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Leap Zagreb and leap hyper-Zagreb indices of Jahangir and Jahangir derived graphs

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Abstract: Topological indices are numerical parameters of a graph which characterize its topology. The second degree of a vertex in a graph is equal to the number of its second neighbors. In this paper, we will compute leap Zagreb indices and leap hyper-Zagreb indices of Jahangir graph and its line graph based on the 2-distance degree of the vertices. Moreover we will compute the same indices for the subdivision graph and the line graph of the subdivision of Jahangir graph.

Keywords: Leap Zagreb indices, leap hyper-Zagreb indices, F-leap indices, Jahangir graph, second degree.

1. Introduction and Preliminaries

Chemical graph theory, a branch of mathematical chemistry, applies graph theory to mathematical modeling of chemical phenomena. Topological indices are real numbers that are presented as graph parameters (e.g. the degree of vertices, distances, etc.) introduced during studies conducted on the molecular graphs in chemistry and can describe some physical and chemical properties of molecules.

Let G be a simple and connected graph with order $V(G)$ and size $E(G) \subseteq V(G) \times V(G)$. The degree d_u of any vertex u is defined as the number of vertices joined to that vertex u. The distance between two vertices u and v of a graph G is the number of edges in a shortest path connecting them and is denoted by $d(u,v)$. The subdivision graph $S(G)$ is a graph resulting from the subdivision of all the edges of G. The line graph $L(G)$ of a graph G whose vertices are the edges of G, with $uv \in E(L(G))$ when u and v have a common end point in G. In structural chemistry, line graph of a graph G is very useful. The first topological indices on the basis of line graph was introduced by Bertz in 1981 (see [1]). The topological indices of Jahangir graph and its line graph are computed by many researchers (see [2–5]).

For a vertex v in G, the open $k-$neighborhood of v is defined as $N_k(v/G) = \{u \in V(G) : d(u,v) = k\}$, where k is a positive integer. The $k-$distance degree, denoted by $d_k(v/G)$, of a vertex $v \in V(G)$ is the number of $k-$neighbors of v in G, *i.e.*, $d_k(v/G) = |N_k(v/G)|$. It is clear that $d_1(v/G) = d_v$ for every $v \in V(G)$. The $2-$distance degree of a vertex v consists of all vertices at distance two to v.

Recently, leap Zagreb indices of a graph have been introduced by Naji *et al.*, (see [6]), depending on $2-$distance degree of the vertices. The leap Zagreb indices have several chemical applications. The first leap Zagreb index has very good correlation with physical properties of chemical compounds like boiling point, entropy, $DHVAP$, $HVAP$ and accentric factor (see [7]). Shiladhar *et al.*, computed leap Zagreb indices of some wheel related graphs (see [8]). For more details about leap indices, see [9–12].

For a graph G, the first, second, and third leap Zagreb indices are defined as:

$$LM_1(G) = \sum_{v\in V(G)} (d_2(v/G))^2, \quad (1)$$

$$LM_2(G) = \sum_{uv\in E(G)} d_2(u/G)(d_2(v/G), \quad (2)$$

$$LM_3(G) = \sum_{v\in V(G)} d_1(v/G)d_2(v/G). \quad (3)$$

The first and second leap-hyper Zagreb indices were introduced by Kulli [13] as:

$$HLM_1(G) = \sum_{uv\in E(G)} [d_2(u/G) + d_2(v/G)]^2, \quad (4)$$

$$HLM_2(G) = \sum_{uv\in E(G)} [d_2(u/G)d_2(v/G)]^2. \quad (5)$$

Later on, Basavanagoud and Chitra computed leap hyper-Zagreb indices of some nanostructures [14]. In 2019, Kulli *et al.*, computed leap hyper-Zagreb indices of certain windmill graphs [15].

2. Leap Zagreb and leap hyper-Zagreb indices of Jahangir graph $J_{n,m}$

In this section, we will compute leap Zagreb indices and hyper-leap Zagreb indices of Jahangir graph $J_{n,m}$. The Jahangir graph $J_{n,m}$ consists of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} at distance to each other [16].

