	0.012293	0.103472
5.2. "	$y = 0.2602 + 0.2647x' + 0.0548x'^2$	0.9494	0.9013	−0.000167	0.011872	0.046564
Mean value of statistical indicator		**0.9395**	**0.8832**	**−0.000054**	**0.019452**	**0.032485**

Besides the high level of performance for any of the sixty obtained correlations, the next other features can be stated about the comparison on one another. On account of the performance level of the overall ten equations per station, according to decreasing mean value of R or R^2, the six stations rank as follows: Rwegura, Ruvyironza, Gitega-Zege, Bujumbura-Airport, Musasa, Imbo-SEMS. According to increasing mean value of $RMSE$, the ranking of those stations is almost the same as previously, except that there is change of places between Ruvyironza and Bujumbura, together with between Musasa and Imbo-SEMS. At its turn, the ranking of the six stations is the following, according to increasing mean absolute value of MBE and thus to decreasing performance level of the overall ten equations per station: Gitega-Zege, Imbo-SEMS, Bujumbura-Airport, Rwegura, Musasa, Ruvyironza. According to increasing mean value of the t-statistics, the stations' ranking is almost the same as the previous one, except change of places between Imbo-SEMS and Bujumbura, together with between Rwegura and Musasa. Moreover, on account of the overall six stations, the ten developed equations per station rank as shown in **Table 9**, according to decreasing mean value of R or R^2, increasing mean absolute value of MBE, together with increasing mean value of $RMSE$ and t, thus according to decreasing performance level.

On the one hand, in the ranking according to mean value of R or R^2 and of $RMSE$, quadratic, linear and logarithmic correlations hold successively the six first positions, whereas power and exponential correlations hold the four last positions. On the other hand, in the ranking according to mean absolute value of MBE and mean value of t, power, logarithmic and linear correlations are successively in the six first positions, whereas exponential and quadratic correlations hold successively the four last positions. In terms of examples, curves of the obtained linear, quadratic and logarithmic empirical equations, respectively, which fit variation of measured average daily clearness index versus each of the two kinds of RSD, are shown for one station in **Figure 5** (a.1)-(a.2), (b.1)-(b.2) and (c.1)-(c.2), respectively.

Table 9. Ranking of the ten correlations per station on account of the overall six stations (equation numbers from Table 4 are used to mark these correlations).

Rank → **According to ↓**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Decreasing R_{mean} or R^2_{mean}	(5.2)	(1.2)	(2.1)	(2.2)	(5.1)	(1.1)	(4.2)	(4.1)	(3.1)	(3.2)
Increasing $\lvert MBE\rvert_{mean}$	(4.1)	(4.2)	(2.2)	(1.2)	(2.1)	(1.1)	(5.2)	(5.1)	(3.2)	(3.1)
Increasing $RMSE_{mean}$	(5.2)	(5.1)	(1.2)	(2.2)	(1.1)	(2.1)	(3.2)	(4.2)	(4.1)	(3.1)
Increasing t_{mean}	**(4.2)**	**(2.2)**	**(2.1)**	**(1.2)**	**(4.1)**	**(1.1)**	**(3.2)**	**(3.1)**	**(5.2)**	**(5.1)**

