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Buys-Ballot Estimates for Overall Sample Variances and Their Statistical Properties: A Mixed Model Case

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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ABSTRACT

This article proposes an estimation procedure of overall sample variances for the mixed model while comparing them with those of the additive and multiplicative models. The estimation procedure is based on the row, column and overall means and variances of time series data arranged in a Buys-Ballot table. The procedure assumes that (1) the underlying distribution of the variable, X_{ij}, $i = 1, 2, ..., m$, $j = 1, 2, ..., s$, under study is normal. (2) the trending curve is either linear or quadratic and (3) the decomposition method is either multiplicative or mixed. Statistical properties of the overall sample variances of the Buys-Ballot table are also considered in this study. Result indicates that, under the stated assumptions, the expected value of the overall sample variances involve sum of square and cross-product of trend parameters and seasonal indices.

Keywords: Time series decomposition; linear trend; mixed model; overall sample variance; suitable model; buys-ballot estimates.

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1. INTRODUCTION

Dozie [1] proposed an estimation method based on the row, column and overall means and variances of the Buys-Ballot table for the mixed model in time series decomposition. This method was initially developed for short period of time in which the trend-cycle component (M_t) is jointly combined and can be represented by linear equation.

$$M_t = a + b_t \text{, t = 1, 2, .. n}$$

where a is the intercept, b is the slope and t is the time point.

The models most commonly used for time series decomposition are the

Additive Model:

$$X_t = T_t + S_t + C_t + e_t \quad (1)$$

Multiplicative Model:

$$X_t = T_t \times S_t \times C_t \times e_t \quad (2)$$

and Mixed Model

$$X_t = T_t \times S_t \times C_t + e_t \quad (3)$$

If short period of time are involved, the cyclical component is superimposed into the trend [2] and the observed time series $(X_t, \, t = 1, 2, ..., n)$ can be decomposed into the trend-cycle component (M_t), seasonal component (S_t) and the irregular/residual component (e_t). Therefore, the decomposition models are

Additive Model:

$$X_t = M_t + S_t + e_t \quad (4)$$

Multiplicative Model:

$$X_t = M_t \times S_t \times e_t \quad (5)$$

and Mixed Model

$$X_t = M_t \times S_t + e_t . \quad (6)$$

It is always assumed that the seasonal effect, when it exists, has period *s*, that is, it repeats after *s* time periods.

$$S_{t+s} = S_t \text{, for all } t \quad (7)$$

For Equation (4), it is convenient to make the further assumption that the sum of the seasonal components over a complete period is zero, ie ,

$$\sum_{j=1}^{s} S_{t+j} = 0 . \quad (8)$$

Similarly, for Equations (5) and (6), the convenient variant assumption is that the sum of the seasonal components over a complete period is *s*.

$$\sum_{j=1}^{s} S_{t+j} = s . \quad (9)$$

Descriptive methods involve the separation of an observed time series into components representing trend (long term direction), the seasonal (systematic, calendar related movements), cyclical (long term oscillations or swings about the trend) and irregular (unsystematic, short term fluctuations) components. Description includes the examination of trend, seasonality, cycles, changes in level, trend and scale and so on that may influence the series. This is also very vital preliminary to modelling, when it has to be decided whether and how to seasonally adjust, to transform, and to deal with outliers and whether to fit a model. In the examination of trend, seasonality and cycles, a time series is often described as having trends, seasonal effects, cyclic pattern and irregular or random component. An important aspect of descriptive time series analysis is the choice of model in time series decomposition. As the literature indicates, choice of model in descriptive time series has attracted so much research attention. Different approaches to determine choice of model like the use of sequence plot (time plot) as well as other techniques have continue to evolve. Among them are the use of the coefficient of variation of seasonal differences (CV) and seasonal quotient by Puerto and Rivera [3]. The test for constant variances by Iwueze and Nwogu [4]. Proposed Chi-Square test by Nwogu, et al, [5] and Dozie, et al, [6] etc.

Iwueze and Nwogu [4] stated that when the trend-cycle component is linear, the column variances of the Buys-Ballot table are constant for the additive model, but contain the seasonal component for the multiplicative model. Thus, choice between additive and multiplicative models reduces to test for constant variance to identify the additive model. They observed that any of the tests for constant variance can be used to identify a series that admits the additive model. This is an improvement over what is in existence. However, this approach can only identify the additive model (when the column variance is constant), but does not tell the analyst the alternative model when the variance is not constant. The implication of this is that when the test for constant variance says the appropriate model for a study series is not the additive model; an analyst still faces the challenge of distinguishing between mixed model and the multiplicative model.

Iwueze and Nwogu [4] proposed a test for choice of model based on Chi-Square distribution. Although time series data does not satisfy all the assumptions of most common statistical test, the Chi-Square test appears to be the most efficient among them. The proposed test is able to distinguish between the mixed and multiplicative models with a high degree of confidence.