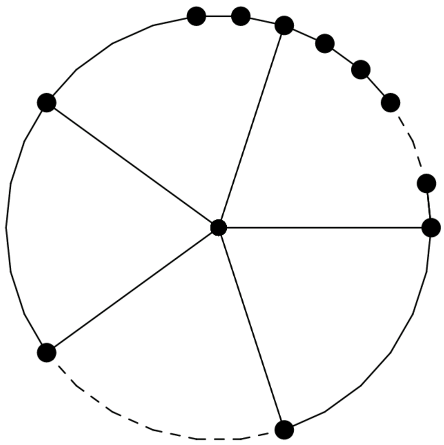

Figure 1. The Jahangir graph $J_{n,m}$

Theorem 1. *Let $J_{n,m}$ be the Jahangir graph, then*

1. $LM_1(J_{n,m}) = m^3 + 6m^2 + 4mn + 7m.$
2. $LM_2(J_{n,m}) = 2m^3 + 8m^2 + 4mn + 2m.$
3. $LM_3(J_{n,m}) = 5m^2 + 4mn + 3m.$
4. $HLM_1(J_{n,m}) = 11m^3 + 22m^2 + 16mn + 19m.$
5. $HLM_2(J_{n,m}) = 4m^5 + 8m^4 + 22m^3 + 36m^2 + 16mn + 26m.$

Proof. Let $J_{n,m}$ be a Jahangir graph with $m(n+1)$ edges and $mn+1$ vertices as shown in Figure 1. For an edge $uv \in E(J_{n,m})$, the $2-$distance degree of a vertex u and vertex v is denoted by $d_2(u/J_{n,m})$ and $d_2(v/J_{n,m})$ respectively. The edge partition of $J_{n,m}$ with respect to $2-$distance degree of vertices of $J_{n,m}$ is shown in Table 1. The vertex partition of graph $J_{n,m}$ based on the $1-$distance degree of a vertex u and $2-$distance degree of a vertex u is shown in Table 2.

Table 1. The edge partition of $J_{n,m}$, where $d_2(u/J_{n,m})d_2(v/J_{n,m}) \in E(J_{n,m})$

No. of edges	$d_2(u/J_{n,m})$	$d_2(v/J_{n,m})$
$m(n-4)$	2	2
$2m$	2	3
$2m$	3	$m+1$
m	$2m$	$m+1$

Table 2. The vertex partition of $J_{n,m}$ where $d_1(v/J_{n,m}), d_2(v/J_{n,m}) \in V(J_{n,m})$

No. of vertices	$d_1(v/J_{n,m})$	$d_2(v/J_{n,m})$
1	m	$2m$
$m(n-3)$	2	2
$2m$	2	3
m	3	$m+1$

1. Using Formula (1) and Table 2, we have

$$\begin{aligned} LM_1(J_{n,m}) &= 1.(2m)^2 + m(n-3)(2)^2 + 2m(3)^2 + m(m+1)^2 \\ &= m^3 + 6m^2 + 4mn + 7m. \end{aligned}$$

2. Using Formula (2) and Table 1, we have

$$\begin{aligned} LM_2(J_{n,m}) &= m(n-4)(2.2) + 2m(2.3) + 2m(3.(m+1)) + m(2m.(m+1)) \\ &= 2m^3 + 8m^2 + 4mn + 2m. \end{aligned}$$

3. Using Formula (3) and Table 2, we have

$$\begin{aligned} LM_3(J_{n,m}) &= 1.(2m.m) + m(n-3)(2.2) + m(3.(m+1)) + 2m(3.2) \\ &= 5m^2 + 4mn + 3m. \end{aligned}$$

4. Using Formula (4) and Table 1, we have

$$\begin{aligned} HLM_1(J_{n,m}) &= m(n-4)(2+2)^2 + 2m(2+3)^2 + 2m(3+(m+1))^2 + m(2m+(m+1))^2 \\ &= 11m^3 + 22m^2 + 16mn + 19m. \end{aligned}$$

5. Using Formula (5) and Table 1, we have

$$\begin{aligned} HLM_2(J_{n,m}) &= m(n-4)(2.2)^2 + 2m(2.3)^2 + 2m(3.(m+1))^2 + m(2m.(m+1))^2 \\ &= 4m^5 + 8m^4 + 22m^3 + 36m^2 + 16mn + 26m. \end{aligned}$$

□

3. Leap Zagreb and leap hyper-Zagreb indices of the line graph of Jahangir graph $J_{n,m}$

In this section, we will compute leap Zagreb indices and hyper-leap Zagreb indices of the line graph of Jahangir graph $J_{n,m}$.

