The Effects of Dark Energy on Gravitation

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Authors' contributions

This work was carried out in collaboration among all authors. Author EAE designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author OO managed the analyses of the study. Authors JNA and FC managed the literature searches. All authors read and approved the final manuscript.

Short Research Article

ABSTRACT

Dark energy is thought to be very homogenous, not very dense and it is not known to interact through any of the fundamental forces other than gravity. In this research work, we use analytical method to find a model that may possibly relate dark energy to gravitation. After some plausible analyses, we obtain a relation, $\mu = \frac{2G^2 m_1 m_2 \Lambda}{\dot{R}(t) H_o R_o^3}$ where μ is Gravitational potential energy, m_1 is the mass of galaxy, m_2 is mass of earth, Λ is dark energy, $\dot{R}(t)$ is receding velocity, H_o is Hubble's constant and R_o is the distance measured from the point of observation and the galaxy. Moreover, we estimate $K = \frac{2G^2 m_1 m_2}{R^3}$ for various galaxies, finally, the relation $\mu \propto \Lambda$, indicates a direct relationship between Dark energy and Gravitation.

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1. INTRODUCTION

Most of the energy in the universe consist of some form of dark energy that is gravitationally self-repulsive and that is causing the expansion of the universe to accelerate in which the possible features causing this expansion are; the vacuum energy density (cosmological constant) and quintessence, a time evolving, spatially, inhomogeneous component with negative pressure. Although this hypothesized dark energy cannot be seen with sophisticated instrument, but its effects are constantly felt in the universe as it accelerate the expansion of the universe [1]. It is under the purview of this project/ research work to explore and find the effects of this vacuum energy on gravitation and also to explicitly explain the features of gravitation that qualifies it as anti-dark energy. This task tends to be actualized by studying extensively the characteristics of both dark energy and gravity. The basis of the theory of dark energy begins with redshift. The expansion of the universe is detected through the distortion of the light emitted by the receding galaxies due to Doppler shifts; wavelengths received from galaxies moving away from the Milky Way are elongated and contain less energy. The observed redshift indicates that most galaxies are moving away from the Milky Way [2]. Among the most Direct evidence from precise measurements of the cosmological distance redshift relation of Type-Ia supernovae has indicated with high confidence that the expansion of the universe is accelerate [3]. The data has also implied evidence for a universe endowed with a positive "vacuum energy" density (not necessarily constant). This sort of positive "cosmological constant" (or "vacuum energy"), whose origin is totally unknown, dominates (as compared to matter) the present expansion of the universe. That sort of energy is the so-called "dark energy" [4]. Evidence that dark energy contributes with a large fraction to the energy content of the universe has also been given by measurements of the cosmic microwave background [5]. While dark energy has been invoked to account for observational data, its nature and origin remain totally unexplained discussed possibilities for it is the cosmological constant ("vacuum energy") or some sort of dynamical field, which is usually taken as a scalar field.

2. GRAVITY

Hernandez and Wang in 2014 [6] stated in their article that gravity is one of the four fundamental interactions/forces of nature and is certainly the first interaction/forces that people studied over centuries dating back to Aristotle, Galileo, Johannes Kepler, Isaac Newton and Albert Einstein. It was Albert Einstein who first derived the basic law of gravity. The Einstein gravitational field equations were obtained by postulating the principle of equivalence and the principle of general relativity. In mathematical terms, the principle of equivalence says that the space time is a 4-dimensional Riemannian Manifold with metric tensor of M being the gravitational potential [7].

2.1 Gravitational Potential Energy

This is the potential energy of a body which has a mass relative to another massive object due to gravity. It is relative to another massive object due to gravity. It is potential energy associated with the gravitational field. Gravitational energy is dependent on the masses of two bodies, their distance apart and the gravitational constant (G). [8] for simplicity sake, the concept of analytical method of mathematical analysis will be employed which will involve the analysis of physical variables from Newtonian physics to a higher level of consideration. Gravitational force as a conservative field makes integration over a closed path valid that is, work done is independent of the path taken [9].

In the Newtonian physics, the gravitational force cutting between two objects separated by a distance is given as

$$F = \frac{Gm_1m_2}{r^2} \tag{1}$$

Where F is gravitational force in Newton, m_1 is mass of object 1 m_2 is Mass of object 2, r is the distance separating the two objects [9].

