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A Twin Prime Analog of Goldbach's Conjecture

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Abstract

Starting from the observation that the ratio of the number of twin primes to the number of primes up to a given number *n*, is similar to the ratio of the number of primes to the number of positive integers in the same interval, and from Goldbach's conjecture that any even integer greater than 2 can be expressed as the sum of two primes, it is conjectured that any even integer strictly larger than a prime P is the sum of two twin ranks or, equivalently, any prime number is the sum of two twin ranks minus 1. This conjecture was verified up to the 10000th prime, and no counterexample was found.

Keywords: Goldbach's conjecture; distribution of primes; twin primes.

1 Introduction

In a previous paper [1] it was shown that by analyzing the twin primes from the point of view of their analogues, the twin ranks, one arrives at several characteristics potentially useful for solving the Twin Prime Conjecture [2]. Based on this approach, it was concluded [3] that there is a limit on the numbers of twin ranks up to a prime P and, hence, on the number of twin primes up to $6P$. Here we present an important characteristic of the twin ranks, namely the fact that any prime number can be constructed from only two twin ranks.

2 Definitions

- Twin prime pair: Two consecutive prime numbers P_1 and P_2 characterized by the equality $P_2 = P_1 + 2$.

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- Twin index: The number $K = P_1 + 1$ between a pair of twin primes P_1 and P_2.
- Twin rank: A positive integer $k = K/6$.

3 Analysis

According to Goldbach's conjecture [4], any even integer greater than 2 can be expressed as the sum of two primes. Can it be expressed as a sum of two twin ranks? In order to answer this question, let us follow Brun [5] and write the following two sequences of numbers where the twin ranks were written in bold:

1 **2** **3** 4 **5** 6 **7** 8 9 **10** 11 **12** 13 14 15 16 **17** **18** 19 20 21 22 **23** 24 **25** 26 27 28 29 **30** 31
61 60 59 **58** 57 56 55 54 53 **52** 51 50 49 48 **47** 46 **45** 44 43 42 41 **40** 39 **38** 37 36 35 34 **33** **32** 31

We note that the sum of a number on the first line with the number immediately below is always 62. Of these numbers three pairs (3_59, 19_43 and 31_31) are primes and two pairs (10_52 and 30_32) are twin ranks, because (as per the above definition) when multiplied by 6 they give the twin indices 60, 312, 180 and 192 corresponding to the twin prime pairs 59_61, 311_313, 179_181 and 191_193. (For more about twin ranks see ref. [1]). Since the number of twin primes in a large enough interval is much smaller than the number of primes, one would not expect them to be able to express all even numbers in the set of positive integers as a sum of two terms. Indeed, the number 16 for example cannot be expressed as the sum of two twin ranks. One has:

1 **2** **3** 4 **5** 6 **7** 8 9 **10** 11 **12** 13 14 15
15 14 13 **12** 11 **10** 9 8 **7** 6 **5** 4 **3** **2** **1**

However, if we reduce the size of the set we want to express as a sum of two twin ranks to a smaller set, we might have a chance to succeed. Let us find such a set.

According to the prime number theorem [6] the number of primes less than or equal to some integer *n* is on the order of

$$\pi(n) \propto n / \ln n \tag{1}$$

Also, according to the Hardy-Littlewood "conjecture B" [7], the number of twin prime pairs less than *n* is on the order of

$$\pi_2(n) \propto n / \ln^2 n \tag{2}$$

It follows that the ratio $\pi_2(n)/\pi(n) \approx 1/\ln n$ of the number of twin primes to the number of primes up to a given number *n* is similar to the ratio of the number of primes to the number of positive integers in the same interval.

Consequently, we limit our search to a number of positive integers equal to the number of primes, more specifically to the set of even positive integers situated next to a prime number. Indeed, numerical computations seem to suggest the following

Conjecture 1: Any prime number is the sum of two twin ranks minus 1.

$$P = k_i + k_j - 1 \tag{3}$$

4 Results

We verified this conjecture up to the 10000th prime $P_z = 104729$, without finding a counterexample. Of course, 10^5 means nothing in number theory, but since the number of ways available to represent a number as the sum of two other numbers increases considerably as one goes up on the number series, there is a fair chance of the conjecture being true.

The variants $P = k_i + k_j$ and $P = k_i + k_j + 1$ are not true. The same about $n = k_i + k_j$ and $n = k_i + k_j \pm 1$ involving the whole set of positive integers, but here is an interesting fact: For $2n - 1 = k_i + k_j$ all counterexamples up to $n = 2 \times 10^5$ were prime numbers. From this observation one may be tempted to suggest the following

Conjecture 2: Any odd number which is not a prime is the sum of two twin ranks.

As expected, for large enough even numbers of the form $n = P + 1$ the ratio of the numbers of twin ranks N_k that satisfy Conjecture 1 to the numbers of primes N_p that satisfy Goldman's conjecture is on the order of $1/\ln n$. One has:

$$N_k \propto N_p / \ln n \tag{4}$$

Because all twin prime indices are situated in the number series next to a prime, if Conjecture 1 is true, then one can enounce immediately the following

Corollary 1: Any twin index is the sum of two twin ranks.

Indeed, with $K = P + 1$ and $P = k_i + k_j - 1$, it is easy to see that:

$$K = k_i + k_j \tag{5}$$

Since the twin indices and the twin ranks are related by $K = 6k$, this is a much weaker statement than the original conjecture and has a chance to be proved. Such a proof will lead at once to the following

Corollary 2: Any twin prime is the sum of two twin ranks ± 1.

Indeed, with $K = k_i + k_j$ and $K = P_1 + 1 = P_2 - 1$, it is easy to see that $P_1 = k_i + k_j - 1$ and $P_2 = k_i + k_j + 1$.

5 Concluding Remarks

As Hardy put it sometime ago [8] "…there are theorems, like 'Goldbach's Theorem,' which have never been proved and which any fool could have guessed." We completely agree with this statement. The reason we ventured to suggest the above conjectures is because very few facts are known about twin ranks and any observation linking them to the prime numbers can be useful.

Competing Interests

Author has declared that no competing interests exist.

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