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Method of Finding Initial Solution for the Geometric Programming Problem

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Short Research Article

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Abstract

The paper proposes a new method for finding an initial solution in the problem of geometric programming. The paper describes the conditions, under which the geometric programming problem of obtaining a positive solution of the matrix equation is solved. This equation describes the orthogonality and normalization conditions. The authors gave an example of application of the method in case of solving the problem of minimizing the risk of the object safety violation for the level crossing (technical object with safety requirements).

Keywords: Geometric programming; posynomial; objective function; dual problem; matrix; positive solution; orthogonality; normalization.

1 Introduction

A number of practical problems are well approximated to geometric programming [1-5]. We can solve them by quantifying into a mathematical optimization model. For example, geometric programming technique is extremely useful in engineering analysis and design of electrical circuits (e.g., VLSI circuit component sizing) [2] and in solving water resources optimization problems [3]. It has been applied to a wide variety of problems in economics as well [4].

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2 Main Results

Let us consider the problem of geometric programming [5] for finding the minimum of the objective function (criterion)

$$y = f(\boldsymbol{x}) = u_1(\mathbf{x}) + u_2(\mathbf{x}) + \cdots + u_n(\mathbf{x}). \tag{1}$$

Let $u_i(\mathbf{x})$ be a *posynomial*

$$u_i(\boldsymbol{x}) = C_i \cdot \prod_{j=1}^{m} x_j^{a_{ij}},\ C_i > 0,\ i = 1,2,\dots,n,\ a_{ij} \in \mathbb{R},$$

and the vector $\mathbf{x} = (x_1, x_2, \dots, x_m)$ of some components (parameters) x_j is positive. An example of this would be a common risk of the technical object safety violation; the criterion would be expressed as a sum of particular risks $u_i(\mathbf{x})$ on a certain threats of safety violation. In this case, $\mathbf{x}$ would be the vector of parameters of a business object protection system. In the theory of reliability, the function (1) is met as a common rate of failure, which expressed as a sum of separate rates of failure $u_i(\mathbf{x})$. The coefficients a_{ij} and C_i are got by methods of linear regression analysis (see [6]).

The matrix $A = (a_{ij})$ is called an exponent matrix. Suppose that the matrix A has the rank $r(A) = m$. This can be written as a block matrix

$$A = \binom{B}{H}. \tag{2}$$

Here, basis B is a $m \times m$ matrix whose determinant $\det B \neq 0$, and a submatrix H contains d rows of the matrix A, which doesn't belong to the basis B. The difficulty level is characterized by this number $d = n - m$. In the paper, we consider the case $d > 1$. The case $d = 1$ was considered in [7].

Let us consider a $d \times m$ matrix $Q = -H \cdot B^{-1}$ and a $d \times n$ matrix S= (Q, I_d), where I_d is the identity $d \times d$ matrix.

According to [8], the minimum $y_* = f(x_*)$ of the objective function (1) and components x_{*j} of optimal vector $\boldsymbol{x}_* = (x_{*1}, x_{*2}, \dots, x_{*m})$ are given in analytical form:

$$y_* = \prod_{i=1}^{n}\left(\frac{C_i}{\delta_i^*}\right)^{\delta_i^*},\ x_{*j} = \prod_{\nu=1}^{m}\left(\frac{\delta_\nu^* y_*}{C_\nu}\right)^{k_{j\nu}},\ j = 1, 2, \dots, m. \tag{3}$$

Here, numbers $\delta_i = \delta_i^*$ are elements of vector $\delta = \left(\delta_1, \delta_2, \dots, \delta_{m,} \delta_{m+1}, \dots, \delta_n\right) = (\delta_{(m)}, \delta_{(d)})$, where $\delta_{(m)} = (\delta_1, \delta_2, \dots, \delta_m)$ and $\delta_{(d)} = (\delta_{m+1}, \delta_{m+2}, \dots, \delta_{m+d})$, which are found from the equation system:

$$\delta_{(m)} = (\delta_1, \delta_2, \dots, \delta_m) = \delta_{(d)} \cdot Q;\ \delta_{(d)} \cdot \mu = 1;\ \delta_{m+i} = \frac{C_{m+i}}{\lambda} \cdot \prod_{j=1}^{m}\left(\frac{C_j}{\lambda \cdot \delta_j}\right)^{s_{ij}}; \tag{4}$$

$$i = 1, 2, \dots, d.$$

The column vector $\mu = \begin{pmatrix} \mu_1 \\ \cdots \\ \mu_d \end{pmatrix}$ consists of components μ_i, each of which is the sum of elements of row with number i of the matrix $S = (s_{ij})$.