The gravitational potential energy of the universe can be deduced from equation (1) by integrating with respect to displacement; thus, we have

$$W = \int \frac{Gm_1m_2dr}{r^2} \tag{2}$$

$$W = Gm_1m_2 \int_\infty^r \frac{1}{r2} dr \tag{3}$$

$$W = Gm_1m_2 \int_\infty^r r^{-2} dr \tag{4}$$

Evaluating the integral, we have:

$$w = Gm_1m_2 \left[\frac{r^{-1}}{-1}\right]_\infty^r \tag{5}$$

$$W = \left[\frac{-Gm_1m_2}{r}\right]_\infty^r \tag{6}$$

Substituting for r and∞, then we have [8].

$$W = \frac{-Gm_1m_2}{r} - \left(-\frac{Gm_1m_2}{\infty}\right) \tag{7}$$

$$W = \frac{-Gm_1m_2}{r} + \frac{Gm_1m_2}{\infty} \tag{8}$$

But, $\frac{1}{\infty} = 0$

Let µ = W

$$\mu = \frac{-Gm_1m_2}{r} \tag{9}$$

where, G is gravitational constant, m_1 is mass of the galaxy, m_2 is mass of the observer, r is the distance between the galaxy and the observer, µ is the Gravitational potential energy.

2.2 Gravitational Energy and Hubble's Law

2.2.1 Hubble's law

Hubble's law state that the distance to a given galaxy is proportional to the receding velocity as measured by the Doppler redshift. [10]. Hubble discovered that all galaxies were moving away from us. He found that the velocity of recession from us is proportional to the distance of galaxy from us. Mathematically, Hubble's law is expressed as;

V α r
$$V = Hr \tag{10}$$

where V is the receding velocity, H is Hubble's constant; r is Hubble's distance [10].

Hubble's equation can be written as a function of time as it is observed today.

$$V = H_oR(t) = \dot{R}(t) \tag{11}$$

$$H_oR(t) = \dot{R}(t) \tag{12}$$

$$R(t) = \frac{(\dot{R})t}{H_o} \tag{13}$$

Where R is Hubble's radius $\dot{R}(t)$ is receding velocity. This equation can be written as

$$\frac{1}{R(t)} = \frac{H_o}{\dot{R}(t)} \tag{14}$$

Substituting equation (3.14) into equation (3.9) we obtain

$$\mu = \frac{-Gm_1m_2Ho}{\dot{R}(t)} \tag{15}$$

where $r \equiv R(t)$

H_o = Expansion rate as observed today.

2.3 Gravitational Energy and Gravitational Redshift

2.3.1 Redshift

This is the stretching of light emitted from galaxies due to the emitting object's recession. Cosmic expansion causes the distance to increase [11].

According to the principle of equivalence, from general relativity, any frequency shift which can be shown to arise from acceleration of a radiating source could also be produced by the appropriate gravitational field [12].

The gravitational red ship is given by Carrol et al. [10].

$$Z = \frac{Gm_1}{rc^2} \tag{16}$$

Where c is velocity of light, G is gravitational constant, m_1 is mass of galaxy, r is radius of galaxy.

Recall that for small value of z,

$$z = \frac{v}{c} \tag{17}$$

(where $v = \dot{R}(t), v = z = \frac{\Delta\lambda}{\lambda} = \frac{v}{c}$ also from Hubble's law , $V = H_0r$). Substitute for $v = zc$ and make r the subject,

$$r = \frac{ZC}{Ho} \tag{18}$$

Substituting r in equation (16)

$$Z = \frac{Gm_1Ho}{ZC^3} \tag{19}$$

Making Ho the subject of the formula

$$H_o = \frac{Z^2C^3}{GM} \tag{20}$$

Substituting H_o in equation (15)

$$\mu = \frac{-Gm_1m_2Ho}{\dot{R}(t)}$$

$$\mu = \frac{-Gm_1m_2z^2c^2}{\dot{R}(t)Gm_1} \tag{21}$$

Equation (21) becomes

$$\mu = \frac{-M_2 z^2 c^3}{R(t)} \qquad (22)$$

2.4 Negative Pressure

The biggest mystery of the cosmic acceleration is not that two-third of the universe is made of stuff that we cannot see. To examine this strange property of dark energy, it is helpful to introduce a quantity which represents dark energy Λ = pdark/ρdark, density of dark energy in the universe [13].