2. METHODOLOGY

The estimation procedure for overall variances and their statistical properties for the mixed model in this study are done using Buys-Ballot procedure often referred to in the literature. This method adopted in this study assumed that the series are arranged in a Buys-Ballot table with m rows and s columns. For details of this method see Wei [7], Iwueze et.al [8], Nwogu et.al [5], Dozie and Ihekuna [9], Dozie et.al [6], Dozie and Nwanya [10], Dozie [1], Dozie and Ijeomah [11], Dozie and Ibebuogu [12], Dozie and Uwaezuoke [13], Dozie and Ihekuna [14] Dozie and Ibebuogu [15].

2.1 Estimation Procedure of Overall Variances for Mixed Model

This method is developed for short term of period in which the trend and cyclical component are jointly combined. Discussion of this method is restricted to a case in which the trend is a straight line. Length of periodic interval is taken to be s. $\sum_{j=1}^{s} S_{t+j} = s$, and $e_t \sim N(0,1)$. Using the Buys-Ballot table, with m-rows and s-columns $t = (i-1)s + j$, $i = 1,2,\dots,m$, $j = 1,2,\dots,s$. Therefore

$$X_{(i-1)s+j} = M_{(i-1)s+j} \times S_{(i-1)s+j} + e_{(i-1)s+j} \quad (10)$$

For convenience, le $X_{ij} = X_{(i-1)s+j}$, $M_{ij} = M_{(i-1)s+j}$ and $e_{ij} = e_{(i-1)s+j}$. Hence,

$$M_{ij} = a + b[(i-1)s + j]$$

and

$$X_{ij} = M_{ij} S_j + e_{ij}$$

$$= \{a + b[(i-1)s + j]\} S_j + e_{ij}$$

$$\hat{\sigma}_x^2 = \frac{1}{n-1}\sum_{i=1}^{m}\sum_{j=1}^{s}\left(X_{ij} - \bar{X}_{..}\right)^2 \quad (11)$$

$$= \frac{1}{n-1}\sum_{i=1}^{m}\sum_{j=1}^{s}\left\{\left[a + bs(i-1) + bj\right]S_j + e_{ij} - a - \frac{bs(m-1)}{2} - \frac{b}{s}\sum_{j-1}^{s} jS_j - \bar{e}_{..}\right\}^2$$

$$(n-1)\sigma_{\times}^2 = \sum_{i=1}^{m}\sum_{j=1}^{s}\left\{a\left(S_j - 1\right) + bs\left[(i-1)S_j - \frac{m-1}{2}\right] + b\left(jS_j - C_1\right) + \left(e_{ij} - \bar{e}_{..}\right)\right\}^2$$

where

$$C_1 = \frac{1}{s}\sum_{j=1}^{s} jS_j \qquad (12)$$

$$(n-1)\sigma_{\times}^2 = \sum_{i-1}^{m}\sum_{j-1}^{s}\left\{a^2\left(S_j-1\right)^2+(bs)^2\left[(i-1)S_j-\frac{m-1}{2}\right]^2+b^2\left(jS_j-C_1\right)^2+\left(e_{ij}-\bar{e}..\right)\right\}^2$$

$$+2abs=\left(S_j-1\right)\left[(i-1)S_j-\frac{m-1}{2}\right]+2ab\left(S_j-1\right)\left(jS_j-C_1\right)+2b^2s\left[(i-1)S_j-\frac{m-1}{2}\right]$$

$$\left(jS_j-C_1\right)$$

$$+2\left\{a\left(S_j-1\right)+bs\left[(i-1)S_j-\frac{m-1}{2}\right]+b\left(jS_j-C_1\right)\left(e_{ij}-\bar{e}..\right)\right\}$$

$$(n-1)E\left(\sigma_{\times}^2\right)=a^2m\sum_{j=1}^{s}\left(S_j-1\right)^2+(bs)^2\left[\frac{m(m-1)(2m-1)}{6}\sum_{j=1}^{s}S_j^2-(ms)\left(\frac{m-1}{2}\right)^2\right]$$

$$+b^2m\sum_{j-1}^{s}\left(jS_j-C_1\right)^2+\sum_{i=1}^{m}\sum_{j=1}^{s}E\left(e_{ij}\right)^2-nE\left(\bar{e}\right)^2+\sum_{i=1}^{m}\sum_{j=1}^{s}2abs\left(S_j-1\right)\left[(i-1)S_j-\frac{m-1}{2}\right]$$

$$+2abm\sum_{j=1}^{s}\left(S_j-1\right)\left(jS_j-C_1\right)+2b^2s\sum_{j=1}^{s}\left[S_j\sum_{i=1}^{m}(i-1)-m\frac{(m-1)}{2}\right]\left(jS_j-C_1\right)$$