In expression (4), the first and the second ratios are called the orthogonality and normalization conditions respectively, and the last one – third – expresses the condition of optimality.

In [8], it was proven that the condition (4) expresses the uniqueness of the solution $\delta > 0$ if it exists. To get δ in [8], the iteration method of solving was applied, which uses simple matrix operations at each step. The convergence of this method has been proven.

In the formulas (4), λ denotes the maximum value

$$V^* = V(\delta^*) = \prod_{i=1}^{n}(\frac{C_i}{\delta_i^*})^{\delta_i^*} \tag{5}$$

of the dual function $V(\delta) = \prod_{i=1}^{n}(\frac{C_i}{\delta_i})^{\delta_i}$ over the area $\delta_+ = \{\delta_{(d)} > 0 \colon \delta_{(m)} = \delta_{(d)} \cdot Q, \delta_{(d)} \cdot \mu = 1\}$.

The following two systems of equations are used for finding the initial solution:

$$\delta_{(m)} = \delta_{(d)} \cdot Q \text{ and } \delta_{(d)} \cdot \mu = 1. \tag{6}$$

That is, we want to find a positive solution to the system of linear algebraic equations. It is known that this problem still has no general solution, which would also be acceptable computationally. In this paper, we solve the problem under the assumptions that reflect its specific features.

Consider the equation

$$\delta \cdot A = 0 \implies \delta \cdot \begin{pmatrix} B \\ H \end{pmatrix} = 0. \tag{7}$$

According to the theory of generalized inverse matrices [9], the general solution of equation (7) is given by the formula

$$\delta = \eta' \cdot P(A), \text{ where } P(A) = (I_n - A \cdot A^C) \tag{8}$$

by selecting all possible values of an arbitrary vector η' of dimension n. Here, A^C represents the so-called S - inverse matrix for A (see [9]). And in the our case (see (2)), we use the relation $A^C = (B^{-1}, O)$ to determine A^C. Because $A^C \cdot A = (B^{-1}, O) \cdot \begin{pmatrix} B \\ H \end{pmatrix} = I_m$, A^C is the left inverse of A. From this and (8), it follows that

$$\delta = \eta' \cdot \left(I_n - \begin{pmatrix} B \\ H \end{pmatrix} (B^{-1}, \quad O_{m\times d}) \right) = \eta' \cdot \left(I_n - \begin{pmatrix} I_m & O_{m\times d} \\ H \cdot B^{-1} & O_d \end{pmatrix} \right) =$$
$$= \eta' \cdot \begin{pmatrix} O_m & O_{m\times d} \\ -H \cdot B^{-1} & I_d \end{pmatrix} \text{ or } = \eta \cdot (Q \quad I_d)_{d\times n}, \tag{9}$$

where $\eta = (\eta_1, \eta_2, \dots, \eta_d)$ is an arbitrary vector.

From (9), it follows that $\delta = (\eta \cdot Q, \eta)$ and, therefore,

$$\eta = (\delta_{m+1}, \delta_{m+2}, \dots, \delta_{m+d}) \equiv (\delta_{(d)}). \tag{10}$$

This is an arbitrary vector η with a set of components $\delta_{m+1}, \delta_{m+2}, \dots, \delta_{m+d}$ with numbers $m + 1, m + 2, \dots, m + d = n$ of the vector δ.

Let us say that numbers $\delta_1, \delta_2, \dots, \delta_m$ and $\delta_{m+1}, \delta_{m+2}, \dots, \delta_{m+d}$ are the basic and free elements of the vector δ respectively.

Theorem 1.