$$\Lambda = \frac{p_{dark}}{\rho_{dark}} \qquad (23a)$$

In general relativity, the rate of change in the cosmic expansion is proportional to (p_{total} + $3\rho_{total}$); where ρ_{total} is the density of all matter and energy in the universe and p_{total} is the corresponding pressure. To account for the acceleration expansion, however, this quantity must be positive since p_{total} is a positive quantity, and the mean pressure due to both ordinary and dark matter is negligible because it is cold or non-relativistic. We arrive at the requirement that

$$3\Lambda p_{dark} + \rho_{dark} < 0 \qquad (23b)$$

for an accelerating expression, since

$$\rho dark \simeq {}^2\!/_3 \, \rho total \qquad (23c)$$

We find that $\Lambda > \frac{1}{2}$ where Λ is dark energy, so the pressure of the dark energy is negative[14].

The cosmological constant (dark energy) has negative pressure equal to its energy density and so causes the expansion of the universe to accelerate. The reason a cosmological constant (dark energy) has negative pressure can be seen in classical thermodynamics [15]. In general, energy must be lost from inside container (the container must do work on its environment) in order for the volume to increase, specifically, a change in volume dv requires work done equal to a change of energy pdv, where p is the pressure. But the amount of energy in a container full of vacuum actually increase when the volume increase, because the energy is equal to ρv where ρ is the energy density of the dark energy (cosmological constant) therefore pressure p is negative, p = -ρ [14].

Dark energy could as well be energy of vacuum, given that vacuum energy has:

$$P = \rho \Lambda. \qquad (23d)$$

The energy density required to explain the accelerated expansion is about [16].

$$\rho \Lambda \simeq \rho c = \frac{3H_0^2}{8\pi G} \simeq 10^{-29} \, kg\Big/_{cm^3}$$

Dark energy in terms of pressure and volume is represented as

$$\Lambda = -pdv \qquad (23e)$$

where p is pressure, v is the volume [14]

$$p \simeq -\rho$$

Where ρ is the energy density of the dark energy therefore

$$p\Lambda = \rho_c \qquad (24)$$

But the energy density required to explain the accelerated expansion is about three quarters of the critical density $p_{\Lambda \simeq \rho_c}$

$$\rho_c = \frac{3H_0^2}{8\pi G} \qquad (25)$$

$$\Lambda = -\rho_c \, dv \qquad (26)$$

Where ρ_c is critical density of dark energy.

Substituting equation (25) in (26), we have

$$\Lambda = \frac{-3H_0^2}{8\pi G} dv \qquad (27)$$

Assuming a spherical universe, we have the volume of the universe

$$v = \frac{4\pi r^3}{3} \qquad (28)$$

Substituting equation (28) in (27) we obtain,

$$\Lambda = \left(\frac{-3H_0^2}{8\pi G}\right)\frac{4\pi r^3}{3}$$
$$\Lambda = \frac{-12H_0^2 \pi r^3}{24\pi G} \qquad (29)$$

Therefore,

$$\Lambda = \frac{-H_0^2 r^3}{2G} \qquad (30)$$

Equation (30) above is an equation of dark energy where Λ is dark energy parameter. Let us assume that $r = R$ (Hubble's radius)

From equation 30, we have,

$$H_o^2 = \frac{-2G\Lambda}{R^3} \qquad (31)$$

Note that equation (15) can be written in the form of

$$\mu = \frac{-Gm_1m_2H_o^2}{\dot{R}(t)H_o} \qquad (32)$$

Substituting (31) in (32)

$$\mu = \frac{-Gm_1m_2}{\dot{R}(t)H_o}\left(\frac{-2G\Lambda}{R^3}\right)$$
$$\mu = \frac{2G^2m_1m_2\Lambda}{\dot{R}(t)H_oR_o^3} \qquad (33)$$

Equation (33) is the derived equation that correlates dark energy and gravitational potential energy.