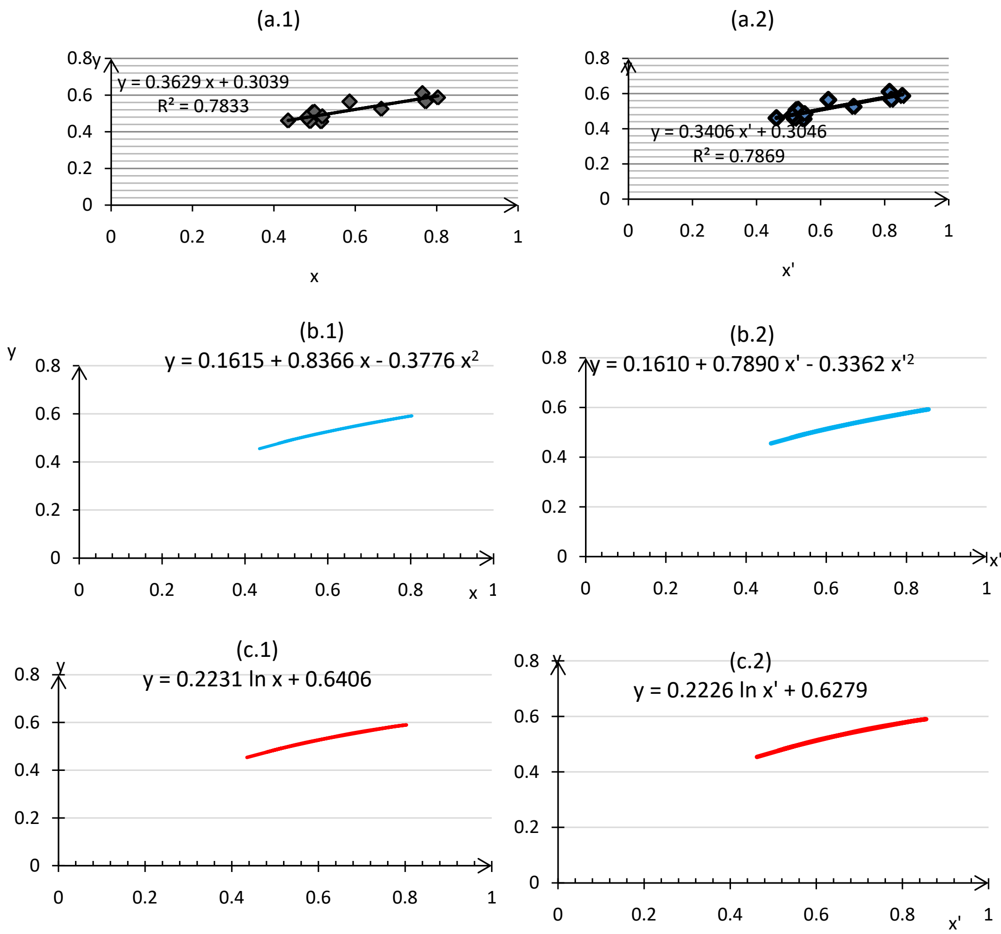

Figure 5. Illustration of model fitting for variation of average clearness index with respect to RSD for one station, e.g.: Musasa.

Table 10. Values of statistical indicators proposed in some other works on the same kinds of empirical correlations as in this study.

N° Location (Region)	Eq. n°	R^2	*MBE*	*RMSE*	*t*	Ref. n°	Year
1. Constanta (Romania)	(1.1); (5.1)	[0.787; 0.794]	---	---	---	[18]	2002
2. Six sites (Egypt)	(1.1); (2.1); (5.1)	---	0.001	0.06	---	[27]	2005
3. Kuala Terengganu (Malaysia)	(1.1); (2.1); (3.1); (5.1)	0.71	[0.0000; 0.0065]	[0.0337; 0.0420]	[0.0015; 0.9800]	[30]	2011
4. Katmandu (Himalaya)	(1.1)	0.71	0.055	0.071	---	[23]	2012
5. Osogbo, Osun State (Nigeria)	(1.1); (5.1)	[0.677; 0.760]	---	---	---	[12]	2013
6. Anantapur (India)	(1.1)	[0.70; 0.86]	---	---	---	[11]	2018
7. Maiduguri (Nigeria)	**(1.1); (2.1); (3.1); (4.1); (5.1)**	**[0.592; 0.777]**	---	---	**[0.0392; 0.1912]**	[10]	**2023**

Anyhow, in the ranking of the ten correlations per station according to any of the considered statistical indicators, three correlations out of five relating the clearness index ($\overline{k}_t$) to the second kind of RSD ($\overline{s}'_r$) appear in the four first positions, whereas the two remaining correlations appear in the four last positions (out of ten). This last feature suggests that using the second kind of RSD is slightly more advantageous than using the first one ($\overline{s}_r$) to estimate monthly average daily clearness index ($\overline{k}_t$) for the stations of this study.