Let the vector δ be selected from the orthogonality condition (7). Then the following statements will be true:

a) Vector δ is written as

$$\delta = \delta_{(d)} \cdot S \quad (11)$$

or

$$\delta_{(m)} = \delta_{(d)} \cdot Q, \quad (12)$$

where $\delta_{(m)}$ and $\delta_{(d)}$ are composed of basic and free (arbitrary) elements of the vector δ. In the expression (11) and (12), an arbitrary row $\delta_{(d)}$ can be selected only as a positive one: $\left(\delta = \delta_{(d)}\, S > 0\right) \Longrightarrow\ \delta_{(d)} > 0$.

b) Normalization conditions type of $\delta_1 + \delta_2 + \cdots + \delta_n = 1$ is written by free elements of the vector δ as

$$\mu_1 \cdot \delta_{m+1} +\ \mu_2 \cdot \delta_{m+2} + \cdots + \mu_d \cdot \delta_{m+d} = \delta_{(d)} \cdot \mu = 1. \quad (13)$$

Moreover, the number $\delta_{m+i} > 0$ and the coefficient μ_i is equals to the sum of row elements s_i of the matrix

$$S\text{, i.e., } \mu_i = s_{i1} + s_{i2} + \cdots + s_{im} + 1. \quad (14)$$

c) The set δ_+ of all admissible vectors δ is given by (11) by specifying all possible values of its free elements $\delta_{m+i} > 0$ satisfying the condition (13).

The dual problem (5) has a feasible solution δ if and only if there exists a row $\delta_{(d)} > 0$, for which $\delta_{(m)} > 0$.

d) If matrix Q comprises of at least one column $q^i < 0, i = 1, 2, \dots, m$ then the dual problem (5) hasn't solution.

Proof.

a) Relation (11) follows directly from (9) and formula (13) follows from the fact that $\left(\delta = \delta_{(d)} \cdot S\right) \Leftrightarrow \left(\delta = \delta_{(d)} \cdot (Q, I) = (\delta_{(d)} \cdot Q, \delta_{(d)})\right) \Longrightarrow \left(\delta_{(m)} = \delta_{(d)} \cdot Q\right)$.

Here, an arbitrary vector $\delta_{(d)}$ can be selected only as a positive one, because the task is to find out the conditions of orthogonality of vectors $\delta = \delta_{(d)} \cdot S > 0$. Hence, we conclude that $\delta_{(d)} > 0$.

b) The normalization conditions $\sum_{i=1}^{n} \delta_i = 1$ can be written as

$\delta \cdot \mathbf{1} = 1 = \delta_{(d)} S \cdot \mathbf{1} = \delta_{(d)} \cdot \mu$, where **1** is a column of ones from which we obtain the relation (14).

c) The set of all vectors $\delta > 0$ is found by the formula (11) by giving all possible values of its free elements $\delta_{m+i} > 0$, satisfying the condition (13), because the expression (12) is the general solution of the homogeneous equation $\delta \cdot A = 0$.

From this it follows, that the task (7) has a positive solution if and only if there exists a row $\delta_{(d)} > 0$, for which $\delta_{(m)} = \delta_{(d)} \cdot Q > 0$.

d) Let the matrix Q contains a column $q^j < 0$. Then according to (11), we obtain the expression $\delta_j = \delta_{(d)} \cdot q^j$, in which, as shown above, $\delta_{(d)} > 0$, hence, in view of the $q^j < 0$ it follows, that $\delta_j < 0$. Hence, we conclude that if the matrix Q comprises at least one column $q^j < 0$, then the problem (7) has no solutions.

Theorem 2.

Let in the equation $\delta_{(d)} \cdot \mu = 1$, vector $\mu > 0$ and each column q^j of matrix Q have at least one positive element. Then for the dual problem there exists a positive solution δ, given by the conditions:

1. We define the free variables row as

$$\delta_{(d)} = \frac{1}{\sigma} \cdot \mu^T , \tag{15}$$

where number $\sigma = \mu^T \cdot \mu = \mu_1^2 + \mu_2^2 + \cdots + \mu_d^2 > 0$, ($T$ denotes transposition). In this case, the following conditions are true: $\delta_{(d)} > 0$, $\delta_{(d)} \cdot \mu = 1$.

2. Let the row $\frac{1}{\sigma}\mu^T \cdot Q \equiv (\omega_1, \omega_2, \dots, \omega_m) > 0$, i.e., all $\omega_j > 0, j = 1, 2, \dots, m$. Then the following inequality is true:

$$\delta_{(m)} = \frac{1}{\sigma} \cdot \mu^T \cdot Q > 0, \tag{16}$$

which means that the theorem is carried out.

3. If in the row $(\omega_1, \omega_2, \dots, \omega_m)$ there is the element $\omega_j < 0$, and q^j is corresponding to its column of the matrix $Q \equiv (q^1\, q^2 \dots q^m)$, then we use the expression for the element δ_j of the row $\delta_{(m)}$ in the form of

$$\delta_j = \omega_j + \eta \cdot q^j - \eta \cdot \mu \cdot \omega_j = -\omega_j \cdot (\, \eta \cdot \mu - 1) + \eta \cdot q^j, j = 1, 2, \dots, m, \tag{17}$$

where $\eta = (\eta_1, \eta_2, \dots, \eta_d)$ is an arbitrary vector.