3. RESULTS AND DISCUSSION

From equation (33), we have

$$\mu = \frac{2G^2m_1m_2\Lambda}{\dot{R}(t)H_oR_o^3}$$
$$\text{Let } K = \frac{2G^2m_1m_2}{R^3} \qquad (34)$$

Equation (33) can be written as

$$\mu = \frac{K\Lambda}{\dot{R}(t)H_o} \qquad (35)$$

Assuming that m_1 is the mass of the earth which is the location of the observer and m_2 is the mass of the various galaxies, and R is the respective distance of the galaxies from earth.

3.1 Milky Way Galaxy

Considering the nearest galaxy, which is the milky way galaxy, the mass of the Milky way is approximately $1.5x10^{30}$ MΘ which is equivalent to 3×10^{42} kg [16]. N/B 1 solar mass MΘ = $2x10^{30}$ kg. the distance of the Milky Way galaxy from the center of the earth is approximately $26 \times 10^2 ly$ [17].

The Milky Way galaxy contains the sun and its solar system and therefore Earth. Most things visible to the naked eye in the sky are part of it, including the Milky Way composing the zone of avoidance (the area of the sky that is obscured by the Milky Way) [18]. We measure the distances of some galaxies from the earth as a reference point.

3.2 Large Magelian Cloud Galaxy

The local group is the galaxy group that includes the Milky Way. It is a satellite galaxy of the Milky Way galaxy [19]. It has a distance of 163 X $10^5 ly$

From the Earth [20]. The LMC is the third close galaxy to the Milky Way. Substituting the values of the constants in equation (34) for Large Magelan Cloud, we have, Mass of LMC = $m_2 = 2X10^{40}$ kg (10 MΘ), = 163 X $10^{21}M$ (0.163 $X10^6$ ly) where the values of the gravitational constant and the mass of the earth remain the same, substituting the values in equation (34), we have,

$2.5X10^{-26}N^2MKg^{-2}$ Therefore equation (35) becomes,

$$\mu = \frac{2.5X10^{-26}\Lambda}{\dot{R}(t)H_o} \qquad (36)$$

3.3 Andromeda Galaxy

The Andromeda galaxy, also known as messier 31, is a spiral galaxy approximately (2.56 million light years) from the earth [21]. Its name stems from the area of the Earth's sky in which it appears as the constellation of Andromeda. This virial mass of the Andromeda galaxy is of the same order of magnitude as that of the Milky way, at a trillion solar masses (10^{12}MΘ).

Mass of the Andromeda galaxy = $3x10^{42}$ kg ($1.5X10^{12}$MΘ),

Distance of the Andromeda from the earth = $2.56X$ $10^{22}m$ (2.56 $X10^6 ly$) [22] substituting the values in equation (4.0), we have K = 95.03x $10^{-4}N^2MKg^{-2}$, equation (35) for Andromeda galaxy becomes

$$\mu = \frac{95.03 \, X10^{-4}\Lambda}{\dot{R}(t)H_o} \qquad (37)$$

The Table 1 shows the values of k for some selected galaxies and their respective distances from the earth.

3.4 Segue 2 Galaxy

Segue 2 is a dwarf spheroid galaxy situated in the constellation Aries and discovered in 2009 in the data obtained by Sloan Digital sky survey [23]. The galaxy is located at the distance of about 110, 000ly from the planetary system

which comprises of the sun and the earth (Milky Way galaxy) [23]. It is classified as a dwarf spheroid galaxy meaning that it has an approximately round shape with the half-light radius of about 43 pc [23]. The mass of segue 2 is $5.5X10^5 m\theta$. The name is due to the fact that it was found by the SEGUE program, the Sloan Extension for galactic understanding and exploration[24].