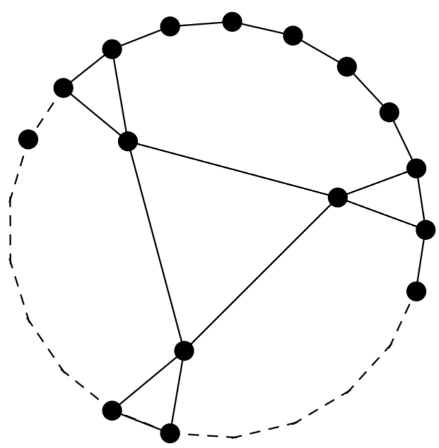

Figure 2. The line graph of Jahangir graph $J_{n,m}$

Theorem 2. *Let $L(J_{n,m})$ be the line graph of the Jahangir graph $J_{n,m}$, then*

1. $LM_1(L(J_{n,m})) = m^3 + 2m^2 + 4mn + 11m$.
2. $LM_2(L(J_{n,m})) = 2m^4 + 3m^3 + 12m^2 + 4mn - m$.
3. $LM_3(L(J_{n,m})) = 2m^3 + 8m^2 + 4mn + 2m$.
4. $HLM_1(L(J_{n,m})) = 8m^4 + 13m^3 + 30m^2 + 16mn + 5m$.
5. $HLM_2(L(J_{n,m})) = 8m^6 + 16m^4 + 27m^3 + 38m^2 + 16mn + 11m$.

Proof. Let $L(J_{n,m})$ be a line graph of Jahangir graph $J_{n,m}$ with $\frac{m^2+2mn+3m}{2}$ edges and $m(n+1)$ vertices is shown in Figure 2. For an edge $uv \in E(L(J_{n,m}))$, the 2-distance degree of a vertex u and vertex v is denoted by $d_2(u/L(J_{n,m}))$ and $d_2(v/L(J_{n,m}))$ respectively. The edge partition of $L(J_{n,m})$ with respect to 2-distance degree of vertices of $L(J_{n,m})$ is shown in Table 3. The vertex partition of graph $L(J_{n,m})$ based on the 1-distance degree of a vertex u and 2-distance degree of a vertex u is shown in Table 4.

Table 3. The edge partition of $L(J_{n,m})$, where $d_2(u/L(J_{n,m}))d_2(v/L(J_{n,m})) \in E(L(J_{n,m}))$

Number of edges	$d_2(u/L(J_{n,m}))$	$d_2(v/L(J_{n,m}))$
$m(n-5)$	2	2
$2m$	2	2
$2m$	2	3
m	3	3
$2m$	3	$m+1$
$\frac{m(m-1)}{2}$	$m+1$	$m+1$

Table 4. The vertex partition of $L(J_{n,m})$ where $d_1(v/L(J_{n,m}))$, $d_2(v/L(J_{n,m})) \in V(L(J_{n,m}))$

No. of 1−distance degree vertices	$d_1(u/L(J_{n,m}))$	No. of 2−distance degree vertices	$d_2(u/L(J_{n,m}))$
$m(n-4)$	2	$m(n-4)$	2
$2m$	2	$2m$	3
$2m$	3	m	$2m$
m	$m+1$	$2m$	$m+1$

1. Using Formula (1) and Table 4, we have

$$\begin{aligned} LM_1(L(J_{n,m})) &= m(n-4)(2)^2 + 2m(2)^2 + 2m(3)^2 + m(m+1)^2 \\ &= m^3 + 2m^2 + 4mn + 11m. \end{aligned}$$

