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*Article*

# K Banhatti and K hyper-Banhatti indices of nanotubes

**Muhammad Shahzad Anjum[1] and Muhammad Usman Safdar[2,*]**

1 Department of Mathematics, The University of Lahore (Pakpattan Campus), Lahore Pakistan.; mshahzadanjum138@gmail.com
2 Department of Mathematics, University of Engineering and Technology, Lahore Pakistan.
* Correspondence: usmansafdar32@gmail.com

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**Abstract:** Nanomaterials are compound substances or materials that are produced and utilized at an exceptionally little scale. Nanomaterials are created to display novel attributes contrasted with a similar material without nanoscale highlights, for example, expanded quality, synthetic reactivity or conductivity. Topological indices are numbers related to molecular graphs that catch symmetry of molecular structures and give it a scientific dialect to foresee properties, such as: boiling points, viscosity, the radius of gyrations and so on. In this paper, we aim to compute topological indices of $TUC_4[m,n]$, $TUZC_6[m,n]$, $TUAC_6[m,n]$, $SC_5C_7[p,q]$, $NPHX[p,q]$, $VC_5C_7[p,q]$ and $HC_5C_7[p,q]$ nanotubes. We computed first and second K Banhatti indices, first and second K hyper-Banhatti indices and harmonic Banhatti indices of understudy nanotubes. We also computed multiplicative version of these indices. Our results can be applied in physics, chemical, material, and pharmaceutical engineering.

**Keywords:** Nanomaterial, molecular graph, Banhatti index, chemical graph theory.

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## 1. Introduction

Chemical reaction network theory deals with an attempt to model the behavior of real world chemical systems. From the very beginning of its foundation, it is hot cake for research community; especially due to its importance in two important branches i.e. biochemistry and theoretical chemistry. It has also a significant place in pure mathematics particularly due to its mathematical structures.

Cheminformatics is an upcoming and progressive area that deals with the relationships of qualitative structure activity (QSAR) and structure property (QSPR) and also predicts the biochemical activities and properties of nanomaterial. In these studies, for the prediction of bioactivity of the chemical compounds, some physcio-chemical properties and topological indices are used see [1–4].

Mathematical chemistry is the branch of chemistry which discusses the chemical structures with the aid of mathematical tools. Molecular graph is a simple connected graph in chemical graph theory. This graph consists of atoms and chemical bonds and they are represented by vertices and edges respectively. The distance between two vertices $u$ and $v$ is represented as $d(u,v)$ and it is the shortest length between $u$ and $v$ in graph $G$. The degree of vertex is basically the number of vertices of $G$ adjacent to a given vertex $v$ and will be denoted by $d_v$.

The topological index of a molecule can be used to quantify the molecular structure. To be simple, the topological index can be considered a function that assign each molecular structure to real number. Boiling point, heat of evaporation, heat of formation, chromatographic retention times, surface tension, vapor pressure etc can be predicted by using topological indices. First and second Zagreb indices are degree based graph invariants have been studies extensively since 1970's.

The first and second K Banhatti indices were introduced by Kulli in [5] as

$$B_1(G) = \sum_{ue} [d_G(u) + d_G(e)]$$

and

$$B_2(G) = \sum_{ue} [d_G(u)\, d_G(e)].$$

The first and second multiplicative K Banhatti indices were introduced by Kulli in [6] as

$$BII_1(G) = \prod_{ue} [d_G(u) + d_G(e)]$$

and

$$BII_2(G) = \prod_{ue} [d_G(u) \, d_G(e)] \, .$$

The following K hyper-Banahatti indices are defined in [6] as

$$HB_1(G) = \sum_{ue} [d_G(u) + d_G(e)]^2$$

and

$$HB_2(G) = \sum_{ue} [d_G(u) \, d_G(e)]^2 \, .$$

The first and second multiplicative K hyper-Banhatti indices are defined as

$$HBII_1(G) = \prod_{ue} [d_G(u) + d_G(e)]^2$$

and

$$HBII_2(G) = \prod_{ue} [d_G(u) \, d_G(e)]^2 \, .$$

The K harmonic Banhatti index is defined as

$$H_b(G) = \sum_{ue} \left[ \frac{2}{d_G(u) + d_G(e)} \right] .$$

The multiplicative K harmonic Banhatti index is defined as

$$HII_b(G) = \prod_{ue} \left[ \frac{2}{d_G(u) + d_G(e)} \right] .$$

In this paper we compute several Banhatti type indices of $TUC_4[m,n]$, $TUZC_6[m,n]$, $TUZC_6[m,n]$, $SC_5C_7[p,q]$, $NPHX[p,q]$, $VC_5C_7[p,q]$ and $HC_5C_7[p,q]$ nanotubes.

## 2. Main Results

### 2.1. Banhatti indices of $TUC_4[m,n]$

In the nanoscience, $TUC_4[m,n]$ nanotubes (where $m$ and $n$ are denoted as the number of squares in a row and the number of squares in a column respectively.) are plane tiling of $C_4$. This tessellation of $C_4$ can cover either a torus or a cylinder. The 3D representation of $TUC_4[m,n]$ is described in Figure 1.

**Theorem 1.** *Let G be the $TUC_4[m,n]$ nanotube. Then we have*

1. $B_1(TUC_4[m,n]) = 40mn + 2m$.
2. $B_2(TUC_4[m,n]) = 96mn - 26m$.
3. $HB_1(TUC_4[m,n]) = 400mn - 144m$.
4. $HB_2(TUC_4[m,n]) = 2304mn - 1630m$
5. $H_b(TUC_4[m,n]) = \frac{4}{5}mn + \frac{559}{630}m$.

**Proof.** Let $G = TUC_4[m,n]$. The edge set of $UC_4[m,n]$ can be partitioned as follows:
$E_6 = \{uv \in E(G) : d_G(u) = d_G(v) = 3\}$,
$E_7 = \{uv \in E(G) : d_G(u) = 3, d_G(v) = 4\}$,
$E_8 = \{uv \in E(G) : d_G(u) = d_G(v) = 4\}$,
such that $|E_6| = 2m$, $|E_7| = 2m$ and $|E_8| = m(2n-3)$.
The edge degree partition of $V$ is given in Table 1. Now

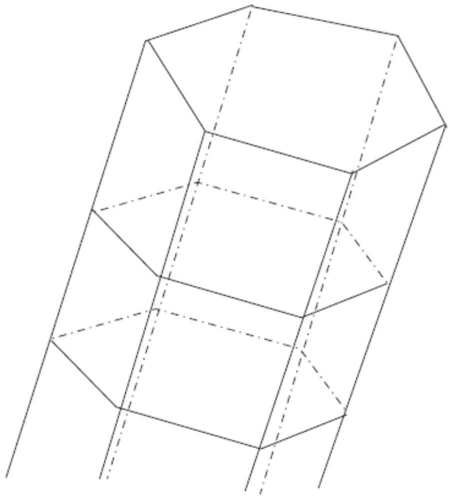

**Figure 1.** Graph of $TUC_4[6,n]$.

**Table 1.** Edge degree partition of $TUC_4[m,n]$.

| $d_G(u), d_G(v) : e = uv \in E(G)$ | $(3,3)$ | $(3,4)$ | $(4,4)$ |
|---|---|---|---|
| $d_G(e)$ | 4 | 5 | 6 |
| Number of edges | $2m$ | $2m$ | $m(2n-3)$ |

1. First K Banhatti index of $TUC_4[m,n]$ is

$$\begin{aligned} B_1(TUC_4[m,n]) &= (2m)[(3+4)+(3+4)] + (2m)[(3+5)+(4+5)] + (m(2n-3))[(4+6)+(4+6)] \\ &= 40mn + 2m. \end{aligned}$$

2. Second K Banhatti index of $TUC_4[m,n]$ is

$$\begin{aligned} B_2(TUC_4[m,n]) &= (2m)[(3\times 4)+(3\times 4)] + (2m)[(3\times 5)+(4\times 5)] + (m(2n-3))[(4\times 6)+(4\times 6)] \\ &= 96mn - 26m. \end{aligned}$$