Mass of Segue 2 galaxy $= 11\ X10^{33} kg$ ($5.5\ X10^5 m\theta$) and the distance of segue 2 galaxy from the center of the earth is $11X10^{20} m$ (11X 10^4 ly) [23]. Substituting the values in equation (34), we have $K = 4.39X10^{-23} N^2 MKg^{-2}$, equation (35) for segue 2 galaxy becomes

$$\mu = \frac{4.39X10^{-23}\Lambda}{\dot{R}(t)H_o} \tag{38}$$

3.5 Canis Major Galaxy

This is a disputed dwarf Irregular galaxy in the local group located in the same part of thr sky as constellation canis Major [22]. The mass of cains major is approximately 1 billion solar mass ($1X10^9 0$) ($2X10^{39}\ Kg$ and its distances from the earth is approximately $0.025X10^6$ ly ($2.5X10^{20}$). Subtituting the constant in (4.0), we have K= $6.79\ X10^{-18}$) $N^2 m^2 Kg^{-2}$ eqn. 35 for canis Major Dwarf becomes $\mu = 6.79\ X\ 10^{-18}$

3.6 Segue I

This is a dwarf spheroidal galaxy or globular cluster situated in the leo constellation [25]. The mass of Segue I is approximately $6X10^5 0$($12X10^{35}$ Kg) and its distances from the earth is about $0.075X10^6$ light years ($7.5X10^{20}$ m). Substituting the values in eqn(34). We have K=$1.51X10^{-20} N^2 MKg^{-2}$ equation (35) for segue 2 galaxy be becomes

$$\mu = \frac{1.51X10^{-20}\Lambda}{\dot{R}(t)H_0} \tag{39}$$

3.7 Small Magelian Cloud

This is a dwarf galaxy near the Milky Way [26]. The SMC contains a central bar structure and astronomers speculate that it was disturbed by the milky way to become somewhat irregular. It has a solar mass of about $6.5X10^9$ solar mass ($13X10^{39}$ Kg) and its distance from the center of the earth is 0.206.ly [22].

Substituting the values in equation 34 we have K= $7.90X10^{-20} N^2 MKg^{-2}$ equation (35) for Segue 1 galaxy becomes

$$\mu = \frac{7.90X10^{-20}\Lambda}{\dot{R}(t)H_0} \tag{40}$$

3.8 Willman Galaxy

This is an Ultra low-luminosity dwarf galaxy or a stain cluster. As of 2007, Willman galaxy declared the least massive galaxy known. $0.4\ X\ 10^6$ [27] solar mass ($0.8\ X\ 10^{36}$) and its distance from the center of the earth is about $0.12X10^6$ ly ($1.2X10^{21}$ m)

Substituting the values in equation 34, we obtain k=$2.46X10^{-23} N^2 M^2 Kg^{-2}$ equation (35) for Willman galaxy

$$\mu = \frac{2.46X10^{-23}\Lambda}{\dot{R}(t)H_0} \tag{41}$$

3.9 Triangulum Ii

Triangulum II is a dwarf galaxy close to the Milky Way galaxy. It contains only 1000 stars, yet is quite massive, having a solar mass of $50X10^6$ solar mass [28] ($1.0\ X\ 10^{41}$ kg) and it has an approximate distance of $9.8\ X\ 10^4$ ly ($9.8\ X\ 10^{20}$ m) from the center of the earth. Substituting these values in equation (34), we obtained K = $5.60\ X\ 10^{-18}\ N^2 MKg^{-2}$. Equation (35)for triangullum II galaxy becomes

$$\mu = \frac{5.60\ X10^{-18}}{\dot{R}(t)H_0} \tag{42}$$

The values of the different Galaxies K- parameter values as computed in the table below is also demonstrated in the chart below; see Chart 1.

It can be deduced from equation (35) that the receding velocity of the redshift between the galaxies is inversely proportional to the gravitational potential energy at a constant parameter K, which means when dark energy increases, more vacuum is created because dark energy is a function of space time and when vacuum is created, particles and anti-particles are also created then gravitational potential set in because gravity is a function of mass. So equation (35) implies that dark energy cannot exist without gravitational potential energy so they are intrinsic properties of each other. It can also be observed from equation (35) that the velocity $\dot{R}(t)$ of the redshift from the galaxies is

Table 1. Different galaxies and their K- values

| Galaxy	Distance from	the Earth	$K = \frac{2G^2 M1 M2}{R^3}$ $N^2 M Kg^{-2}$
	light year 10^6 (Ly)	Metre (M)	
Segue 2	0.11 [22]	11×10^{20}	4.39×10^{-23}
Large Magelian Cloud	0.163 [21]	1.63×10^{21}	2.5×10^{-19}
Andromeda	2.56 [20]	2.56×10^{22}	9.50×10^{-21}
Canis Major Dwarf	0.025[25]	$2.5 \, X 10^{20}$	6.79×10^{-18}
Segue I	0.075[25]	7.5×10^{20}	1.51×10^{-20}
Small Magelian Cloud	0.206[22]	2.06×10^{21}	7.90×10^{-20}
Willman Galaxy	0.120[27]	1.2×10^{21}	2.40×10^{-23}
Triangulum II Galaxy	0.098[23]	9.8×10^{20}	5.60×10^{-18}