$$=a^2m\sum_{j=1}^{s}\left(S_j-1\right)^2+(bs)^2\left\{\frac{m(m-1)(2m-1)}{6}\left[s+\sum_{j=1}^{s}\left(S_j-1\right)^2\right]-ms\left(\frac{m-1}{2}\right)^2\right\}$$

$$+b^2m\sum_{j=1}^{s}\left(jS_j-C_1\right)^2+\left(n\sigma^2-\frac{n\sigma^2}{n}\right)+2abs\sum_{j=1}^{s}\left(S_j-1\right)\left[\frac{m(m-1)}{2}\left(S_j-1\right)\right]$$

$$+2abm\sum_{j=1}^{s}\left(S_j-1\right)\left(jS_j-C_1\right)+2b^2s\sum_{j=1}^{s}\left[\frac{m(m-1)}{2}S_j-m\left(\frac{m-1}{2}\right)\right]\left(jS_j-C_1\right)$$

$$=a^2m\sum_{j=1}^{s}\left(S_j-1\right)^2+(bs)^2\left\{\frac{m(m-1)(2m-1)}{6}\sum_{j=1}^{s}\left(S_j-1\right)^2+\frac{sm(m-1)(2m-1)}{6}-ms\left(\frac{m-1}{2}\right)^2\right\}$$

$$+2abs\frac{m(m-1)}{2}\sum_{j=1}^{s}\left(S_j-1\right)^2+b^2m\sum_{j=1}^{s}\left(jS_j-C_1\right)^2+2abm\sum_{j=1}^{s}\left(S_j-1\right)\left(jS_j-C_1\right)+2b^2s\frac{m(m-1)}{2}$$

$$\sum_{j=1}^{s}\left(S_j-1\right)\left(jS_j-C_1\right)+(n-1)\sigma^2$$

$$(n-1)E\left(\sigma_{\times}^{2}\right)=\frac{ms(m-1)(m+1)}{12}(bs)^{2}+m\left\{\left[a+bs\left(\frac{m-1}{2}\right)\right]^{2}+(bs)^{2}\frac{(m-1)(m+1)}{12}\right\}$$

$$\sum_{j-1}^{s}\left(S_{j}-1\right)^{2}$$

$$+b^{2}m\sum_{j=1}^{s}\left(jS_{j}-C_{1}\right)^{2}+2bm\left[a+bs\frac{(m-1)}{2}\right]\sum_{j=1}^{s}\left(S_{j}-1\right)\left(jS_{j}-C_{1}\right)+(n-1)\sigma^{2}$$

$$=\frac{n(n-s)(n+s)}{12}b^{2}+ms\left\{\left(a+bs\frac{(m-1)}{2}\right)^{2}+\frac{(n-s)(n+s)}{12}b^{2}\right\}\sigma_{\times}^{2}+b^{2}sm\sigma^{2}$$

$$+2bsm\left[a+bs\frac{(m-1)}{2}\right]\operatorname{cov}\left(S_{j},C_{1}\right)+(n-1)\sigma^{2}$$

$$E\left(\sigma_{\times}^{2}\right)=\frac{n}{n-1}\left\{\frac{b^{2}\left(n^{2}-s^{2}\right)}{12}+\left\{\left[a+b\left(\frac{n-s}{2}\right)\right]^{2}+b^{2}\frac{\left(n^{2}-s^{2}\right)}{12}\right\}\sigma_{\times}^{2}+b^{2}\sigma_{\times}^{2}+2b\left[a+b\left(\frac{n-s}{2}\right)\right]\operatorname{cov}\left(S_{j},C_{1}\right)\right\}$$

$+\sigma^{2}$. Where $\sigma_{\times}^{2}=\frac{1}{s}\sum_{j=1}^{s}\left(S_{j}-1\right)^{2}+\sigma_{\times}^{2}=\frac{1}{s}\sum_{j=1}^{s}\left(jS_{j}-C_{1}\right)^{2}$

$$Cov\left(S_{j},C_{1}\right)=\frac{1}{s}\sum_{j=1}^{s}\left(S_{j}-1\right)\left(jS_{j}-C_{1}\right)$$

$$\sigma_{\times}^{2}=\frac{n}{n-1}\left\{\frac{b^{2}\left(n^{2}-s^{2}\right)}{12}+\left[a^{2}+ab(n-s)+\frac{b^{2}(n-s)(2n-s)}{12}\right]\operatorname{var}\left(S_{j}\right)+b^{2}\operatorname{var}\left(jS_{j}\right)+2b\left[a+b\left(\frac{n-s}{2}\right)\right]\right\}$$

$$Cov\left(S_{j},jS_{j}\right)+\sigma^{2} \tag{13}$$

2.2 Basic Properties of Overall Sample Variances for Mixed Model

1) A function of weighted average of the square of the seasonal component (S_j^2).
2) The error variance is not known, it requires to be estimated from data.
3) The expected value involve sum of squares and cross product of trend-cycle and seasonal indices
4) A function of both row and column specific
5) A product of variance of S_j and co-variance of jS_j