3. First K hyper-Banhatti index of $TUC_4[m,n]$ is

$$\begin{aligned} HB_1(TUC_4[m,n]) &= (2m)\left[(3+4)^2+(3+4)^2\right] + (2m)\left[(3+5)^2+(4+5)^2\right] \\ &\quad + (m(2n-3))\left[(4+6)^2+(4+6)^2\right] \\ &= 400mn - 144m. \end{aligned}$$

4. Second K hyper-Banhatti index of $TUC_4[m,n]$ is

$$\begin{aligned} HB_2(TUC_4[m,n]) &= (2m)\left[(3\times 4)^2+(3\times 4)^2\right] + (2m)\left[(3\times 5)^2+(4\times 5)^2\right] \\ &\quad + (m(2n-3))\left[(4\times 6)^2+(4\times 6)^2\right] \\ &= 2304mn - 1630m. \end{aligned}$$

5. K Banhatti harmonic index of $TUC_4[m,n]$ is

$$H_b(TUC_4[m,n]) = (2m)\left[\left(\frac{2}{3+4}\right)+\left(\frac{2}{3+4}\right)\right] + (2m)\left[\left(\frac{2}{3+5}\right)+\left(\frac{2}{4+5}\right)\right]$$

$$+m\,(2n-3)\left[\left(\frac{2}{4+6}\right)+\left(\frac{2}{4+6}\right)\right]$$
$$=\frac{4}{5}mn+\frac{559}{630}m.$$

□

**Theorem 2.** *Let G be the $TUC_4\,[m,n]$ nanotube. Then we have*

1. $BII_1\,(TUC_4\,[m,n]) = 7^{4m}\times 8^{2m}\times 9^{2m}\times 10^{2m(2n-3)}$.
2. $BII_2\,(TUC_4\,[m,n]) = 12^{4m}\times 15^{2m}\times 20^{2m}\times 24^{2m(2n-3)}$.
3. $HBII_1\,(TUC_4\,[m,n]) = 7^{8m}\times 8^{4m}\times 9^{4m}\times 10^{4m(2n-3)}$.
4. $HBII_2\,(TUC_4\,[m,n]) = 12^{8m}\times 15^{4m}\times 20^{4m}\times 24^{4m(2n-3)}$.
5. $HII_b\,(TUC_4\,[m,n]) = (\frac{2}{7})^{4m}\times\left(\frac{1}{4}\right)^{2m}\times(\frac{2}{9})^{2m}\times\left(\frac{1}{5}\right)^{2m(2n-3)}$.

**Proof.** 1. First multiplicative K Banhatti index of $TUC_4\,[m,n]$ is

$$\begin{aligned}BII_1\,(TUC_4\,[m,n]) &= \left[(3+4)^{(2m)}\times(3+4)^{(2m)}\right]\times\left[(3+5)^{(2m)}\times(4+5)^{(2m)}\right]\\&\quad\times\left[(4+6)^{(m(2n-3))}\times(4+6)^{(m(2n-3))}\right]\\&= 7^{4m}\times 8^{2m}\times 9^{2m}\times 10^{2m(2n-3)}.\end{aligned}$$

2. Second multiplicative K Banhatti index of $TUC_4\,[m,n]$ is

$$\begin{aligned}BII_2\,(TUC_4\,[m,n]) &= \left[(3\times4)^{(2m)}\times(3\times4)^{(2m)}\right]\times\left[(3\times5)^{(2m)}\times(4\times5)^{(2m)}\right]\\&\quad\times\left[(4\times6)^{(m(2n-3))}\times(4\times6)^{(m(2n-3))}\right]\\&= 12^{4m}\times 15^{2m}\times 20^{2m}\times 24^{2m(2n-3)}.\end{aligned}$$

3. First multiplicative K hyper-Banhatti index of $TUC_4\,[m,n]$ is

$$\begin{aligned}HBII_1\,(TUC_4\,[m,n]) &= \left[\left((3+4)^2\right)^{(2m)}\times\left((3+4)^2\right)^{(2m)}\right]\times\left[\left((3+5)^2\right)^{(2m)}\times\left((4+5)^2\right)^{(2m)}\right]\\&\quad\times\left[\left((4+6)^2\right)^{(m(2n-3))}\times\left((4+6)^2\right)^{(m(2n-3))}\right]\\&= 7^{8m}\times 8^{4m}\times 9^{4m}\times 10^{4m(2n-3)}.\end{aligned}$$

4. Second multiplicative K hyper-Banhatti index of $TUC_4\,[m,n]$ is

$$\begin{aligned}HBII_2\,(TUC_4\,[m,n]) &= \left[\left((3\times4)^2\right)^{(2m)}\times\left((3\times4)^2\right)^{(2m)}\right]\times\left[\left((3\times5)^2\right)^{(2m)}\times\left((4\times5)^2\right)^{(2m)}\right]\\&\quad\times\left[\left((4\times6)^2\right)^{(m(2n-3))}\times\left((4\times6)^2\right)^{(m(2n-3))}\right]\\&= 12^{8m}\times 15^{4m}\times 20^{4m}\times 24^{4m(2n-3)}.\end{aligned}$$

5. Multiplicative K harmonic Banhatti index of $TUC_4\,[m,n]$ is

$$\begin{aligned}HII_b\,(TUC_4\,[m,n]) &= \left[\left(\frac{2}{3+4}\right)^{(2m)}\times\left(\frac{2}{3+4}\right)^{(2m)}\right]\times\left[\left(\frac{2}{3+5}\right)^{(2m)}\times\left(\frac{2}{4+5}\right)^{(2m)}\right]\\&\quad\times\left[\left(\frac{2}{4+6}\right)^{m(2n-3)}\times\left(\frac{2}{4+6}\right)^{m(2n-3)}\right]\\&= \left(\frac{2}{7}\right)^{4m}\times\left(\frac{1}{4}\right)^{2m}\times\left(\frac{2}{9}\right)^{2m}\times\left(\frac{1}{5}\right)^{2m(2n-3)}.\end{aligned}$$

**Table 2.** dge degree partition of $TUZC_6[m,n]$.

| $d_G(u), d_G(v) : e = uv \in E(G)$ | $(3,3)$ | $(2,3)$ |
|---|---|---|
| $d_G(e)$ | 4 | 3 |
| Number of edges | $3mn - 2m$ | $4m$ |

□

### 2.2. Banhatti indices of $TUZC_6[m,n]$

The zigzag nanotube $TUZC_6[m,n]$, where $m$ is the number of hexagons in the first row and $n$ is the number of hexagons in the first column. The molecular structures of $TUZC_6[m,n]$ can be referred to Figure 2.

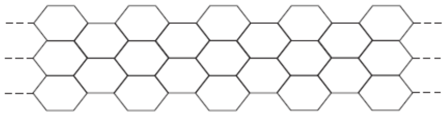

**Figure 2.** The 3D lattice of the zigzag $TUZC_6[10,7]$.

**Theorem 3.** *Let G be the zigzag nanotube $TUZC_6[m,n]$. Then we have*

1. $B_1(TUZC_6[m,n]) = 42mn + 16m$.
2. $B_2(TUZC_6[m,n]) = 72mn + 12m$.
3. $HB_1(TUZC_6[m,n]) = 294mn + 48m$.
4. $HB_2(TUZC_6[m,n]) = 864mn - 108m$.
5. $H_b(TUZC_6[m,n]) = \frac{12}{7}mn - \frac{188}{108}m$.