Finally, as evidenced by examples of **Table 10**, results of this study match adequately data from some other works on the same kind of empirical correlations and statistical indicators.

5. Conclusions

Five couples of empirical Ångström-Prescott type correlations have been developed in this paper to estimate GSR on a horizontal surface using sunshine duration data for each of six selected stations in Burundi. In terms of long-term monthly average daily data, any of those correlations relates the clearness index ($y = \overline{k}_t$) to one of the two RSD ($x = \overline{s}_r$ and $x' = \overline{s}'_r$) defined in sections 1 and 2.2. The main conclusions of this study are as follows:

1) Higher mean values of RSD and clearness index are observed during the dry period, *i.e.* in June, July and August at any of the six stations.

2) According to annual mean values of the clearness index and RSD, sky over these stations is fairly clear (or partially cloudy) for a high number of days in the year.

3) Results of the statistical indicators *R*, R^2, *MBE*, *RMSE* and *t*-statistics lead to assert that any of the developed sixty equations fits very adequately measured data. Therefore, they all are recommended to predict GSR on a horizontal surface for the relevant locations. Nevertheless, according to mean values of *R* (or R^2) and *RMSE*, quadratic, linear and logarithmic correlations perform slightly better than power and exponential ones.

4) Using $x' = \overline{s}'_r$ as RSD is slightly more advantageous than using $x = \overline{s}_r$ to predict $y = \overline{k}_t$ for stations of this study.

5) Values of statistical indicators of this study are in good agreement with data from other works on the same kinds of correlations.

Acknowledgements

The authors would like to acknowledge the management of the Geographical Institute of Burundi (IGEBU, located in Gitega) for making the data of GSR and SD available.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] (2017) International Rankings of Burundi-2017; Burundi Data Portal. https://burundi.opendataforafrica.org/data#menu=topic

[2] Ma, C.C.Y. and Iqbal, M. (1984) Statistical Comparison of Solar Radiation Correlations. *Solar Energy*, **33**, 143-148. https://doi.org/10.1016/0038-092X(84)90231-7

[3] Iqbal, M. (1983) An Introduction to Solar Radiation. Academic Press, Toronto.

[4] Bashahu, M. and Nkundabakura, P. (1994) Analysis of Daily Global Irradiation Data for Five Sites in Rwanda and One in Senegal. *Renewable Energy*, **4**, 425-435. https://doi.org/10.1016/0960-1481(94)90050-7

[5] Bashahu, M. and Mpanzimana, D. (2009) Estimation of the Monthly Average Daily Global Solar Irradiation with Climatological Parameters at Some Burundian Stations. *International Review of Physics*, **3**, 237-243.

[6] Bristol, K.L. and Campbell, G.S. (1994) On the Relationship between Incoming Solar Radiation and Daily Maximum and Minimum Temperature. *Agricultural and Forest Meteorology*, **31**, 159-166. https://doi.org/10.1016/0168-1923(84)90017-0

[7] Grossi Gallegos, H., Righini, R. and Raichijk, C. (2006) Analysis of Alternative for the Assessment of the Solar Resource in Argentina. *Proceedings of the IXth World Renewable Energy Congress* (*WREC IX*), Florence, 19-25 August, Paper ID: RT 7 (CD-ROM).