The summary of the overall variances for the mixed model when trend cycle component is linear are in Table 1 while comparing them with those of the additive and multiplicative model. From Table 1, the overall variances are not same for the three models. In particular, while the overall variance of the Buys-Ballot table is a product of weighted average of the square of the seasonal component (S_j^2) for the mixed model. The overall variance, on the other hand, is for the multiplicative model, depends on the column j only through the square of the seasonal component (S_j^2) and trend parameters through the square of the seasonal mean for multiplicative model. It is a product of trending curves and the seasonal component (S_j^2) for the additive model.

Table 1. Estimates of overall variances for the mixed, multiplicative and additive models

Sample variance	Buys-ballot estimates for overall variances		
	Mixed model	**Multiplicative model**	**Additive model**
$\hat{\sigma}^2_{..}$	$\frac{n}{n-1}\left[\begin{array}{l}\frac{b^2(n^2-s^2)}{12}+ \\ \left[a^2+2ab\left(\frac{n-s}{2}\right)+\frac{b^2(n-s)(2n-s)}{2}\right]\operatorname{var}(S_j) \\ +2b\left[a+b\left(\frac{n-s}{2}\right)\right]\operatorname{cov}(S_j, jS_j) \\ +b^2\operatorname{var}(jS_j)+\sigma_1^2\end{array}\right]$	$\frac{1}{n-1}\left[\begin{array}{l}\frac{b^2n(n-s)(n+s)}{12}+ \\ m\left[a^2+ab(n-s)+\frac{b^2(n-s)(2n-s)}{6}\right] \\ \sum_{j=1}^{s}(S_j-1)^2+mb^2\sum\left(jS_j-\frac{c_1}{s}\right)^2 \\ +2b\left[ma+\frac{nb(m-1)}{2}\right] \\ \sum(S_j-1)\left(jS_j-\frac{c_1}{s}\right)\end{array}\right]$	$b^2\left(\frac{n(n+1)}{12}\right)$ $+\frac{1}{n-1}\left\{2bm\sum_{j=1}^{s}jS_j\right.$ $\left.+m\sum_{j=1}^{s}S_j^2\right\}$

Source: Iwueze and Nwogu (2014), Nwogu, et al, (2019) and Dozie, et al, (2020)

2.3 Method of Choosing the Mixed Model

Iwueze and Nwogu [4], Nwogu, et al [5] and Dozie, et al [6] discussed the estimation procedure for the seasonal variances of the Buys-Ballot table for additive, multiplicative and mixed models for linear trending curve are given in equations (14), (15) and (16) respectively.

$$\hat{\sigma}_j^2 = b^2\left(\frac{n(n+s)}{12}\right) \tag{14}$$

$$\hat{\sigma}_j^2 = \left[b^2\left(\frac{n(n+s)}{12}\right)\right]S_j^2 \tag{15}$$

$$\hat{\sigma}_j^2 = b^2\,\frac{n(n+s)}{12}\,S_j^2+\sigma_1^2 \tag{16}$$

The column variance is, for additive model and equation (14), is a product of trending series only. For the multiplicative and equation (15), is a quadratic function of the season j and square of the seasonal effect $S_{j,}^2$ and for the mixed model and equation (16), a constant multiple of square of the seasonal effect only. Therefore, the test for choice between the mixed and multiplicative model is based on the column variances σ_j^2 of the Buys-Ballot table. Hence, the null hypothesis to be tested

H0**:** $\sigma_j^2 = \sigma_{0j}^2$

and the model is mixed, against the alternative

H1**:** $\sigma_j^2 \neq \sigma_{0j}^2$

and the model is not mixed, where

$\sigma_j^2 = (j=1,\ 2,\ldots,s)$ is the original variance of the jth column.

$$\sigma_{0j}^2 = \frac{b^2\,n(n+s)}{12}\,S_j^2+\sigma_1^2 \tag{17}$$

and σ_1^2 is the error variance, assumed equal to 1.

The test statistic

$$\chi_c^2 = \frac{(m-1)\sigma_j^2}{\sigma_{0j}^2} \tag{18}$$

follows the chi-square distribution with $m-1$ degrees of freedom, m represents the number of period and s is the number of columns, the interval $\left[\chi^2_{\frac{\alpha}{2},(m-1)}, \chi^2_{1-\frac{\alpha}{2},(m-1)}\right]$ contains the statistic (18) with 100 (1- α)% degree of confidence.