**Proof.** Let $G = TUZC_6[m,n]$. The edge set of $TUZC_6[m,n]$ can be divided into following classes:
$E_5 = \{uv \in E(G) : d_G(u) = 2, d_G(v) = 3\}$,
$E_6 = \{uv \in E(G) : d_G(u) = d_G(v) = 3\}$,
such that $|E_5| = 4m$ and $|E_6| = 3mn - 2m$.
The edge degree partition is given in Table 2. Now

1. First K Banhatti index of $TUZC_6[m,n]$ is

$$\begin{aligned} B_1(TUZC_6[m,n]) &= (3mn-2m)[(3+4)+(3+4)] + (4m)[(2+3)+(3+3)] \\ &= 42mn + 16m. \end{aligned}$$

2. Second K Banhatti index of $TUZC_6[m,n]$ is

$$\begin{aligned} B_2(TUZC_6[m,n]) &= (3mn-2m)[(3\times 4)+(3\times 4)] + (4m)[(2\times 3)+(3\times 3)] \\ &= 72mn + 12m. \end{aligned}$$

3. First K hyper-Banhatti index of $TUZC_6[m,n]$ is

$$\begin{aligned} HB_1(TUZC_6[m,n]) &= (3mn-2m)\left[(3+4)^2+(3+4)^2\right] + (4m)\left[(2+3)^2+(3+3)^2\right] \\ &= 294mn + 48m. \end{aligned}$$

4. Second K hyper-Banhatti index of $TUZC_6[m,n]$ is

$$\begin{aligned} HB_2(TUZC_6[m,n]) &= (3mn-2m)\left[(3\times 4)^2+(3\times 4)^2\right] + (4m)\left[(2\times 3)^2+(3\times 3)^2\right] \\ &= 864mn - 108m. \end{aligned}$$

5. K harmonic Banhatti index of $TUZC_6\,[m,n]$ is

$$\begin{aligned} H_b\left(TUZC_6\,[m,n]\right) &= (3mn-2m)\left[\left(\frac{2}{3+4}\right)+\left(\frac{2}{3+4}\right)\right]+(4m)\left[\left(\frac{2}{2+3}\right)+\left(\frac{2}{3+3}\right)\right] \\ &= \frac{12}{7}mn-\frac{188}{108}m. \end{aligned}$$

□

**Theorem 4.** *Let G be the zigzag nanotube $TUZC_6\,[m,n]$. Then we have*

1. $BII_1\left(TUZC_6\,[m,n]\right)=5^{4m}\times 6^{4m}\times 7^{2m(3n-2)}$.
2. $BII_2\left(TUZC_6\,[m,n]\right)=3^{8m}\times 6^{4m}\times 12^{2m(3n-2)}$.
3. $HBII_1\left(TUZC_6\,[m,n]\right)=5^{8m}\times 6^{8m}\times 7^{4m(3n-2)}$.
4. $HBII_2\left(TUZC_6\,[m,n]\right)=3^{16m}\times 6^{8m}\times 12^{4m(3n-2)}$.
5. $HII_b\left(TUZC_6\,[m,n]\right)=\left(\frac{1}{3}\right)^{4m}\times\left(\frac{2}{5}\right)^{4m}\times\left(\frac{2}{7}\right)^{2m(3n-2)}$.

**Proof.** 1. First multiplicative K Banhatti index of $TUZC_6\,[m,n]$ is

$$\begin{aligned} BII_1\left(TUZC_6\,[m,n]\right) &= \left[(3+4)^{(3mn-2m)}\times(3+4)^{(3mn-2m)}\right]\times\left[(2+3)^{(4m)}\times(3+3)^{(4m)}\right] \\ &= 5^{4m}\times 6^{4m}\times 7^{2m(3n-2)}. \end{aligned}$$

2. Second multiplicative K Banhatti index of $TUZC_6\,[m,n]$ is

$$\begin{aligned} BII_2\left(TUZC_6\,[m,n]\right) &= \left[(3\times 4)^{(3mn-2m)}\times(3\times 4)^{(3mn-2m)}\right]\times\left[(2\times 3)^{(4m)}\times(3\times 3)^{(4m)}\right] \\ &= 3^{8m}\times 6^{4m}\times 12^{2m(3n-2)}. \end{aligned}$$

3. First multiplicative K hyper-Banhatti index of $TUZC_6\,[m,n]$ is

$$\begin{aligned} HBII_1\left(TUZC_6\,[m,n]\right) &= \left[\left((3+4)^2\right)^{(3mn-2m)}\times\left((3+4)^2\right)^{(3mn-2m)}\right] \\ &\quad\times\left[\left((2+3)^2\right)^{(4m)}\times\left((3+3)^2\right)^{(4m)}\right] \\ &= 5^{8m}\times 6^{8m}\times 7^{4m(3n-2)}. \end{aligned}$$

4. Second multiplicative K hyper-Banhatti index of $TUZC_6\,[m,n]$ is

$$\begin{aligned} HBII_2\left(TUZC_6\,[m,n]\right) &= \left[\left((3\times 4)^2\right)^{(3mn-2m)}\times\left((3\times 4)^2\right)^{(3mn-2m)}\right] \\ &\quad\times\left[\left((2\times 3)^2\right)^{(4m)}\times\left((3\times 3)^2\right)^{(4m)}\right] \\ &= 3^{16m}\times 6^{8m}\times 12^{4m(3n-2)}. \end{aligned}$$

5. Multiplicative K harmonic Banhatti index of $TUZC_6\,[m,n]$ is

$$\begin{aligned} HII_b\left(TUZC_6\,[m,n]\right) &= \left[\left(\frac{2}{3+4}\right)^{(3mn-2m)}\times\left(\frac{2}{3+4}\right)^{(3mn-2m)}\right]\times\left[\left(\frac{2}{2+3}\right)^{(4m)}\times\left(\frac{2}{3+3}\right)^{(4m)}\right] \\ &= \left(\frac{1}{3}\right)^{4m}\times\left(\frac{2}{5}\right)^{4m}\times\left(\frac{2}{7}\right)^{2m(3n-2)}. \end{aligned}$$

□

**Table 3.** Edge degree partition of $TUAC_6[m,n]$.

| $d_G(u), d_G(v) : e = uv \in E(G)$ | $(2,2)$ | $(3,3)$ | $(2,3)$ |
|---|---|---|---|
| $d_G(e)$ | 2 | 4 | 3 |
| Number of edges | $m$ | $3mn - m$ | $2m$ |

### 2.3. Banhatti indices of $TUAC_6[m,n]$

The armchair nanotube $TUAC_6[m,n]$, where $m$ is the number of hexagons in the first row and $n$ is the number of hexagons in the first column. The molecular structures of $TUAC_6[m,n]$ can be referred to Figure 3.

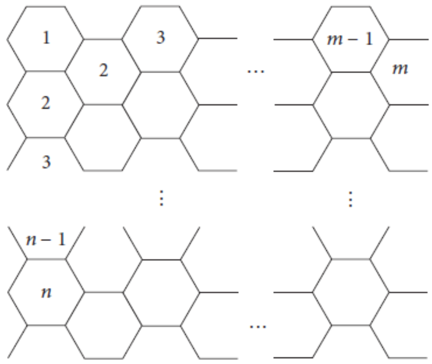


**Figure 3.** The 3D lattice of the armchair $TUAC_6[m,n]$.

**Theorem 5.** *Let G be the armchair nanotube $TUAC_6[m,n]$. Then we have*

1. $B_1(TUAC_6[m,n]) = 42mn + 16m$.
2. $B_2(TUAC_6[m,n]) = 72mn + 14m$.
3. $HB_1(TUAC_6[m,n]) = 294mn + 56m$.
4. $HB_2(TUAC_6[m,n]) = 864mn - 22m$.
5. $H_b(TUAC_6[m,n]) = \frac{12}{7}mn + \frac{199}{105}m$.

**Proof.** Let $G = TUAC_6[m,n]$. we have edge set of $TUAC_6[m,n]$ can be partitioned as follows:
$E_4 = \{uv \in E(G) : d_G(u) = d_G(v) = 2\}$,
$E_5 = \{uv \in E(G) : d_G(u) = 2, d_G(v) = 3\}$,
$E_6 = \{uv \in E(G) : d_G(u) = d_G(v) = 3\}$, such that $|E_4| = m$, $|E_5| = 2m$ and $|E_6| = 3mn - m$.
The edge degree partition is given in Table 3. Now

1. First K Banhatti index of $TUAC_6\,[m,n]$ is

$$\begin{aligned} B_1\left(TUAC_6\,[m,n]\right) &= (m)\left[(2+2)+(2+2)\right] + (3mn-m)\left[(3+4)+(3+4)\right] \\ &\quad + (2m)\left[(2+3)+(3+3)\right] \\ &= 42mn + 16m. \end{aligned}$$

2. Second K Banhatti index of $TUAC_6\,[m,n]$ is

$$\begin{aligned} B_2\left(TUAC_6\,[m,n]\right) &= (m)\left[(2\times 2)+(2\times 2)\right] + (3mn-m)\left[(3\times 4)+(3\times 4)\right] \\ &\quad + (2m)\left[(2\times 3)+(3\times 3)\right] \\ &= 72mn + 14m. \end{aligned}$$