[8] Fan, J., Chen, B., Wu, L., Zhang, F., Lu, X. and Xiang, Y. (2018) Evaluation and Development of Temperature-Based Empirical Models for Estimating Daily Global Solar Radiation in Humid Regions. *Energy*, **144**, 903-914. https://doi.org/10.1016/j.energy.2017.12.091

[9] Chen, R., Ersi, K., Yang, J., Lu, S. and Zhao, W. (2004) Validation of Five Global Radiation Models with Measured Daily Data in China. *Energy Conversion and Management*, **45**, 1759-1769. https://doi.org/10.1016/j.enconman.2003.09.019

[10] Akpootu, D.O., *et al.* (2023) Sunshine and Temperature Based Models for Estimating Global Solar Radiation in Maiduguri, Nigeria. *Saudi Journal of Engineering and Technology*, **8**, 82-90. https://doi.org/10.36348/sjet.2023.v08i05.001

[11] Reddy, M.V., Suvarana, R.P., Kumari, S.P. and Gopal, K.R. (2018) Estimation of Global Solar Radiation Using Sunshine and Temperature Based Models Over Anantapur Region. *International Journal of Creative Research Thoughts* (*IJCRT*), **6**, 607-618.

[12] Ohunakin, O.S., Adaramola, M.S., Ogewola, O.M. and Fegenle, R.O. (2013) Correlations for Estimating Solar Radiation Using Sunshine Hours and Temperature Measurement in Osogbo, Osun State, Nigeria. *Frontiers in Energy*, **7**, 214-222. https://doi.org/10.1007/s11708-013-0241-2

[13] Mohamed, N. (2010) Etude Comparative des Méthodes d'Estimation du Rayonnement Solaire. Mémoire de Magister, Université Ferhat Abbas, Sétif.

[14] Al-Salihi, A.M., Kadun, M.M. and Mohamed, A.J. (2010) Estimation of Global Solar Radiation on Horizontal Surface Using Routine Meteorological Measurements for Different Cities in Iraq. *Asian Journal of Scientific Research*, **3**, 240-248. https://doi.org/10.3923/ajsr.2010.240.248

[15] Fan, J., Wang, X., Wu, L., Zhang, F., Lu, X. and Xiang, Y. (2018) New Combined Models for Estimating Daily Global Solar Radiation Based on Sunshine Duration in Humid Regions: A Case Study in South China. *Energy Conversion and Management*, **156**, 618-625. https://doi.org/10.1016/j.enconman.2017.11.085

[16] Sarkar, M.N.I. and Sifat, A.I. (2016) Global Solar Radiation Estimation from Com-

monly Available Meteorological Data for Bangladesh. *Renewables*: *Wind*, *Water and Solar*, **3**, Article No. 6. https://doi.org/10.1186/s40807-016-0027-3

[17] Ångström, A. (1924) Solar and Terrestrial Radiation. *Quarterly Journal of Royal Meteorological Society*, **50**, 121-126. https://doi.org/10.1002/qj.49705021008

[18] Paltineau, C., Mihailescu, I.F., Torica, V. and Albu, A.N. (2002) Correlation between Sunshine Duration and Global Solar Radiation in South-Eastern Romania. *International Agrophysics*, **16**, 139-145.

[19] Prescott, J.A. (1940) Evaporation from a Water Surface in Relation to Solar Radiation. *Transactions of the Royal Society of South Australia*, **64**, 114-125.

[20] Andretta, A., Bartoli, B., Coluzzi, B., Cuomo, V., Francesca, M. and Serio, C. (1981) Global Solar Radiation Estimation from Relative Sunshine Hours in Italy. *Journal of Applied Meteorology and Climatology*, **2**, 1377-1384. https://doi.org/10.1175/1520-0450(1982)021<1377:GSREFR>2.0.CO;2

[21] Abdul-Aziz, J., Nagi, A.A. and Zumailan, A.A.R. (1993) Global Solar Estimation from Relative Sunshine Hours in Yemen. *Renewable Energy*, **3**, 645-653. https://doi.org/10.1016/0960-1481(93)90071-N

[22] Drif, M. and Chikh, M. (2000) Estimation de l'Irradiation Solaire par la Logique Floue. *Revue des Energies Renouvelables*, Chemss, 105-110. https://api.semanticscholar.org/CorpusID:171259650

[23] Poudyal, K., Bhattarai, B.K., Sapkota, B. and Kjeldstal, B. (2012) Estimation of Global Solar Radiation Using Sunshine Duration in Himalaya Region. *Research Journal of Chemical Sciences*, **2**, 20-25.