3. SIMULATIONS RESULTS USING THE MIXED MODEL

The simulated series used consists 100 data sets of 120 observations each simulated from the mixed model: $X_t = (a + bt) \times S_t + e_t$, using the MINITAB 17.0 version software. The trend-cycle component is used with $a = 1,\ b = 0.02,\quad e_t \sim N(0,1),$ $S_1 = 0.98,\ S_2 = 0.80, S_3 = 0.88,\ S_4 = 1.04,$

$S_5 = 0.96,\ S_6 = 1.22,\ S_7 = 1.27,\ S_8 = 1.32,\ S_9 = 0.96,\ S_{10} = 0.80,\ S_{11} = 0.88\ S_{12} = 0.87,\ S = 12$

The Buys-Ballot table of the series listed as monthly and seasonal data is shown in Table 2 The results of the calculated values of the statistic from the simulated time series data are shown in Table 2. The critical values at 5% level of significance and which for $m - 1 = 9$ degrees of freedom, equal to 2.7 and 19.0. The null hypothesis that the time series data accepts mixed model is rejected, if the calculated value of the test statistic shown in (18) is not within the range, otherwise, do not reject the null hypothesis. When compared with the range (2.7 and 19.0), 100 out of the 100 calculated values of test statistic from the stimulated time series data contained in Table 2 lie within the range, suggesting that the proposed test successfully recorded 100% of times for mixed model.

Table 2. Calculated chi-square for mixed model

Col	Series									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	11.2121	12.1287	6.9876	8.9087	7.7709	10.9060	14.2310	12.5432	9.9876	7.7876
2	10.1212	10.8037	4.9876	3.8469	6.9436	5.4959	5.4413	12.9076	15.0742	14.8701
3	13.7630	5.1598	11.9903	8.6702	14.7272	8.7078	11.9401	6.8329	8.4794	9.8765
4	11.9876	11.8765	13.9876	10.8293	4.9423	11.2650	7.9545	9.3044	13.3286	12.9876
5	16.8765	10.8765	8.1209	10.5545	9.2798	17.1571	3.9333	5.2740	4.9656	12.9938
6	8.5543	16.4874	6.8760	3.3076	7.5989	9.7034	7.8557	15.1125	7.5076	7.6820
7	8.0987	10.7654	9.9001	10.5750	6.5760	4.9320	6.0845	9.1149	6.6597	12.1276
8	6.4432	2.5709	15.9770	9.7204	9.0275	13.3809	11.6644	8.7687	18.1803	12.9876
9	7.8876	4.3121	4.8592	9.8997	3.3922	4.2077	8.2510	4.6430	8.8565	8.9871
10	7.9877	4.5750	5.2015	8.1676	5.4410	11.3758	8.8120	13.1021	6.3318	7.4324
11	9.7355	11.9050	12.1345	6.8173	0.7856	4.7580	6.8402	6.0106	5.0302	8.8765
12	7.8516	9.8700	10.7143	4.7000	8.1478	8.0776	3.2408	12.7302	6.2707	11.0090
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

Table 2. Calculated chi-square for mixed model (cont.)

Col	Series									
	11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	12.6094	11.2685	5.9455	12.3441	9.5303	10.2527	11.9166	8.9109	13.4996	6.7858
2	8.2284	10.0976	5.4781	12.1527	6.7144	8.8954	7.5931	7.3266	10.3937	6.3749
3	7.8765	5.5312	6.2797	15.7758	9.4296	5.5272	4.8595	10.4684	5.0037	12.0421
4	5.3209	9.5869	10.2811	9.0821	12.0272	8.1651	5.9885	11.6237	8.5224	7.9510
5	6.9873	4.7852	6.8220	7.7652	3.9335	6.2330	8.0709	4.5609	7.6014	13.8613
6	13.1399	5.7359	6.9559	6.8765	13.9755	9.0818	7.1403	13.1169	4.8691	5.9653
7	6.8764	7.9960	14.7312	13.3212	8.2492	11.2175	10.9352	16.8010	6.7297	8.7209
8	15.0076	5.4835	7.6453	6.7654	12.3519	5.6445	8.2929	5.1867	4.7716	12.7810
9	6.7421	6.6425	6.7641	13.4532	5.9355	8.0525	8.9194	4.1364	10.5184	9.6881
10	11.0909	7.5444	12.1063	5.6543	8.6373	11.5941	9.8175	15.5784	5.3195	4.6014
11	4.4324	16.6570	7.5044	3.3156	15.2454	6.3591	4.1399	4.2341	7.3711	10.6682
12	9.4321	9.9693	15.2131	9.9019	5.3212	6.7654	10.3212	12.3212	11.9074	7.5432
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

Table 2. Calculated chi-square for mixed model (cont.)