3. First K hyper-Banhatti index of $TUAC_6\,[m,n]$ is

$$\begin{aligned} HB_1\left(TUAC_6\,[m,n]\right) &= (m)\left[(2+2)^2+(2+2)^2\right]+(3mn-m)\left[(3+4)^2+(3+4)^2\right] \\ &\quad +(2m)\left[(2+3)^2+(3+3)^2\right] \\ &= 294mn+56m. \end{aligned}$$

4. Second K hyper-Banhatti index of $TUAC_6\,[m,n]$ is

$$\begin{aligned} HB_2\left(TUAC_6\,[m,n]\right) &= (m)\left[(2\times 2)^2+(2\times 2)^2\right]+(3mn-m)\left[(3\times 4)^2+(3\times 4)^2\right] \\ &\quad +(2m)\left[(2\times 3)^2+(3\times 3)^2\right] \\ &= 864mn-22m. \end{aligned}$$

5. K harmonic Banhatti index of $TUAC_6\,[m,n]$ is

$$\begin{aligned} H_b\left(TUAC_6\,[m,n]\right) &= (m)\left[\left(\frac{2}{2+2}\right)+\left(\frac{2}{2+2}\right)\right]+(3mn-m)\left[\left(\frac{2}{3+4}\right)+\left(\frac{2}{3+4}\right)\right] \\ &\quad +(2m)\left[\left(\frac{2}{2+3}\right)+\left(\frac{2}{3+3}\right)\right] \\ &= \frac{12}{7}mn+\frac{199}{105}m. \end{aligned}$$

□

**Theorem 6.** *Let G be the armchair nanotube $TUAC_6\,[m,n]$. Then we have*

1. $BII_1\left(TUAC_6\,[m,n]\right)=2^{4m}\times 5^{2m}\times 6^{2m}\times 7^{2m(3n-1)}$.
2. $BII_2\left(TUAC_6\,[m,n]\right)=2^{4m}\times 3^{4m}\times 6^{2m}\times 12^{2m(3n-1)}$.
3. $HBII_1\left(TUAC_6\,[m,n]\right)=2^{8m}\times 5^{4m}\times 6^{4m}\times 7^{4m(3n-1)}$.
4. $HBII_2\left(TUAC_6\,[m,n]\right)=2^{8m}\times 3^{8m}\times 6^{4m}\times 7^{4m(3n-1)}$.
5. $HII_b\left(TUAC_6\,[m,n]\right)=\left(\frac{1}{2}\right)^{2m}\times\left(\frac{1}{3}\right)^{2m}\times\left(\frac{2}{5}\right)^{2m}\times\left(\frac{2}{7}\right)^{2m(3n-1)}$.

**Proof.** 1. First multiplicative K Banhatti index of $TUAC_6\,[m,n]$ is

$$\begin{aligned} BII_1\left(TUAC_6\,[m,n]\right) &= \left[(2+2)^{(m)}\times(2+2)^{(m)}\right]\times\left[(3+4)^{(3mn-m)}\times(3+4)^{(3mn-m)}\right] \\ &\quad \times\left[(2+3)^{(2m)}\times(3+3)^{(2m)}\right] \\ &= 2^{4m}\times 5^{2m}\times 6^{2m}\times 7^{2m(3n-1)}. \end{aligned}$$

2. Second multiplicative K Banhatti index of $TUAC_6\,[m,n]$ is

$$\begin{aligned} BII_1\left(TUAC_6\,[m,n]\right) &= \left[(2\times 2)^{(m)}\times(2\times 2)^{(m)}\right]\times\left[(3\times 4)^{(3mn-m)}\times(3\times 4)^{(3mn-m)}\right] \\ &\quad \times\left[(2\times 3)^{(2m)}\times(3\times 3)^{(2m)}\right] \\ &= 2^{4m}\times 3^{4m}\times 6^{2m}\times 12^{2m(3n-1)}. \end{aligned}$$

3. First multiplicative K hyper-Banhatti index of $TUAC_6\,[m,n]$ is

$$\begin{aligned} HBII_1\left(TUAC_6\,[m,n]\right) &= \left[\left((2+2)^2\right)^{(m)}\times\left((2+2)^2\right)^{(m)}\right] \\ &\quad \times\left[\left((3+4)^2\right)^{(3mn-m)}\times\left((3+4)^2\right)^{(3mn-m)}\right] \end{aligned}$$

$$\times\left[\left((2+3)^2\right)^{(2m)}\times\left((3+3)^2\right)^{(2m)}\right]$$
$$= 2^{8m}\times 5^{4m}\times 6^{4m}\times 7^{4m(3n-1)}.$$

4. Second multiplicative K hyper-Banhatti index of $TUAC_6\,[m,n]$ is

$$\begin{aligned}HBII_2\left(TUAC_6\,[m,n]\right) &= \left[\left((2\times 2)^2\right)^{(m)}\times\left((2\times 2)^2\right)^{(m)}\right]\\ &\times\left[\left((3\times 4)^2\right)^{(3mn-m)}\times\left((3\times 4)^2\right)^{(3mn-m)}\right]\\ &\times\left[\left((2\times 3)^2\right)^{(2m)}\times\left((3\times 3)^2\right)^{(2m)}\right]\\ &= 2^{8m}\times 3^{8m}\times 6^{4m}\times 12^{4m(3n-1)}.\end{aligned}$$

5. Multiplicative K harmonic Banhatti index of $TUAC_6\,[m,n]$ is

$$\begin{aligned}HII_b\left(TUAC_6\,[m,n]\right) &= \left[\left(\frac{2}{2+2}\right)^{(m)}\times\left(\frac{2}{2+2}\right)^{(m)}\right]\times\left[\left(\frac{2}{3+4}\right)^{(3mn-m)}\times\left(\frac{2}{3+4}\right)^{(3mn-m)}\right]\\ &\times\left[\left(\frac{2}{2+3}\right)^{(2m)}\times\left(\frac{2}{3+3}\right)^{(2m)}\right]\\ &= \left(\frac{1}{2}\right)^{2m}\times\left(\frac{1}{3}\right)^{2m}\times\left(\frac{2}{5}\right)^{2m}\times\left(\frac{2}{7}\right)^{2m(3n-1)}.\end{aligned}$$

□

### 2.4. Banhatti indices of $NPHX[p,q]$

H-Naphtalenic nanotubes $NPHX[p,q]$ (where $p$ and $q$ are denoted as the number of pairs of hexagons in first row and the number of alternative hexagons in a column, respectively) are a trivalent decoration with sequence of $C_6$, $C_6$, $C_4$, $C_6$, $C_6$, $C_4$, ... in the first row and a sequence of $C_6$, $C_8$, $C_6$, $C_8$, ... in the other rows. In other words, this nanolattice can be considered as a plane tiling of $C_4$, $C_6$, and $C_8$. Therefore, this class of tiling can cover either a cylinder or a torus 4.

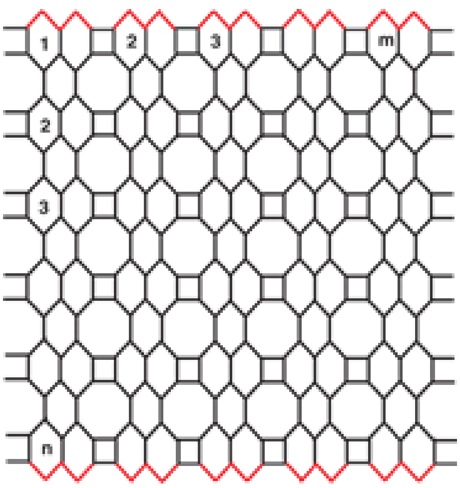


**Figure 4.** Naphthylenic nanotubes.

**Theorem 7.** *Let G be the H-Naphtalenic nanotube $NPHX[m,n]$. Then we have*

1. $B_1(NPHX[m,n]) = 210mn - 52m$.

**Table 4.** Edge Edge degree partition of $NPHX[m,n]$.

| $d_G(u), d_G(v) : e = uv \in E(G)$ | $(3,3)$ | $(2,3)$ |
|---|---|---|
| $d_G(e)$ | 4 | 3 |
| Number of edges | $15mn - 10m$ | $8m$ |

2. $B_2(NPHX[m,n]) = 360mn - 120m$.
3. $HB_1(NPHX[m,n]) = 1470mn - 492m$.
4. $HB_2(NPHX[m,n]) = 4320mn - 1944m$.
5. $H_b(NPHX[m,n]) = \frac{60}{7}mn - \frac{33}{7}m$.