If the column q^j has its components $q_i^j < 0$ then in (17) we believe $\eta_i = 0$. The rest of the elements of the row $\eta > 0$ we believe are positive: $\eta_s > 0$. In this case, the number $\eta \cdot q^j > 0$ for any $\eta_s > 0$.

Numbers $\eta_s > 0$ are chosen by the condition $\eta \cdot \mu - 1 > 0$ and we arrive at the inequalities

$$\eta \cdot q^j > 0, -\omega_j \cdot (\eta \cdot \mu - 1) > 0,$$

whence follows the relation (16).

Proof.

a) According to the theory of generalized inverse matrices [9], the set of solutions of equation $\delta_{(d)} \cdot \mu = 1$ can be found by the formula

$$\delta_{(d)} = \frac{1}{\sigma} \cdot \mu^T + \eta \cdot (I_d - \frac{1}{\sigma} \cdot \mu \cdot \mu^T) \tag{18}$$

by specifying all possible values of the random vector η.

Provided that $\eta = 0$, we get one of the solutions $\delta_{(d)} = \frac{1}{\sigma} \cdot \mu^T$, satisfying the relations $\delta_{(d)} > 0$, $\delta_{(d)} \cdot \mu = 1$. Further, according to (18), the arbitrary vector $\eta = (\eta_1, \eta_2, \dots, \eta_d) > 0$ is chosen.

b) For the row $\delta_{(m)}$ of basic variables $\delta_j, (j = 1, 2, \dots, m)$, we have

$$\delta_{(m)} = \delta_{(d)} \cdot Q = \mu^T \cdot Q \cdot \frac{1}{\sigma} + \eta \cdot Q - \eta \cdot \mu \cdot \mu^T \cdot Q \cdot \frac{1}{\sigma}, \quad (19)$$

and $\delta_j = \omega_j + \eta \cdot q^j - \eta \cdot \mu \cdot \omega_j = -\omega_j \cdot (\eta \cdot \mu - 1) + \eta \cdot q^j$.

Let the row $(\omega_1, \omega_2, \dots, \omega_m) > 0$. Then we have the inequality (16), and in this case the theorem is true.

c) If the row $(\omega_1, \omega_2, \dots, \omega_m)$ has elements $\omega_j < 0$ and q^j is corresponding to its column of the matrix Q, we obtain an expression for element δ_j of the row $\delta_{(m)}$ in the form of relation (17).

If the column q^j has component $q_i^j < 0$, then in (17) we believe $\eta_i = 0$. The remaining components of the row η remain positive: $\eta_s > 0$. Since according to the hypothesis, the column q^j contains at least one element $q_i^j > 0$, then in (17) the number $\eta \cdot q^j > 0$ for any $\eta_s > 0$. Choosing numbers η_s from the condition $\eta \cdot \mu - 1 > 0$ we arrive at the inequalities: $\eta \cdot q^j > 0$, $-\omega_j\,(\eta \cdot \mu - 1) > 0$, so, $\delta_j > 0$, whence follows the relation (16).

3 Numerical Example

As example of using our method, let us consider a level crossing as a technical object with safety requirements. The method, based on the theory of geometric programming, allows us to solve the problem of minimizing common risk of the object safety violation in a simple analytic form due to the choice of object protection parameter set.

It is known that a large proportion of traffic incidents are committed on level crossing. Thus a task of ensuring safety on level crossing is relevant. Let us consider a simplified formulation and solution of the task, since the total volume of its solution is problematic and it is beyond the scope of this article. Let's consider the following safety threats:

- U_1 - drive over level crossing at red traffic light by a law abiding drivers (group I);
- U_2 - drive over level crossing at red traffic light by a criminal drivers (group II);
- U_3 - a vehicle's collision on the level crossing that doesn't stop the tracks;
- U_4 - a collision with a train and other traffic incidents, which lead to a stop of transport on the tracks.