Col	Series									
	21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
1	9.4321	10.6872	10.3019	9.8740	12.9731	8.4321	10.9098	11.1023	11.5324	8.8765
2	9.8764	7.6571	5.1852	3.8616	12.1936	16.0880	6.3393	9.3057	7.9772	11.7654
3	15.7739	9.2675	7.5613	7.3520	9.9086	3.5049	7.1022	7.4936	9.2753	11.8765
4	7.9491	8.8898	6.4351	3.4607	4.3421	8.9271	5.1558	6.6446	9.9960	4.9875
5	15.4095	7.8317	12.7285	2.6768	5.5439	8.9074	8.3408	14.2036	6.3303	5.5432
6	4.4069	11.7922	7.8317	16.1478	8.8765	10.8279	17.3949	7.7486	8.6958	6.1019
7	10.5531	10.0569	9.3096	14.4791	12.9097	4.7670	7.9232	10.5640	13.9507	8.9876
8	9.1184	5.5631	12.5454	7.8327	11.8765	9.5254	8.5931	9.3800	10.2167	8.9876
9	8.0141	8.2588	7.5039	6.1646	9.1121	5.1337	5.9292	8.7776	6.7719	8.2804
10	7.0412	3.1853	8.2615	11.9400	7.7875	5.5111	7.5531	13.3698	15.8669	11.7257
11	2.3845	8.1368	4.6635	3.9207	6.9876	14.2902	6.2990	8.0857	17.5143	9.1542
12	8.7773	15.5007	9.8291	3.6141	12.9876	10.7897	12.2908	10.2846	13.0388	13.5793
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

Table 2. Calculated chi-square for mixed model (cont.)

Col	Series									
	31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
1	10.3301	12.2052	13.6108	12.4196	10.2948	10.7634	13.7346	9.9785	8.0880	7.2840
2	12.5402	8.0585	4.8702	12.7460	3.7832	3.6429	6.8156	7.5950	12.7919	10.6303
3	5.3371	6.7496	7.6270	9.8716	6.0977	4.2432	13.7521	13.4111	8.3039	4.9887
4	9.4842	14.6409	2.7816	9.4014	6.2389	12.0595	15.8052	5.1196	6.9820	9.5517
5	3.4534	8.2087	11.4106	6.0613	5.8798	6.6919	8.6991	12.1725	12.3841	11.2771
6	19.0150	8.7136	5.6496	11.7791	12.0007	17.3150	10.3676	17.1883	8.5542	9.1544
7	13.7678	17.1864	9.5216	10.3132	5.1484	7.9064	13.3484	6.0798	10.6810	10.6176
8	11.9285	9.2793	9.5300	13.9811	6.4705	8.3198	10.1218	4.7634	8.9523	9.3645
9	12.2726	7.0765	12.9150	10.2232	9.3524	6.5209	5.8827	10.9141	14.8175	10.1636
10	4.3265	9.1949	6.7857	12.5474	0.8313	17.9135	2.8196	8.8829	7.6546	6.6831
11	10.4583	3.4714	9.7626	9.9101	10.5449	6.0276	2.9715	12.7119	7.4249	8.9223
12	8.4716	4.9938	9.8488	8.7529	8.1471	4.4695	5.8214	3.4978	3.6641	9.1728
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

Table 2. Calculated chi-square for mixed model (cont.)

Col	Series									
	41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
1	11.3301	6.2052	3.6108	10.4196	10.1232	10.9875	13.2121	9.0987	8.6543	7.4321
2	10.5402	8.0585	4.8702	12.1231	3.7832	3.6429	6.8156	7.5950	12.7919	10.6303
3	5.6543	6.7496	7.6270	9.0987	6.0977	4.2432	13.7521	13.2123	8.3039	4.9887
4	9.4764	14.6409	2.7816	9.9765	6.2389	12.0595	15.8052	5.5432	6.9820	9.5517
5	3.9876	8.2087	11.4106	6.9876	5.8798	6.6919	8.6991	12.8765	12.3841	11.2771
6	11.0150	8.7136	5.6496	10.7791	12.0007	17.3150	10.3676	10.1883	8.5542	9.1544
7	10.7678	17.1864	9.5216	11.3132	5.1484	7.9064	13.3484	8.0798	10.6810	10.6176
8	13.9285	9.2793	9.5300	3.9811	6.4705	8.3198	10.1218	6.7634	8.9523	9.3645
9	12.0032	7.0765	12.9150	10.9876	9.3524	6.5209	5.8827	11.9141	14.8175	10.1636
10	4.4321	9.1949	6.7857	12.5474	10.8313	17.9135	2.8196	10.8829	7.6546	6.6831
11	10.4583	3.4714	9.7626	9.9101	10.1010	6.0276	2.9715	11.7119	7.4249	8.9223
12	8.9876	4.7432	11.8488	7.7529	14.1471	5.4695	8.8214	3.4978	13.6641	3.1728
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

Table 2. Calculated chi-square for mixed model (cont.)