**Proof.** Let $G = NPHX[m,n]$, then we have edge division of edge set $E(NPHX[m,n])$ as follows: $E_5 = \{uv \in E(G) : d_G(u) = 2, d_G(v) = 3\}$,
$E_6 = \{uv \in E(G) : d_G(u) = d_G(v) = 3\}$,
such that $|E_5| = 8m$ and $|E_6| = 15mn - 10m$.
The edge degree partition is given in Table 4. Now

1. First K Banhatti index of $NPHX[m,n]$ is

$$\begin{aligned} B_1(NPHX[m,n]) &= (15mn - 10m)[(3+4) + (3+4)] + (8m)[(2+3) + (3+3)] \\ &= 210mn - 52m. \end{aligned}$$

2. Second K Banhatti index of $NPHX[m,n]$ is

$$\begin{aligned} B_2(NPHX[m,n]) &= (15mn - 10m)[(3\times 4) + (3\times 4)] + (8m)[(2\times 3) + (3\times 3)] \\ &= 360mn - 120m. \end{aligned}$$

3. First K hyper-Banhatti index of $NPHX[m,n]$ is

$$\begin{aligned} HB_1(NPHX[m,n]) &= (15mn - 10m)\left[(3+4)^2 + (3+4)^2\right] + (8m)\left[(2+3)^2 + (3+3)^2\right] \\ &= 1470mn - 492m. \end{aligned}$$

4. Second K hyper-Banhatti index of $NPHX[m,n]$ is

$$\begin{aligned} HB_2(NPHX[m,n]) &= (15mn - 10m)\left[(3\times 4)^2 + (3\times 4)^2\right] + (8m)\left[(2\times 3)^2 + (3\times 3)^2\right] \\ &= 4320mn - 1944m. \end{aligned}$$

5. K harmonic Banhatti index of $NPHX[m,n]$ is

$$\begin{aligned} H_b(NPHX[m,n]) &= (15mn - 10m)\left[\left(\frac{2}{3+4}\right) + \left(\frac{2}{3+4}\right)\right] + (8m)\left[\left(\frac{2}{2+3}\right) + \left(\frac{2}{3+3}\right)\right] \\ &= \frac{60}{7}mn - \frac{33}{7}m. \end{aligned}$$

□

**Theorem 8.** *Let G be the H-Naphtalenic nanotube $NPHX[m,n]$. Then we have*

1. $BII_1(NPHX[m,n]) = 5^{8m} \times 6^{8m} \times 7^{10m(3n-2)}$.
2. $BII_2(NPHX[m,n]) = 6^{8m} \times 9^{8m} \times 12^{10m(3n-2)}$.
3. $HBII_1(NPHX[m,n]) = 5^{16m} \times 6^{16m} \times 7^{20m(3n-2)}$.
4. $HBII_2(NPHX[m,n]) = 6^{16m} \times 9^{16m} \times 12^{20m(3n-2)}$.
5. $HII_b(NPHX[m,n]) = \left(\frac{2}{5}\right)^{8m} \times \left(\frac{1}{3}\right)^{8m} \times \left(\frac{2}{7}\right)^{10m(3n-2)}$.

**Proof.** 1. First multiplicative K Banhatti index of $NPHX[m,n]$ is

$$\begin{aligned} BII_1(NPHX[m,n]) &= \left[(3+4)^{(15mn-10m)} \times (3+4)^{(15mn-10m)}\right] \times \left[(2+3)^{(8m)} \times (3+3)^{(8m)}\right] \\ &= 5^{8m} \times 6^{8m} \times 7^{10m(3n-2)}. \end{aligned}$$

2. Second multiplicative K Banhatti index of $NPHX[m,n]$ is

$$\begin{aligned} BII_2(NPHX[m,n]) &= \left[(3\times 4)^{(15mn-10m)} \times (3\times 4)^{(15mn-10m)}\right] \times \left[(2\times 3)^{(8m)} \times (3\times 3)^{(8m)}\right] \\ &= 6^{8m} \times 9^{8m} \times 12^{10m(3n-2)}. \end{aligned}$$

3. First multiplicative K hyper-Banhatti index of $NPHX[m,n]$ is

$$\begin{aligned} HBII_1(NPHX[m,n]) &= \left[\left((3+4)^2\right)^{(15mn-10m)} \times \left((3+4)^2\right)^{(15mn-10m)}\right] \\ &\quad \times \left[\left((2+3)^2\right)^{(8m)} \times \left((3+3)^2\right)^{(8m)}\right] \\ &= 5^{16m} \times 6^{16m} \times 7^{20m(3n-2)}. \end{aligned}$$

4. Second multiplicative K hyper-Banhatti index of $NPHX[m,n]$ is

$$\begin{aligned} HBII_2(NPHX[m,n]) &= \left[\left((3+4)^2\right)^{(15mn-10m)} \times \left((3+4)^2\right)^{(15mn-10m)}\right] \\ &\quad \times \left[\left((2+3)^2\right)^{(8m)} \times \left((3+3)^2\right)^{(8m)}\right] \\ &= 6^{16m} \times 9^{16m} \times 12^{20m(3n-2)}. \end{aligned}$$

5. Multiplicative K harmonic Banhatti index of $NPHX[m,n]$ is

$$\begin{aligned} HII_b(NPHX[m,n]) &= \left[\left(\frac{2}{3+4}\right)^{(15mn-10m)} \times \left(\frac{2}{3+4}\right)^{(15mn-10m)}\right] \\ &\quad \times \left[\left(\frac{2}{2+3}\right)^{(8m)} \times \left(\frac{2}{3+3}\right)^{(8m)}\right] \\ &= \left(\frac{2}{5}\right)^{8m} \times \left(\frac{1}{3}\right)^{8m} \times \left(\frac{2}{7}\right)^{10m(3n-2)}. \end{aligned}$$

□

### 2.5. Banhatti indices of $SC_5C_7[p,q]$

In nanoscience, $SC_5C_7[p,q]$ (where $p$ and $q$ express the number of heptagons in each row and the number of periods in whole lattice respectively) nanotube is a class of $C_5C_7$-net which is yielded by alternating $C_5$ and $C_7$. The standard tiling of $C_5$ and $C_7$ can cover either a cylinder or a torus and each period of $SC_5C_7[p,q]$ consisted of three rows (more details on pth period can be referred to in Figure 5.

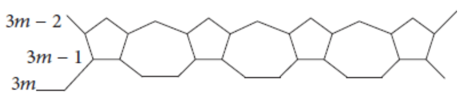


**Figure 5.** ith period of $SC_5C_7[p,q]$ nanotube.

**Theorem 9.** *Let G be the $SC_5C_7[p,q]$ nanotube. Then we have*

**Table 5.** Edge degree partition of $SC_5C_7[p,q]$.

| $d_G(u), d_G(v) : e = uv \in E(G)$ | $(2,2)$ | $(3,3)$ | $(2,3)$ |
|---|---|---|---|
| $d_G(e)$ | 2 | 4 | 3 |
| Number of edges | $p$ | $12pq - 9p$ | $6p$ |

1. $B_1(SC_5C_7[p,q]) = 168pq - 52p$.
2. $B_2(SC_5C_7[p,q]) = 288pq - 118p$.
3. $HB_1(SC_5C_7[p,q]) = 1176pq - 484p$.
4. $HB_2(SC_5C_7[p,q]) = 3456pq - 1858p$.
5. $H_b(SC_5C_7[p,q]) = \frac{48}{7}pq + \frac{9}{35}p$.