The group II consists of a car thieves, a drunk drivers, pursued criminals and other persons whose contact with the police is tantamount to arrest them. The group I consists of violators which do not belong to the group II. Common threat U of object safety consists of at least one of the threat U_1, U_2, U_3, U_4: $U = \sum_1^4 U_i$. In fact, the number of threats n is much more than 4. But to keep things simple we'll assume that $n = 4$. This is enough to illustrate the proposed method of estimating and minimizing the common risk of the object safety violation.

Suppose that the events U_i are independent, $i = 1, 2, 3, 4$, and the probability of common risk of the object safety violation $y = P(U)$ is expressed as a sum of particular risks u_i:

$$y = u_1 + u_2 + u_3 + u_4,$$

where $u_1 = P(U_1)$; $u_2 = P(U_2)(1 - u_1)$, $u_3 = (P(U_3)(1 - u_1)(1 - u_2)$; $u_4 = P(U_4)(1 - u_1)(1 - u_2)(1 - u_3)$.

Let in addition, $u_1 = P(U_1) \approx \frac{M_1}{N_1}$. Here, the fraction $\frac{M_1}{N_1}$ is an estimate of the particular risk u_1, where N_1 is total number of vehicles which passed through the crossing for time *T* (let us say *T=1* day) and M_1 is number of drivers of the group I which passed through the crossing for time *T* at red traffic lights.

Similarly, we can estimate other particular risks, for example, $u_2 = P(U_2)(1 - u_1) \approx \frac{M_2}{N_2}$. The fraction $\frac{M_2}{N_2}$ is an estimate of the particular risk u_2, where N_2 is total number of vehicles which passed through the level crossing for time *T* without violators of the group I. Here, M_2 is a number of drivers of the group II which passed through the level crossing for time *T* at red traffic lights.

Suppose that the technical object has safety protection system. This system includes signal operator, road inspectors, technical means of preventing violations such as barriers, remote control system barriers etc.

Protection system gives us the following parameters:

- x_1 – time of duty by road inspectors;
- x_2 – time between duty;
- x_3 – average time between the opening and closing of the barriers.

In fact, the number of parameters x_i is much more than $m = 3$. But this is enough to illustrate the estimation method of minimizing the common risk of the object safety violation. A table below is a fragment of empirical data for 10 observations, which we used for the calculations. Each row in the table corresponds to time *T=1* day for situation on the level crossing.

Table 1. A fragment of empirical data for 10 observations

Probability of threats U_i (multiplied by 10^3)				**Protection parameters (in hours)**		
u_1	u_2	u_3	u_4	x_1	x_2	x_3
2.10	0.20	5.80	0.51	2.00	4.00	0.25
0.42	0.53	1.09	0.20	2.50	3.00	0.20
0.75	0.29	3.00	0.63	3.00	5.00	0.25
2.00	0.52	0.17	0.20	1.50	2.00	0.10
5.90	0.16	3.80	0.43	1.30	3.00	0.20
1.80	0.35	1.12	0.12	1.30	2.00	0.20
0.30	0.53	3.08	0.52	4.00	5.00	0.25
2.80	0.23	1.28	0.40	1.70	3.00	0.15
4.90	0.13	7.50	1.22	2.00	5.00	0.20
3.30	0.20	1.44	2.80	3.00	6.00	0.10

These data are mainly expert evaluation of road inspectors, ambulance workers and staff which services the technical means of preventing violations. We assume that the vector $x = (x_1, x_2, x_3)$ of protection parameters is positive.

Let $u_i = u_i(x)$ be a posynomial $u_i = u_i(x) = C_i \cdot \prod_{j=1}^{3} x_j^{a_{ij}}$, where $C_i > 0$, $i = 1, 2, 3, 4$. (20)

Here, the matrix $A = (a_{ij})$ is an exponent matrix. Taking the logarithm of both side of (20), we obtain

$$ln u_i(x) = a_{i0} + a_{i1} ln x_1 + a_{i2} ln x_2 + a_{i3} ln x_3 \, , \, i = 1, 2, 3, 4, \, C_i = e^{a_{i0}}. \quad (21)$$

Thus, we can get coefficients a_{ij} by methods of linear regression analysis [6]. Writing the equation (21) for the first row of the Table 1, we obtain the expression for the risk u_1: $ln2{,}1 = a_{10} + a_{11} ln2 + a_{12} ln4 + a_{13} ln0.25$ or $a_{10} + 0.693a_{11} + 1.386a_{12} - 1.386a_{13} = 0.742$. Similarly, writing the equation (21) for next rows of the Table 1 we obtain an algebraic system

$$Fa^1 = w^1, \tag{22}$$

where $a^1 = (a_{10},\ a_{11},\ a_{12},\ a_{13})^T$ is a column of vector of required coefficients in the posynomial $u_1(x) = C_1 \cdot \prod_{j=1}^{3} x_j^{a_{1j}}$, $C_1 = e^{a_{10}}$ and the matrix F is expressed as $F = (\mathbf{1},\ lnx^1,\ lnx^2,\ lnx^3)$.