Source: Computed by the researcher
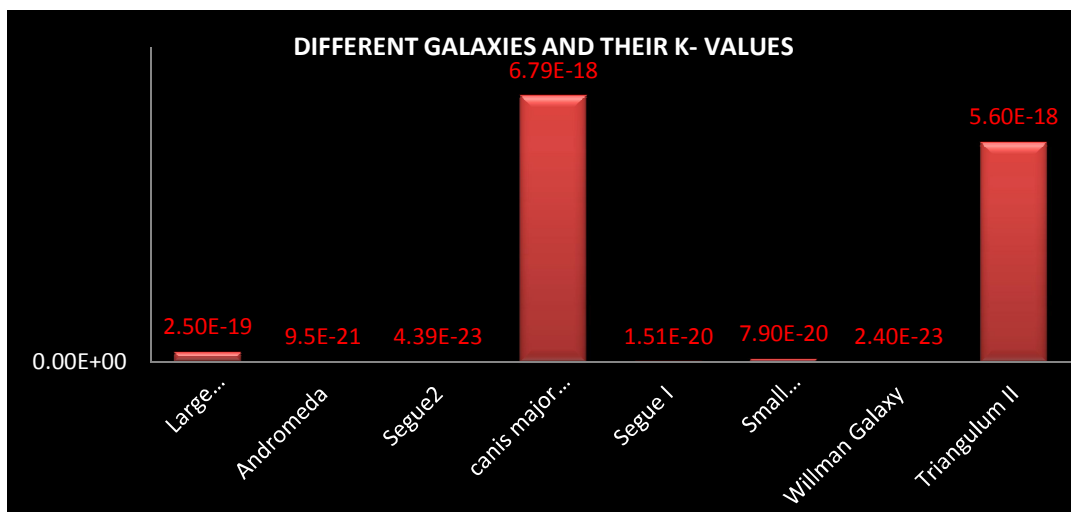
Chart 1. The different galaxies and their K-parameter values gotten from the center of the earth
Source: Computed by the researcher

inversely proportional to the dark energy which agree with the fact that dark energy accelerates the rates of expansion of the universe. From table (1.0), the values of K for the various galaxies as measured from the Center of the Earth is decreasing in the order of magnitude which implies that the value of K decreases with farther in the distance of the galaxies from the Center of the earth.K is a proportionality constant that defines the relationship between the gravitational potential energy, dark energy and the receding velocity.

4. CONCLUSION

In summary, from the mathematical analysis and the result obtained it can be understood that dark energy maintains a direct proportionality with gravitation, which implies that both occurs simultaneously which enables them to keep the universe in balance because the rate of expansion accelerated by dark energy is balanced by gravitation. . From equation (30), it can deduce that the effects of the dark energy increases with an increment in the distance of the galaxies, also the dark energy is gradually increasing the radius of the universe. From the same equation (30), the values of the dark energy can be obtained at a given distance between the galaxies at a particular time of observation. From equation (30), it can be observed that the gravitational potential energy is inversely proportional to the distance separating the galaxies which can be further be explained as gravitational effects been greater between the galaxies at a smaller distance between them.

After some plausible assumptions, we obtain a relation, $\mu = \frac{2G^2 m_1 m_2 \Lambda}{\dot{R}(t) \acute{H_o} R_o^3}$ where μ is Gravitational potential energy, m_1 is the mass of galaxy, m_2 is mass of earth, Λ is dark energy, $\dot{R}(t)$ is receding velocity, H_o is Hubble's constant and R_o is the distance measured from the point of observation

and the galaxy. Moreover, we estimate K = $\frac{2G^2 m_1 m_2}{R^3}$ for various galaxies, finally, the relation μ αΛ, indicates a direct relationship between Dark energy and Gravitation.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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