Col	Series									
	51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
1	10.4321	12.4329	3.6108	12.1214	9.2948	13.7634	3.7346	3.9785	6.0880	3.2840
2	12.5402	8.0585	4.8702	7.7460	3.7832	3.6429	6.8156	7.5950	12.7919	10.6303
3	5.3371	6.7496	7.6270	9.8716	6.0977	4.2432	13.7521	13.4111	8.3039	4.9887
4	9.4842	14.6409	2.7816	6.4014	6.2389	12.0595	15.8052	5.1196	6.9820	9.5517
5	3.4534	8.2087	11.4106	13.0613	5.8798	6.6919	8.6991	12.1725	12.3841	11.2771
6	9.0150	4.7136	15.6496	10.1215	8.0007	7.3150	11.3676	7.1883	8.5542	12.1544
7	13.7678	17.1864	9.5216	7.3132	5.1484	7.9064	13.3484	6.0798	10.6810	10.6176
8	11.9285	9.2793	9.5300	3.9811	6.4705	8.3198	10.1218	4.7634	8.9523	9.3645
9	12.2726	7.0765	12.9150	8.2232	9.3524	6.5209	5.8827	10.9141	14.8175	10.1636
10	4.3265	9.1949	6.7857	10.5474	10.8313	17.9135	2.8196	8.8829	7.6546	6.6831
11	10.4583	3.4714	9.7626	4.9101	10.5449	6.0276	2.9715	12.7119	7.4249	8.9223
12	8.4716	4.9938	9.8488	4.7529	8.1471	4.4695	5.8214	3.4978	3.6641	9.1728
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

Table 2. Calculated chi-square for mixed model (cont.)

Col	Series									
	61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
1	10.4821	15.0921	9.5963	7.7977	7.1774	9.3257	9.4257	3.1520	3.7203	9.0872
2	12.9157	10.8037	14.0504	3.8469	6.9436	5.4959	5.4413	12.9076	15.0742	12.7306
3	13.0416	5.1566	17.2798	7.6702	13.7272	8.7078	13.9401	6.8329	8.4794	6.9740
4	6.9085	12.2505	12.4492	14.8293	4.9423	13.2650	7.9545	7.3044	12.3286	12.3963
5	16.1748	10.4191	8.4801	8.5545	4.2798	7.1571	7.9333	5.2740	4.9656	12.9938
6	8.2899	16.4968	6.5897	3.3076	7.5989	10.7034	7.8557	5.1125	7.5076	6.6820
7	8.0266	10.3324	9.5820	16.5750	16.5760	4.9320	6.0845	9.1149	6.6597	10.0505
8	16.9952	2.5788	5.9770	9.7204	7.0275	13.3809	4.6644	9.7687	7.1803	9.2279
9	7.5575	3.3121	4.8592	7.8997	3.3922	4.2077	8.2510	4.6430	12.8565	8.9871
10	7.0924	14.5750	8.2015	8.1676	15.4410	11.3758	8.8120	13.1021	6.3318	3.9990
11	15.7355	12.9050	10.1345	6.8173	12.7856	4.7580	6.8402	6.0106	5.0302	7.8786
12	3.8516	3.6963	14.7143	12.7000	8.1478	9.0776	13.2408	10.7302	9.2707	9.8515
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

Table 2. Calculated chi-square for mixed model (cont.)

Col	Series									
	71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
1	7.6094	4.2685	9.9455	14.3441	12.5303	3.2527	14.9166	15.9109	11.9987	14.7858
2	11.2284	10.1212	5.4781	12.1527	6.7144	8.8954	7.5931	7.3266	10.3937	6.3749
3	7.0312	5.9872	6.2797	15.7758	9.4296	5.5272	4.8595	10.4684	5.0037	12.0421
4	5.9895	9.5869	10.2811	9.0821	12.0272	8.1651	5.9885	11.6237	8.5224	7.9510
5	6.5379	4.4324	6.8220	7.7652	3.9335	6.2330	8.0709	4.5609	7.6014	13.8613
6	13.8824	5.9762	6.9559	6.2243	13.9755	9.0818	7.1403	13.1169	4.8691	5.9653
7	6.1482	7.7098	14.7312	13.7218	8.2492	11.2175	10.9352	16.8010	6.7297	8.7209
8	15.7446	5.5090	7.6453	6.5535	12.3519	5.6445	8.2929	5.1867	4.7716	17.7810
9	6.3454	6.1212	6.7641	13.7449	5.9355	8.0525	8.9194	4.1364	10.5184	7.6881
10	9.0031	7.9785	12.1063	5.8258	8.6373	11.5941	9.8175	15.5784	5.3195	4.6014
11	4.7313	6.6570	7.5044	3.1541	15.2454	6.3591	4.1399	4.2341	7.3711	10.6682
12	9.2021	7.9693	15.0332	9.3116	5.2108	6.6369	10.7285	10.6694	18.1081	7.5191
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

Table 2. Calculated chi-square for mixed model (cont.)