[24] Govindasamy, T.R. and Chetty, N. (2018) Quantifying the Global Solar Radiation Received in Pietermaritzburg, Kwazulu-Natal to Motivate the Consumption of Solar Technologies. *Open Physics*, **16**, 786-794. https://doi.org/10.1515/phys-2018-0098

[25] Cao, Q., Liu, Y., Lyu, K., Lu, Y., Li, D.H.W. and Yang, L. (2020) Solar Radiation Zoning and Daily Global Radiation Models for Regions with Only Surface Meteorological Measurements in China. *Energy Conversion and Management*, **225**, Article ID: 113447. https://doi.org/10.1016/j.enconman.2020.113447

[26] Almorox, J. and Hontoria, C. (2004) Global Solar Radiation Estimation Using Sunshine Duration in Spain. *Energy Conversion and Management*, **45**, 1529-1535. https://doi.org/10.1016/j.enconman.2003.08.022

[27] El-Metwally, M. (2005) Sunshine and Global Solar Radiation Estimation at Different Sites in Egypt. *Journal of Atmospheric and Solar-Terrestrial Physics*, **67**, 1331-1342. https://doi.org/10.1016/j.jastp.2005.04.004

[28] Zhang, Y. (2006) Comparison of Measured and Estimated Monthly Average Daily Global Radiation in Shanghai, Nanjing and Hangzhou Areas in East China. *Proceedings of the IXth World Renewable Energy Congress* (*WREC IX*), Florence, 19-25 August, Paper ID: RTPC 26 (CD-ROM).

[29] Luis, M.A., Felipe, P.R. and Pilar, N.R. (2008) Estimation of Global Solar Radiation by Means of Sunshine Duration. In: Goswami, D.Y. and Zhao, Y., Eds., *Proceedings of ISES Wold Congress* 2007 (*Vol. I-Vol. II*), Springer, Berlin, 2627-2631. https://doi.org/10.1007/978-3-540-75997-3_530

[30] Muzathik, A.M., *et al.* (2011) Daily Global Solar Radiation Estimate Based on Sunshine Hours. *International Journal of Mechanical and Materials Engineering*, **6**, 75-80.

[31] Khan, M.M. and Ahmad, M.J. (2012) Estimation of Global Solar Radiation Using Clear Sky Radiation in Yemen. *Journal of Engineering Science and Technology Re-

view, **5**, 12-19. https://doi.org/10.25103/jestr.052.03

[32] Gana, N.N. and Akpootu, D.O. (2013) Estimation of Global Solar Radiation Using Four Sunshine Based Models in Kebbi, North-Western, Nigeria. *Advances in Applied Science Research*, **4**, 409-421.

[33] Razika, I., Nabila, I. and Marouane, M. (2015) Estimation of Global Solar Radiation Using Sunshine Duration for M'Sila (Algeria). 2015 3*rd International Renewable and Sustainable Energy Conference* (*IRSEC*), Marrakesh, 10-13 December 2015, 1-6. https://doi.org/10.1109/IRSEC.2015.7455004

[34] Er, Z., Rouabah, Z. and Turna, I.B. (1017) A Modal Study on the Estimation of Daily Solar Radiation from Sunshine Duration. *Acta Physica Polonica A*, **132**, 1070-1075. https://doi.org/10.12693/APhysPolA.132.1070

[35] Anis, M.S., Jamil, B., Ansari, M.A. and Bellos, E. (2019) Generalized Models for Estimation of Global Solar Radiation Based on Sunshine Duration and Detailed Comparison with the Existing: A Case Study for India. *Sustainable Energy Technologies and Assessments*, **31**, 179-198. https://doi.org/10.1016/j.seta.2018.12.009