Moreover, ***1*** is a column of ones; $lnx^j, j = 1, 2, 3$ is a column for values lnx_{js} of the lnx_j for the parameter x_j and 10 observations $s = 1, 2, \ldots, 10$; w^1 is a column for values $lnu_{is}(x)$ of the $lnu_i(x)$; $w^1 = (ln2.1 \quad ln0.42 \ \ldots \quad ln3.3)^T$.

According to the method of least squares (MLS) the solution $a^i = \dot{a}^i$ of the equation (22) is given in the form [6]:

$$\dot{a}^1 = F^+ w^1. \tag{23}$$

Here F^+ is a pseudoinverse matrix of the matrix F. Calculation method for the matrix F^+ is given in the paper [9]. Recall that the pseudoinverse is defined and unique for all matrices whose entries are real or complex numbers. Vector $\dot{a}^1$ is a solution of equation (22) under the condition that the equation is compatibility. In the converse case, $\dot{a}^1$ is the best approximation solution (according to the MLS).

Thus, $\dot{a}^1 = F^+ w^1 = \begin{pmatrix} a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \end{pmatrix} = \begin{pmatrix} -2.08 \\ -4 \\ 3 \\ -1 \end{pmatrix}$, $C_1 = e^{-2.08} = 0.125$ and the polynomial u_1 is expressed as $u_1 = u_1(x) = 0.125x_1^{-4}x_2^3x_3^{-1}$. Using the given calculate scheme for the risks u_2, u_3, u_4, we obtain $u_2 = u_2(x) = 0.8x_1^2x_2^{-2}$, $u_3 = u_3(x) = 6x_1^{-2}x_2^3x_3^2$, $u_4 = u_4(x) = 0.004x_1^{-1}x_2^3x_3^{-1}$. The common risk y at the interval *[0, T]* of time is expressed as

$$y = f(x) = 0.125x_1^{-4}x_2^3x_3^{-1} + 0.8x_1^2x_2^{-2} + 6x_1^{-2}x_2^3x_3^2 + 0.004x_1^{-1}x_2^3x_3^{-1}.$$

Coefficient of variation $\dot{V}$ is used for precision and sufficiency to empirical data: $\dot{V} = \frac{\dot{\sigma}}{\dot{y}} 100\%$. Here, $\dot{y} = \dot{u}_1 + \dot{u}_2 + \dot{u}_3 + \dot{u}_4$. Moreover, $\dot{u}_i, i = 1, 2, 3, 4$ is a sample mean of observations u_{is}, $s = 1, 2, \ldots, 10$; and $\dot{\sigma}^2$ is the sum of the sample variance $\dot{\sigma}_i^2 = \left\|F\dot{a}^i - w^i\right\|^2/(N - m - 1)$, where N is the total number of observations and m is the number of protection parameters ($m = 3$); $\|\cdot\|$ means an Euclidean norm of vector; $\dot{\sigma}_i^2$ is the sample variance according to the MLS-solutions $\dot{a}^i = F^+ w^i$ for the algebraic system $Fa^i = w^i$, $i = 1, 2, 3, 4$.

In our case, $\dot{\sigma} = \sqrt{\dot{\sigma}_1^2 + \dot{\sigma}_2^2 + \dot{\sigma}_3^2 + \dot{\sigma}_4^2}$. We obtained $\dot{V} = 9\%$ as a result of data processing for *N=100*. This good result gives acceptable discrepancy between the experimental and calculated data [9].