Col	Series									
	81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
1	14.9925	16.6872	9.3019	8.4440	6.1231	4.9909	5.5655	12.0012	7.7653	7.7653
2	9.9520	7.6571	5.1852	3.8616	5.1936	16.0880	6.3393	9.3057	7.9772	11.1121
3	15.7739	9.2675	7.5613	7.3520	9.5426	3.5049	7.1022	7.4936	6.2753	3.7839
4	7.9491	8.8898	6.4351	3.4607	4.3826	8.9271	5.1558	6.6446	5.9960	14.3721
5	15.4095	7.8317	12.7285	2.6768	5.3428	8.9074	8.3408	14.2036	6.3303	15.2609
6	4.4069	11.7922	7.8317	16.1478	8.3204	10.8279	17.3949	7.7486	8.6958	12.9722
7	10.5531	10.0569	9.3096	14.4791	7.5278	4.7670	7.9232	10.5640	13.9507	8.6635
8	9.1184	5.5631	12.5454	7.8327	11.9395	9.5254	8.5931	9.3800	10.2167	8.2705
9	8.0141	8.2588	7.5039	6.1646	9.3666	5.1337	5.9292	8.7776	6.7719	8.2804
10	7.0412	3.1853	8.2615	11.9400	7.4869	5.5111	7.5531	13.3698	15.8669	11.7257
11	2.3845	8.1368	4.6635	3.9207	6.7324	14.2902	6.2990	8.0857	17.5143	11.1542
12	8.7773	15.5007	9.8291	3.6141	10.8442	10.7897	12.2908	10.2846	13.0388	10.1221
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

Table 2. Calculated chi-square for mixed model (cont.)

Col	Series									
	91	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
1	4.3301	5.2052	11.6108	8.4196	13.2948	8.7634	9.7346	7.9785	7.0880	5.2840
2	12.5402	8.0585	4.8702	8.7460	3.7832	3.6429	6.8156	7.5950	12.7919	18.6303
3	5.3371	6.7496	7.6270	5.8716	6.0977	4.2432	13.7521	13.4111	8.3039	3.9887
4	9.4842	14.6409	2.7816	6.4014	6.2389	12.0595	15.8052	5.1196	6.9820	13.5517
5	3.4534	8.2087	11.4106	4.0613	5.8798	6.6919	8.6991	12.1725	12.3841	10.2771
6	19.0150	8.7136	5.6496	14.7791	12.0007	17.3150	10.3676	17.1883	8.5542	5.1544
7	13.7678	17.1864	9.5216	11.3132	5.1484	7.9064	13.3484	6.0798	10.6810	14.6176
8	11.9285	9.2793	9.5300	10.9811	6.4705	8.3198	10.1218	4.7634	8.9523	7.3645
9	12.2726	7.0765	12.9150	11.2232	9.3524	6.5209	5.8827	10.9141	14.8175	14.1636
10	4.3265	9.1949	6.7857	10.5474	10.8313	17.9135	2.8196	8.8829	7.6546	6.6831
11	10.4583	3.4714	9.7626	5.9101	10.5449	6.0276	2.9715	12.7119	7.4249	4.9223
12	8.4716	4.9938	9.8488	5.7529	8.1471	4.4695	5.8214	3.4978	3.6641	7.1728
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

4. SUMMARY, CONCLUSION AND RECOMMENDATIONS

This study has examined the procedure for estimation of overall sample variances in time series decomposition and their statistical properties. The rationale for this study is to fill the gap in the existing estimation procedures, by providing analyst with a basis for the estimation of overall variance for mixed model. The emphasis is to obtain the Buys-Ballot estimates of overall sample variances for the mixed model and compare it with those of multiplicative and additive models.

Results from Table 1 indicate that, the overall variances are not same for the three models. In particular, while the overall variance of the Buys-Ballot table is a product of weighted average of the square of the seasonal component (S_j^2) for the mixed model. It depends on the column j only through the square of the seasonal component (S_j^2) and trend parameters through the square of the seasonal mean for multiplicative model. A function of trending curves and the seasonal component (S_j^2) for the additive model.

Empirical example has been used to assess the validity of the proposed test by Nwogu *et.al* [5] and Dozie *et.al* [6]. Result from simulated series indicates that the proposed test is capable of identifying the model correctly as mixed 100 percent of the times.

The study has provided a basis for the estimation of overall sample variance for the mixed model when trending curve is linear. Other trending curves are quadratic, exponential etc are yet to be considered, are recommended for further investigation.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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