**Proof.** Let $G = SC_5C_7[p,q]$. There are following three types of edges of $SC_5C_7[p,q]$, based on the degree of end vertices $E_4(G) = \{uv \in E(G) : d_G(u) = d_G(v) = 2\}$,
$E_5(G) = \{uv \in E(G) : d_G(u) = 2, d_G(v) = 3\}$,
$E_6(G) = \{uv \in E(G) : d_G(u) = d_G(v) = 3\}$,
such that
$|E_4(G)| = p$,$|E_5(G)| = 6p$ and $|E_6(G)| = 12pq - 9p$. The edge degree partition is given in Table 5.
Now

1. First K Banhatti index of $SC_5C_7\,[p,q]$ is

$$\begin{aligned} B_1\left(SC_5C_7\,[p,q]\right) &= (p)\,[(2+2)+(2+2)] + (12pq-9p)\,[(3+4)+(3+4)] + (6p)\,[(2+3)+(3+3)] \\ &= 168pq - 52p. \end{aligned}$$

2. Second K Banhatti index $SC_5C_7\,[p,q]$ is

$$\begin{aligned} B_2\left(SC_5C_7\,[p,q]\right) &= (p)\,[(2\times 2)+(2\times 2)] + (12pq-9p)\,[(3\times 4)+(3\times 4)] + (6p)\,[(2\times 3)+(3\times 3)] \\ &= 288pq - 118p. \end{aligned}$$

3. First K hyper-Banhatti index $SC_5C_7\,[p,q]$ is

$$\begin{aligned} HB_1\left(SC_5C_7\,[p,q]\right) &= (p)\left[(2+2)^2+(2+2)^2\right] + (12pq-9p)\left[(3+4)^2+(3+4)^2\right] \\ &\quad + (6p)\left[(2+3)^2+(3+3)^2\right] \\ &= 1176pq - 484p. \end{aligned}$$

4. Second K hyper-Banhatti index $SC_5C_7\,[p,q]$ is

$$\begin{aligned} HB_2\left(SC_5C_7\,[p,q]\right) &= (p)\left[(2\times 2)^2+(2\times 2)^2\right] + (12pq-9p)\left[(3\times 4)^2+(3\times 4)^2\right] \\ &\quad + (6p)\left[(2\times 3)^2+(3\times 3)^2\right] \\ &= 3456pq - 1858p. \end{aligned}$$

5. K harmonic Banhatti index $SC_5C_7\,[p,q]$ is

$$\begin{aligned} H_b\left(SC_5C_7\,[p,q]\right) &= (p)\left[\left(\frac{2}{2+2}\right)+\left(\frac{2}{2+2}\right)\right] + (12pq-9p)\left[\left(\frac{2}{3+4}\right)+\left(\frac{2}{3+4}\right)\right] \\ &\quad + (6p)\left[\left(\frac{2}{2+3}\right)+\left(\frac{2}{3+3}\right)\right] \\ &= \frac{48}{7}pq + \frac{9}{35}p. \end{aligned}$$

□

**Theorem 10.** *Let G be the $SC_5C_7\,[p,q]$ nanotube. Then we have*

1. $BII_1\left(SC_5C_7\left[p,q\right]\right) = 4^{2p} \times 5^p \times 6^p \times 7^{6p(4q-3)}$.
2. $BII_2\left(SC_5C_7\left[p,q\right]\right) = 4^{2p} \times 6^p \times 9^p \times 12^{6p(4q-3)}$.
3. $HBII_1\left(SC_5C_7\left[p,q\right]\right) = 4^{4p} \times 5^{2p} \times 6^{2p} \times 7^{12p(4q-3)}$.
4. $HBII_2\left(SC_5C_7\left[p,q\right]\right) = 4^{4p} \times 6^{2p} \times 9^{2p} \times 12^{12p(4q-3)}$.
5. $HII_b\left(SC_5C_7\left[p,q\right]\right) = \left(\frac{1}{2}\right)^{2p} \times \left(\frac{2}{5}\right)^p \times \left(\frac{1}{3}\right)^p \times \left(\frac{2}{7}\right)^{6p(4q-3)}$.

**Proof.** Using Table 5, we have

1. First multiplicative K Banhatti index $SC_5C_7\left[p,q\right]$ is

$$\begin{aligned} BII_1\left(SC_5C_7\left[p,q\right]\right) &= \left[(2+2)^{(p)} \times (2+2)^{(p)}\right] \times \left[(3+4)^{(12pq-9p)} \times (3+4)^{(12pq-9p)}\right] \\ &\quad \times \left[(2+3)^{(6p)} \times (3+3)^{(6p)}\right] \\ &= 4^{2p} \times 5^p \times 6^p \times 7^{6p(4q-3)}. \end{aligned}$$

2. Second multiplicative K Banhatti index $SC_5C_7\left[p,q\right]$ is

$$\begin{aligned} BII_2\left(SC_5C_7\left[p,q\right]\right) &= \left[(2\times 2)^{(p)} \times (2\times 2)^{(p)}\right] \times \left[(3\times 4)^{(12pq-9p)} \times (3\times 4)^{(12pq-9p)}\right] \\ &\quad \times \left[(2\times 3)^{(6p)} \times (3\times 3)^{(6p)}\right] \\ &= 4^{2p} \times 6^p \times 9^p \times 12^{6p(4q-3)}. \end{aligned}$$

3. First multiplicative K hyper-Banhatti index $SC_5C_7\left[p,q\right]$ is

$$\begin{aligned} HBII_1\left(SC_5C_7\left[p,q\right]\right) &= \left[\left((2+2)^2\right)^{(p)} \times \left((2+2)^2\right)^{(p)}\right] \times \left[\left((3+4)^2\right)^{(12pq-9p)} \times \left((3+4)^2\right)^{(12pq-9p)}\right] \\ &\quad \times \left[\left((2+3)^2\right)^{(6p)} \times \left((3+3)^2\right)^{(6p)}\right] \\ &= 4^{4p} \times 5^{2p} \times 6^{2p} \times 7^{12p(4q-3)}. \end{aligned}$$

4. Second multiplicative K hyper-Banhatti index $SC_5C_7\left[p,q\right]$ is

$$\begin{aligned} HBII_2\left(SC_5C_7\left[p,q\right]\right) &= \left[\left((2\times 2)^2\right)^{(p)} \times \left((2\times 2)^2\right)^{(p)}\right] \times \left[\left((3\times 4)^2\right)^{(12pq-9p)} \times \left((3\times 4)^2\right)^{(12pq-9p)}\right] \\ &\quad \times \left[\left((2\times 3)^2\right)^{(6p)} \times \left((3\times 3)^2\right)^{(6p)}\right] \\ &= 4^{4p} \times 6^{2p} \times 9^{2p} \times 12^{12p(4q-3)}. \end{aligned}$$

5. Multiplicative K harmonic Banhatti index $SC_5C_7\left[p,q\right]$ is

$$\begin{aligned} HII_b\left(SC_5C_7\left[p,q\right]\right) &= \left[\left(\frac{2}{2+2}\right)^{(p)} \times \left(\frac{2s}{2+2}\right)^{(p)}\right] \times \left[\left(\frac{2}{3+4}\right)^{(12pq-9p)} \times \left(\frac{2}{3+4}\right)^{(12pq-9p)}\right] \\ &\quad \times \left[\left(\frac{2}{2+3}\right)^{(6p)} \times \left(\frac{2}{3+3}\right)^{(6p)}\right] \\ &= \left(\frac{1}{2}\right)^{2p} \times \left(\frac{2}{5}\right)^{p} \times \left(\frac{1}{3}\right)^{p} \times \left(\frac{2}{7}\right)^{6p(4q-3)}. \end{aligned}$$

□

### 2.6. Banhatti indices of $VC_5C_7[p,q]$

The molecular graphs of carbon nanotubes $VC_5C_7[p,q]$ is shown in Figure 6. The structures of this nanotubes consist of cycles $C_5$ and $C_7$ ($C_5C_7$ net which is a trivalent decoration constructed by alternating $C_5$ and $C_7$) by different compound. It can cover either a cylinder or a torus. The 2 dimensional lattice of $VC_5C_7[p,q]$ is shown in Figure 7.