2. Using Formula (2) and Table 3, we have

$$\begin{aligned} LM_2(L(J_{n,m})) &= m(n-5)(2.2) + 2m(2.3) + 2m(3.(m+1)) + m(m+1)^2 + 2m(2m.(m+1)) \\ &\quad + \frac{m(m-1)}{2}(2m.2m) \\ &= 2m^4 + 3m^3 + 12m^2 + 4mn - m. \end{aligned}$$

3. Using Formula (3) and Table 4, we have

$$\begin{aligned} LM_3(L(J_{n,m})) &= m(n-4)(2.2) + m.(2m.(m+1)) + 2m(3.(m+1)) + 2m(3.2) \\ &= 2m^3 + 8m^2 + 4mn + 2m. \end{aligned}$$

4. Using Formula (4) and Table 3, we have

$$\begin{aligned} HLM_1(L(J_{n,m})) &= m(n-5)(2+2)^2 + 2m(2+3)^2 + 2m(3+(m+1))^2 + m(m+1)^2 \\ &\quad +2m(2m+(m+1))^2 + \frac{m(m-1)}{2}(2m+2m)^2 \\ &= 8m^4 + 13m^3 + 30m^2 + 16mn + 5m. \end{aligned}$$

5. Using Formula (5) and Table 3, we have

$$\begin{aligned} HLM_2(L(J_{n,m})) &= m(n-5)(2.2)^2 + 2m(2.3)^2 + 2m(3.(m+1))^2 + m(m+1)^2 \\ &\quad + 2m(2m.(m+1))^2 + \frac{m(m-1)}{2}(2m.2m)^2 \\ &= 8m^6 + 16m^4 + 27m^3 + 38m^2 + 16mn + 11m. \end{aligned}$$

□

4. Leap Zagreb and leap-hyper Zagreb indices of the subdivision of Jahangir Graph $J_{n,m}$

In this section, we will compute leap Zagreb indices and hyper-leap Zagreb indices of the subdivision of Jahangir graph $J_{n,m}$.

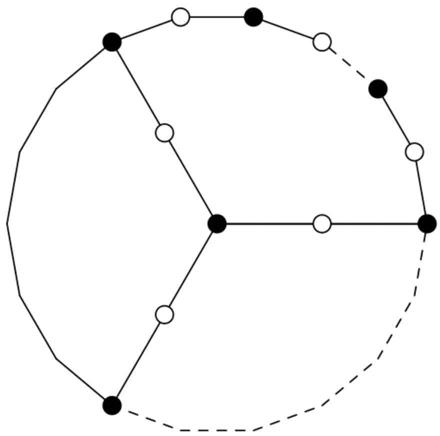

Figure 3. The subdivision of Jahangir graph $J_{n,m}$

Theorem 3. *Let $S(J_{m,n})$ be the subdivision graph of the Jahangir graph $J_{m,n}$, then*

1. $LM_1(S(J_{n,m})) = m^3 + 3m^2 + 8mn + 16m.$
2. $LM_2(S(J_{n,m})) = m^3 + 4m^2 + 8mn + 17m.$
3. $LM_3(S(J_{n,m})) = 3m^2 + 4mn + 11m.$
4. $HLM_1(S(J_{n,m})) = 5m^3 + 12m^2 + 32mn + 75m.$
5. $HLM_2(S(J_{n,m})) = m^5 + 2m^4 + 10m^3 + 18m^2 + 32mn + 179m.$

Proof. Let $S(J_{n,m})$ be a subdivision of Jahangir graph with $2m(n+1)$ edges and $m(2n+1)+1$ vertices is shown in Figure 3. For an edge $uv \in E(S(J_{n,m}))$, the 2−distance degree of a vertex u and vertex v is denoted by $d_2(u/S(J_{n,m}))$ and $d_2(v/S(J_{n,m}))$ respectively. The edge partition of $E(S(J_{n,m}))$ with respect to 2−distance degree of a vertex u in $S(J_{n,m})$ is shown in Table 5. The vertex partition of graph $S(J_{n,m})$ based on the 1−distance degree of a vertex u and 2−distance degree of a vertex u is shown in Table 6.