It should be found the vector $x = x_* > 0$, with components x_{j^*} such that value $y_* = f(x_*)$ is minimal. Using exponents a_{ij}, we can write the exponent matrix *A* as

$$A = (a_{ij}) = \begin{pmatrix} B \\ H \end{pmatrix} = \begin{pmatrix} -4 & 3 & -1 \\ 2 & -2 & 0 \\ -2 & 3 & 2 \\ -1 & 3 & -1 \end{pmatrix}, \text{ where } B = \begin{pmatrix} -4 & 3 & -1 \\ 2 & -2 & 0 \\ -2 & 3 & 2 \end{pmatrix} \text{ and } H = (-1 \quad 3 \quad -1).$$

Note that $detB \neq 0$. It follows that exist an inverse matrix B^{-1}:

$$B^{-1} = \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix} = \begin{pmatrix} -2 & -\frac{9}{2} & -1 \\ -2 & -5 & -1 \\ 1 & 3 & 1 \end{pmatrix}.$$

In our case sub-matrix $H = (-1 \quad 3 \quad -1)$ contains one row of matrix *A*, which do not belong to the sub-matrix *B*. Using the formulas from [8], we get subsidiary variables δ_i. These ones are called dual variables and are found by the formula

$\delta^T = (\delta_1,\ \delta_2,\ \delta_3,\ \delta_4) = \frac{1}{\mu}(-H \cdot B^{-1}, \quad 1) = \frac{1}{\mu}(5,\ \frac{27}{2},\ 3,\ 1)$, where the number $\mu = 5 + \frac{27}{2} + 3 + 1 = \frac{45}{2}$. Therefore $\delta^T = (\delta_1,\ \delta_2,\ \delta_3,\ \delta_4) = \frac{1}{45}(10, 27, 6, 2)$.

Thus, $\delta_1 = \frac{10}{45}, \delta_2 = \frac{27}{45},\ \delta_3 = \frac{6}{45}, \delta_4 = \frac{2}{45}$.

Using the formulas of the paper [10], we can write the minimal value y_* multiplied by 10^3 of the common risk y due to $C_1 = 0.125$, $C_2 = 0.8$, $C_3 = 6.0$, $C_4 = 0.004$. In our case $y_* = \prod_{i=1}^{4} \left(\frac{C_i}{\delta_i}\right)^{\delta_i} = \left(\frac{0.125}{10}\right)^{\frac{10}{45}} \left(\frac{0,8}{27}\right)^{\frac{27}{45}} \left(\frac{6}{6}\right)^{\frac{6}{45}} \left(\frac{0,004}{2}\right)^{\frac{2}{45}} = 1.55$, i.e. minimal value of the common risk is 0.155%. Then the protection parameters $x_{j^*},\ j = 1, 2, 3$, can be found from the equations:

$$x_{1^*} = \prod_{i=1}^{3} \left(\frac{\delta_i \cdot y_*}{C_i}\right)^{k_{1i}} = \left(\frac{10 \cdot 1.55}{45 \cdot 0.125}\right)^{-2} \left(\frac{27 \cdot 1.55}{45 \cdot 0.8}\right)^{-\frac{9}{2}} \left(\frac{6 \cdot 1.55}{45 \cdot 6}\right)^{-1};$$

$$x_{2^*} = \prod_{i=1}^{3} \left(\frac{\delta_i \cdot y_*}{C_i}\right)^{k_{2i}} = \left(\frac{10 \cdot 1.55}{45 \cdot 0.125}\right)^{-2} \left(\frac{27 \cdot 1.55}{45 \cdot 0.8}\right)^{-5} \left(\frac{6 \cdot 1.55}{45 \cdot 6}\right)^{-1};$$

$$x_{3^*} = \prod_{i=1}^{3} \left(\frac{\delta_i \cdot y_*}{C_i}\right)^{k_{3i}} = \left(\frac{10 \cdot 1.55}{45 \cdot 0.125}\right) \left(\frac{27 \cdot 1.55}{45 \cdot 0.8}\right)^{3} \left(\frac{6 \cdot 1.55}{45 \cdot 6}\right).$$

Thus, we get optimal protection parameters (in hours) $x_{1^*} = 1.98, x_{2^*} = 1.84, x_{3^*} = 0.15$. If the proposed model is acceptable with respect to the coefficient of variation, it allows us to solve the problem of minimizing common risk of the object safety violation in a simple analytic form due to the choice of object protection parameter set.

4 Final Remarks

A new method for finding an initial solution in the problem of geometric programming was proposed. The conditions were described, under which the geometric programming problem of obtaining a positive solution of the matrix equation can be solved.

There is an example of solving the problem of minimizing the risk of the technical object safety violation. It is worth noting that the proposed method allows to provide the solution to the technical problem in the form of analytical expression (not in the traditional form of an approximation).

Competing Interests

Authors have declared that no competing interests exist.

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