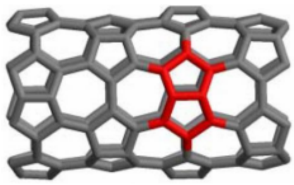

**Figure 6.** Molecular graph of $VC_5C_7[p,q]$.

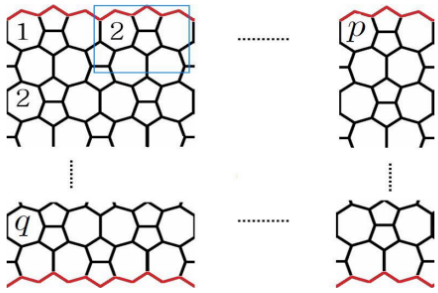


**Figure 7.** 2 dimensional lattice of $VC_5C_7[p,q]$.

**Theorem 11.** *Let G be the $VC_5C_7[p,q]$ nanotube. Then we have*

1. $B_1(VC_5C_7[p,q]) = 336pq + 48p$.
2. $B_2(VC_5C_7[p,q]) = 576pq + 36p$.
3. $HB_1(VC_5C_7[p,q]) = 2352pq + 144p$.
4. $HB_2(VC_5C_7[p,q]) = 6912pq - 324p$.
5. $H_b(VC_5C_7[p,q]) = \frac{96}{7}pq + \frac{188}{35}p$.

**Proof.** Let $G = VC_5C_7[p,q]$. Then the edge set of $VC_5C_7[p,q]$ can be partitioned into following two classes:
$E_6 = \{uv \in E(G) : d_G(u) = d_G(v) = 3\}$,
$E_5 = \{uv \in E(G) : d_G(u) = 2, d_G(v) = 3\}$,
such that $|E_6| = 24pq - 6p$ and $|E_5| = 12p$.
The edge degree partition is given in Table 6. Now

1. First K Banhatti index of $VC_5C_7[p,q]$ is

$$\begin{aligned} B_1(VC_5C_7[p,q]) &= (24pq - 6p)[(3+4)+(3+4)] + (12p)[(2+3)+(3+3)] \\ &= 336pq + 48p. \end{aligned}$$

2. Second K Banhatti index of $VC_5C_7[p,q]$ is

$$\begin{aligned} B_2(VC_5C_7[p,q]) &= (24pq - 6p)[(3\times 4)+(3\times 4)] + (12p)[(2\times 3)+(3\times 3)] \\ &= 576pq + 36p. \end{aligned}$$

3. First K hyper-Banhatti index of $VC_5C_7[p,q]$ is

$$\begin{aligned} HB_1(VC_5C_7[p,q]) &= (24pq - 6p)\left[(3+4)^2+(3+4)^2\right] + (12p)\left[(2+3)^2+(3+3)^2\right] \\ &= 2352pq + 144p. \end{aligned}$$

**Table 6.** Edge degree partition of $VC_5C_7[p,q]$.

| $d_G(u), d_G(v) : e = uv \in E(G)$ | $(3,3)$ | $(2,3)$ |
|---|---|---|
| $d_G(e)$ | 4 | 3 |
| Number of edges | $24pq - 6p$ | $12p$ |

4. Second K hyper-Banhatti index of $VC_5C_7[p,q]$ is

$$\begin{aligned} HB_2(VC_5C_7[p,q]) &= (24pq-6p)\left[(3\times 4)^2+(3\times 4)^2\right]+(12p)\left[(2\times 3)^2+(3\times 3)^2\right] \\ &= 6912pq-324p. \end{aligned}$$

5. K harmonic Banhatti index of $VC_5C_7[p,q]$ is

$$\begin{aligned} H_b(VC_5C_7[p,q]) &= (24pq-6p)\left[\left(\frac{2}{3+4}\right)+\left(\frac{2}{3+4}\right)\right]+(12p)\left[\left(\frac{2}{2+3}\right)+\left(\frac{2}{3+3}\right)\right] \\ &= \frac{96}{7}pq+\frac{188}{35}p. \end{aligned}$$

□

**Theorem 12.** *Let G be the $VC_5C_7[p,q]$ nanotube. Then we have*

1. $BII_1(VC_5C_7[p,q]) = 5^{12p}\times 6^{12p}\times 7^{12p(4q-1)}$.
2. $BII_2(VC_5C_7[p,q]) = 3^{12p}\times 6^{12p}\times 12^{12p(4q-1)}$.
3. $HBII_1(VC_5C_7[p,q]) = 5^{24p}\times 6^{24p}\times 7^{24p(4q-1)}$.
4. $HBII_2(VC_5C_7[p,q]) = 3^{24p}\times 6^{24p}\times 12^{24p(4q-1)}$.
5. $HII_b(VC_5C_7[p,q]) = \left(\frac{2}{7}\right)^{12p(4q-1)}\times\left(\frac{2}{5}\right)^{12p}\times\left(\frac{1}{3}\right)^{12p}$.

**Proof.** 1. First multiplicative K Banhatti index of $VC_5C_7[p,q]$ is

$$\begin{aligned} BII_1(VC_5C_7[p,q]) &= \left[(3+4)^{(24pq-6p)}\times(3+4)^{(24pq-6p)}\right]\times\left[(2+3)^{(12p)}\times(3+3)^{(12p)}\right] \\ &= 5^{12p}\times 6^{12p}\times 7^{12p(4q-1)}. \end{aligned}$$

2. Second multiplicative K Banhatti index of $VC_5C_7[p,q]$ is

$$\begin{aligned} BII_2(VC_5C_7[p,q]) &= \left[(3\times 4)^{(24pq-6p)}\times(3\times 4)^{(24pq-6p)}\right]\times\left[(2\times 3)^{(12p)}\times(3\times 3)^{(12p)}\right] \\ &= 3^{12p}\times 6^{12p}\times 12^{12p(4q-1)}. \end{aligned}$$

3. First multiplicative K hyper-Banhatti index of $VC_5C_7[p,q]$ is

$$\begin{aligned} HBII_1(VC_5C_7[p,q]) &= \left[\left((3+4)^2\right)^{(24pq-6p)}\times\left((3+4)^2\right)^{(24pq-6p)}\right] \\ &\quad\times\left[\left((2+3)^2\right)^{(12p)}\times\left((3+3)^2\right)^{(12p)}\right] \\ &= 5^{24p}\times 6^{24p}\times 7^{24p(4q-1)}. \end{aligned}$$

4. Second multiplicative K hyper-Banhatti index of $VC_5C_7[p,q]$ is

$$\begin{aligned} HBII_2(VC_5C_7[p,q]) &= \left[\left((3\times 4)^2\right)^{(24pq-6p)}\times\left((3\times 4)^2\right)^{(24pq-6p)}\right] \\ &\quad\times\left[\left((2\times 3)^2\right)^{(12p)}\times\left((3\times 3)^2\right)^{(12p)}\right] \\ &= 3^{24p}\times 6^{24p}\times 12^{24p(4q-1)}. \end{aligned}$$

5. Multiplicative K harmonic Banhatti index of $VC_5C_7[p,q]$ is

$$\begin{aligned} HII_b(VC_5C_7[p,q]) &= \left[\left(\frac{2}{3+4}\right)^{(24pq-6p)}\times\left(\frac{2}{3+4}\right)^{(24pq-6p)}\right]\times\left[\left(\frac{2}{2+3}\right)^{(12p)}\times\left(\frac{2}{3+3}\right)^{(12p)}\right] \\ &= \left(\frac{2}{7}\right)^{12p(4q-1)}\times\left(\frac{2}{5}\right)^{12p}\times\left(\frac{1}{3}\right)^{12p}. \end{aligned}$$

**Table 7.** Edge degree partition of $HC_5C_7[p,q]$.

| $d_G(u), d_G(v) : e = uv \in E(G)$ | $(2,2)$ | $(3,3)$ | $(2,3)$ |
|---|---|---|---|
| $d_G(e)$ | 2 | 4 | 3 |
| Number of edges | $p$ | $12pq - 4p$ | $8p$ |

□

### 2.7. Banhatti indices of $HC_5C_7[p,q]$

The molecular graphs of carbon nanotubes $HC_5C_7[p,q]$ is shown in Figure 8. The 2 dimensional lattice of

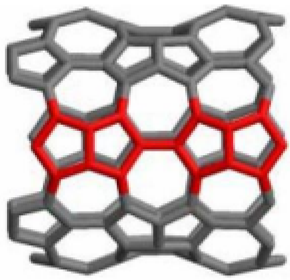

**Figure 8.** Molecular graph of $HC_5C_7[p,q]$.

$HC_5C_7[p,q]$ is shown in Figure 9.