Table 5. The edge partition of $S(J_{n,m})$, where $d_2(u/S(J_{n,m}))d_2(v/S(J_{n,m})) \in E(S(J_{n,m}))$

No. of edges	$d_2(u/S(J_{n,m}))$	$d_2(v/S(J_{n,m}))$
$2m(n-2)$	2	2
$2m$	2	3
$2m$	3	3
m	3	$m+1$
m	m	$m+1$

Table 6. The vertex partition of $S(J_{n,m})$, where $d_1(v/S(J_{n,m})), d_2(v/S(J_{n,m})) \in V(S(J_{n,m}))$

No. of vertices	$d_1(v/S(J_{n,m}))$	$d_2(v/S(J_{n,m}))$
1	m	m
$m(2n-3)$	2	2
$2m$	2	3
m	2	3
m	3	$m+1$

1. Using Formula (1) and Table 6, we have

$$\begin{aligned} LM_1[S(J_{n,m})] &= 1.(m)^2 + m(2n-3)(2)^2 + 2m(3)^2 + m(3)^2 + m(m+1)^2 \\ &= m^3 + 3m^2 + 8mn + 16m. \end{aligned}$$

2. Using Formula (2) and Table 5, we have

$$\begin{aligned} LM_2[S(J_{n,m})] &= 2m(n-2)(2.2) + 2m(2.3) + 2m(3.3) + m(3.(m+1)) + m(m.(m+1)) \\ &= m^3 + 4m^2 + 8mn + 17m. \end{aligned}$$

3. Using Formula (3) and Table 6, we have

$$\begin{aligned} LM_3[S(J_{n,m})] &= 1.(m.m) + m(n-3)(2.2) + 2m(2.3) + m(3.3) + m(2.(m+1)) \\ &= 3m^2 + 4mn + 11m. \end{aligned}$$

4. Using Formula (4) and Table 5, we have

$$\begin{aligned} HLM_1[S(J_{n,m})] &= 2m(n-2)(2+2)^2 + 2m(2+3)^2 + 2m(3+3)^2 + m(3+(m+1))^2 + m(m+(m+1))^2 \\ &= 5m^3 + 12m^2 + 32mn + 75m. \end{aligned}$$

5. Using Formula (5) and Table 5, we have

$$\begin{aligned} HLM_2[S(J_{n,m})] &= 2m(n-2)(2.2)^2 + 2m(2.3)^2 + 2m(3.3)^2 + m(3.(m+1))^2 + m(m.(m+1))^2 \\ &= m^5 + 2m^4 + 10m^3 + 18m^2 + 32mn + 179m. \end{aligned}$$

□

5. Leap Zagreb and leap-hyper Zagreb indices of line graph of the subdivision of Jahangir graph $J_{n,m}$

In this section, we will compute leap Zagreb indices and hyper-leap Zagreb indices of the line graph of the subdivision of Jahangir graph $J_{n,m}$.

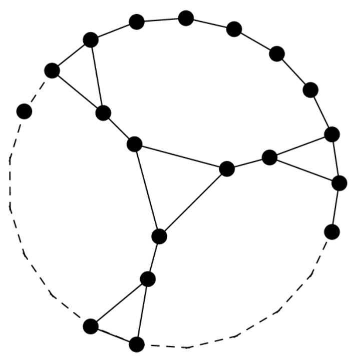

Figure 4. The line graph of the subdivision of Jahangir graph $J_{n,m}$.

Theorem 4. *Let $L[(S(J_{n,m}))]$ be the line graph of the subdivision of Jahangir graph $J_{n,m}$, then*

1. $LM_1[L(S(J_{n,m}))] = 2m^3 + 4m^2 + 8mn + 22m$.
2. $LM_2[L(S(J_{n,m}))] = \frac{1}{2}[m^4 + 3m^3 + 15m^2 + 8mn + 51m]$.
3. $LM_3[L(S(J_{n,m}))] = m^3 + 4m^2 + 8mn + 17m$.
4. $HLM_1[L(S(J_{n,m}))] = \frac{1}{2}[4m^4 + 16m^3 + 44m^2 + 64mn + 224m]$.
5. $HLM_2[L(S(J_{n,m}))] = \frac{1}{2}[m^6 + 5m^5 + 10m^4 + 46m^3 + 77m^2 + 64mn + 507m]$.