[36] Betti, T., Zulin, I., Brkic, S. and Tuka, B. (2020) A Comparison of Models for Estimating Solar Radiation from Sunshine Duration in Croatia. *International Journal of Photoenergy*, **2020**, Article ID: 9605950. https://doi.org/10.1155/2020/9605950

[37] Uçkan, I. and Khuddur K.M. (2022) Improving of Global Solar Radiation Forecast by Comparing Other Meteorological Parameter Models with Sunshine Duration Models. *Environmental Science and Pollution Research*. https://doi.org/10.21203/rs.3.rs-853866/v1

[38] Lan, K., Wang, L., Zhou, Y., Zhand, Z., Fang, S. and Cao, P. (2023) The Applicability of Sunshine-Based Global Solar Radiation Models Modified with Meteorological Factors for Different Climate Zones of China. *Frontiers in Energy Research*, **10**, Article 1010745. https://doi.org/10.3389/fenrg.2022.1010745

[39] Kotu, T.B., Ancha, V.R., Abulkadir Hassen, A.A. and Fanta, S.W. (2023) Prediction of Global Solar Radiation Based on Sunshine Duration Hours in Ethiopia's Amhara Region. In: Woldegiorgis, B.H., Mequanint, K., Getie, M.Z., Mulat, E.G. and Alemayehu Assegie, A., Eds., *Advancement of Science and Technology*, Springer, Cham, 1-31. https://doi.org/10.1007/978-3-031-33610-2_1

[40] Bashahu, M., Nsabimana, J.C. and Manirakiza, F. (2006) Beta Probability Density Functions and Reference Distributions of Relative Sunshine Duration Data from Burundian Stations. *ISESCO Science and Technology Vision*, **2**, 39-43.

[41] Bashahu, M. and Butisi, B. (2010) Statistical Analysis of Daily Global Solar Irradiation Data Using Beta Distributions at Some Burundian Stations. *International Journal of Physical Science*, **2**, 35-41.

[42] Bashahu, M. and Manirakiza, E. (2013) Reference Distributions of Daily Clearness Index Data from Burundian Stations. *International Review of Physics*, **7**, 352-357.

[43] Bashahu, M. and Ntirandekura, D. (2019) Analysis of Sunshine Duration Data Using Two-Parameter Weibull Distributions. *Modern Environmental Science and Engineering*, **5**, 635-644.

[44] Ratto, C.F. (1986) Sun-Earth Astronomical Relationships and the Extraterrestrial Solar Radiation. In: Guzzi, R. and Justus, C.G., Eds., *Physical Climatology for Solar and Wind Energy*, World Scientific, Singapore, 143-148.

[45] Pashiardis, S., Pelengaris, A. and Kalogurou, S.A. (2023) Geographical Distribution of Global Radiation and Sunshine Duration Over the Island of Cyprus. *Applied Science*, **13**, Article 5422. https://doi.org/10.3390/app13095422

[46] Bashahu, M. (2003) Statistical Comparison of Models for Estimating the Monthly

Average Daily Diffuse Radiation at a Subtropical African Site. *Solar Energy*, **75**, 43-51. https://doi.org/10.1016/S0038-092X(03)00213-5

[47] Bashahu, M. and Buseke, M. (2016) Statistical Analysis of Hourly Wind Speed Data from Some Burundian Stations Using Beta Probability Density Functions. *Modern Environmental Science and Engineering*, **2**, 740-746. https://doi.org/10.15341/mese(2333-2581)/11.02.2016/005

[48] Honerkamp, J. (1999) Stochastic Dynamical Systems: Concepts, Numerical Methods, Data Analysis. VCH Publishers, New York.

[49] Critical Values of Student's t Distribution with ν Degrees of Freedom $(t_{1-\alpha}, \nu)$. https://www.itl.nist.gov/div898/handbook/eda/section3/eda3672.htm