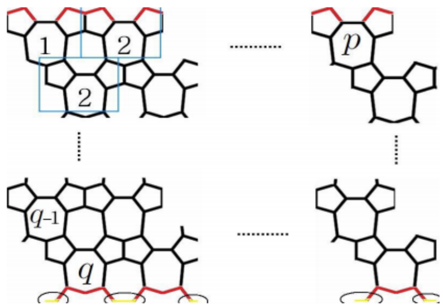


**Figure 9.** 2 dimensional lattice of $HC_5C_7[p,q]$.

**Theorem 13.** *Let G be the $HC_5C_7[p,q]$ nanotube. Then we have*

1. $B_1(HC_5C_7[p,q]) = 168pq + 40p$.
2. $B_2(HC_5C_7[p,q]) = 288pq + 32p$.
3. $HB_1(HC_5C_7[p,q]) = 1176pq + 128p$.
4. $HB_2(HC_5C_7[p,q]) = 3456pq - 184p$.
5. $H_b(HC_5C_7[p,q]) = \frac{48}{7}pq + \frac{219}{35}p$.

**Proof.** Let $G = HC_5C_7[p,q]$. Then the edge set of $HC_5C_7[p,q]$ can be partitioned as follows:
$E_4 = \{uv \in E(G) : d_G(u) = d_G(v) = 2\}$,
$E_5 = \{uv \in E(G) : d_G(u) = 2, d_G(v) = 3\}$,
$E_6 = \{uv \in E(G) : d_G(u) = d_G(v) = 3\}$,
such that $|E_4| = p$, $|E_5| = 8p$ and $|E_6| = 12pq - 4p$.
The edge degree partition is given in Table 7. Now

1. First K Banhatti index of $HC_5C_7[p,q]$ is

$$\begin{aligned} B_1(HC_5C_7[p,q]) &= (p)[(2+2)+(2+2)]+(12pq-4p)[(3+4)+(3+4)] \\ &\quad +(8p)[(2+3)+(3+3)] \\ &= 168pq+40p. \end{aligned}$$

2. Second K Banhatti index of $HC_5C_7[p,q]$ is

$$\begin{aligned} B_2(HC_5C_7[p,q]) &= (p)[(2\times 2)+(2\times 2)]+(12pq-4p)[(3\times 4)+(3\times 4)] \\ &\quad +(8p)[(2\times 3)+(3\times 3)] \\ &= 288pq+32p. \end{aligned}$$

3. First K hyper-Banhatti index of $HC_5C_7[p,q]$ is

$$\begin{aligned} HB_1(HC_5C_7[p,q]) &= (p)\left[(2+2)^2+(2+2)^2\right]+(12pq-4p)\left[(3+4)^2+(3+4)^2\right] \\ &\quad +(8p)\left[(2+3)^2+(3+3)^2\right] \\ &= 1176pq+128p. \end{aligned}$$

4. Second K hyper-Banhatti index of $HC_5C_7[p,q]$ is

$$\begin{aligned} HB_2(HC_5C_7[p,q]) &= (p)\left[(2\times 2)^2+(2\times 2)^2\right]+(12pq-4p)\left[(3\times 4)^2+(3\times 4)^2\right] \\ &\quad +(8p)\left[(2\times 3)^2+(3\times 3)^2\right] \\ &= 3456pq-184p. \end{aligned}$$

5. K harmonic Banhatti index of $HC_5C_7[p,q]$ is

$$\begin{aligned} H_b(HC_5C_7[p,q]) &= (p)\left[\left(\frac{2}{2+2}\right)+\left(\frac{2}{2+2}\right)\right]+(12pq-4p)\left[\left(\frac{2}{3+4}\right)+\left(\frac{2}{3+4}\right)\right] \\ &\quad +(8p)\left[\left(\frac{2}{2+3}\right)+\left(\frac{2}{3+3}\right)\right] \\ &= \frac{48}{7}pq+\frac{219}{35}p. \end{aligned}$$

□

**Theorem 14.** *Let G be the $HC_5C_7[p,q]$ nanotube. Then we have*

1. $BII_1(HC_5C_7[p,q]) = 2^{4p}\times 5^{8p}\times 6^{8p}\times 7^{8p(3q-1)}$.
2. $BII_2(HC_5C_7[p,q]) = 2^{4p}\times 3^{16p}\times 6^{8p}\times 12^{8p(3q-1)}$.
3. $HBII_1(HC_5C_7[p,q]) = 2^{8p}\times 5^{16p}\times 6^{16p}\times 7^{16p(3q-1)}$.
4. $HBII_2(HC_5C_7[p,q]) = 2^{16p}\times 5^{32p}\times 6^{32p}\times 7^{32p(3q-1)}$.
5. $HII_b(HC_5C_7[p,q]) = \left(\frac{1}{2}\right)^{2p}\times\left(\frac{1}{3}\right)^{8p}\times\left(\frac{2}{5}\right)^{8p}\times\left(\frac{2}{7}\right)^{8p(3q-1)}$.

**Proof.** 1. First multiplicative K Banhatti index of $HC_5C_7[p,q]$ is

$$\begin{aligned} BII_1(HC_5C_7[p,q]) &= \left[(2+2)^{(p)}\times(2+2)^{(p)}\right]\times\left[(3+4)^{(12pq-4p)}\times(3+4)^{(12pq-4p)}\right] \\ &\quad \times\left[(2+3)^{(8p)}\times(3+3)^{(8p)}\right] \\ &= 2^{4p}\times 5^{8p}\times 6^{8p}\times 7^{8p(3q-1)}. \end{aligned}$$

2. Second multiplicative K Banhatti index of $HC_5C_7[p,q]$ is

$$\begin{aligned} BII_2(HC_5C_7[p,q]) &= \left[(2\times 2)^{(p)}\times(2\times 2)^{(p)}\right]\times\left[(3\times 4)^{(12pq-4p)}\times(3\times 4)^{(12pq-4p)}\right] \\ &\quad\times\left[(2\times 3)^{(8p)}\times(3\times 3)^{(8p)}\right] \\ &= 2^{4p}\times 3^{16p}\times 6^{8p}\times 12^{8p(3q-1)}. \end{aligned}$$

3. First multiplicative K hyper-Banhatti index of $HC_5C_7[p,q]$ is

$$\begin{aligned} HBII_1(HC_5C_7[p,q]) &= \left[\left((2+2)^2\right)^{(p)}\times\left((2+2)^2\right)^{(p)}\right]\times\left[\left((3+4)^2\right)^{(12pq-4p)}\times\left((3+4)^2\right)^{(12pq-4p)}\right] \\ &\quad\times\left[\left((2+3)^2\right)^{(8p)}\times\left((3+3)^2\right)^{(8p)}\right] \\ &= 2^{8p}\times 5^{16p}\times 6^{16p}\times 7^{16p(3q-1)}. \end{aligned}$$

4. Second multiplicative K hyper-Banhatti index of $HC_5C_7[p,q]$ is

$$\begin{aligned} HBII_2(HC_5C_7[p,q]) &= \left[\left((2\times 2)^2\right)^{(p)}\times\left((2\times 2)^2\right)^{(p)}\right]\times\left[\left((3\times 4)^2\right)^{(12pq-4p)}\times\left((3\times 4)^2\right)^{(12pq-4p)}\right] \\ &\quad\times\left[\left((2\times 3)^2\right)^{(8p)}\times\left((3\times 3)^2\right)^{(8p)}\right] \\ &= 2^{8p}\times 3^{32p}\times 6^{16p}\times 12^{16p(3q-1)}. \end{aligned}$$

5. Multiplicative K harmonic Banhatti index of $HC_5C_7[p,q]$ is

$$\begin{aligned} HII_b(HC_5C_7[p,q]) &= \left[\left(\frac{2}{2+2}\right)^{(p)}\times\left(\frac{2}{2+2}\right)^{(p)}\right]\times\left[\left(\frac{2}{3+4}\right)^{(12pq-4p)}\times\left(\frac{2}{3+4}\right)^{(12pq-4p)}\right] \\ &\quad\times\left[\left(\frac{2}{2+3}\right)^{(8p)}\times\left(\frac{2}{3+3}\right)^{(8p)}\right] \\ &= \left(\frac{1}{2}\right)^{2p}\times\left(\frac{1}{3}\right)^{8p}\times\left(\frac{2}{5}\right)^{8p}\times\left(\frac{2}{7}\right)^{8p(3q-1)}. \end{aligned}$$

□

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