Proof. Let $L(S(J_{n,m}))$ be the line graph of the subdivision of Jahangir graph $J_{n,m}$ with $\frac{m^2+4mn+5m}{2}$ edges and $2m(n+1)$ vertices is shown in Figure 4. For an edge $uv \in E[L(S(J_{n,m}))]$, the $2-distance$ degree of a vertex u and vertex v is denoted by $d_2(u/L(S(J_{n,m})))$ and $d_2(v/L(S(J_{n,m})))$ respectively. The edge partition of $E[L(S(J_{n,m}))]$ with respect to $2-$distance degree of a vertex u in $L(S(J_{n,m}))$ is shown in Table 7. The vertex partition of graph $L(S(J_{n,m}))$ based on the $1-$distance degree of a vertex u and $2-$distance degree of a vertex u is shown in Table 8.

Table 7. The edge partition of $L(S(J_{n,m}))$, where $d_2(u/L(S(J_{n,m})))d_2(v/L(S(J_{n,m}))) \in E(L(S(J_{n,m})))$

No. of edges	$d_2(u/L(S(J_{n,m})))$	$d_2(v/L(S(J_{n,m})])$
$m(2n-5)$	2	2
$2m$	2	3
$3m$	3	3
$2m$	3	$m+1$
$\frac{m(m+1)}{2}$	$m+1$	$m+1$

Table 8. The vertex partition of $L[S(J_{n,m})]$, where $d_1(u/L(S(J_{n,m}))), d_2(u/L(S(J_{n,m}))) \in V(L(S(J_{n,m})))$

No. of vertices	$d_1(u/L(S(J_{n,m})))$	$d_2(u/L(S(J_{n,m})))$
$2m(n-2)$	2	2
$2m$	2	3
$2m$	3	3
m	3	$m+1$
m	m	$m+1$

1. Using Formula (1) and Table 8, we have
$$\begin{aligned} LM_1[L(S(J_{n,m}))] &= 4m(3)^2 + 2m(n-2)(2)^2 + 2m(m+1)^2 \\ &= 2m^3 + 4m^2 + 8mn + 22m. \end{aligned}$$

2. Using Formula (2) and Table 7, we have
$$\begin{aligned} LM_2[L(S(J_{n,m}))] &= m(2n-5)(2)^2 + 2m(2.3) + 3m(3)^2 + 2m(3.(m+1)) + \frac{m(m+1)^3}{2} \\ &= \frac{1}{2}[m^4 + 3m^3 + 15m^2 + 8mn + 51m]. \end{aligned}$$

3. Using Formula (3) and Table 8, we have
$$\begin{aligned} LM_3[L(S(J_{n,m}))] &= 2m(n-2)(2.2) + 2m(2.3) + 2m(3.3) + m(3.(m+1)) + m(m.(m+1)) \\ &= m^3 + 4m^2 + 8mn + 17m. \end{aligned}$$

4. Using Formula (4) and Table 7, we have
$$\begin{aligned} HLM_1[L(S(J_{n,m}))] &= m(2n-5)(2+2)^2 + 2m(2+3)^2 + 3m(3+3)^2 + 2m(3+(m+1))^2 \\ &\quad + \frac{m(m+1)}{2}[(m+1)+(m+1)]^2 \\ &= \frac{1}{2}[4m^4 + 16m^3 + 44m^2 + 64mn + 224m]. \end{aligned}$$

5. Using Formula (5) and Table 7, we have

$$\begin{aligned} HLM_2[L(S(J_{n,m}))] &= m(2n-5)(2.2)^2 + 2m(2.3)^2 + 3m(3.3)^2 + 2m(3.(m+1))^2 \\ &+ \frac{m(m+1)}{2}[(m+1).(m+1)]^2 \\ &= \frac{1}{2}[m^6 + 5m^5 + 10m^4 + 46m^3 + 77m^2 + 64mn + 507m]. \end{aligned}$$

□

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Conflicts of Interest: "The authors declare no conflict of interest."

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