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Prices and Taxes in a Ramsey Climate Policy Model under Heterogeneous Beliefs and Ambiguity

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Abstract: In a Ramsey policy regime, heterogeneity in beliefs about the potential costs of climate change is shown to produce policy ambiguities that alter carbon prices and taxation. Three sources of ambiguity are considered: (i) the private sector is skeptical, with beliefs that are unknown to the government, (ii) private agents have pessimistic doubts about the model, or (iii) the policy authority itself does not trust the extant scientific climate model and fears the worst. These three sources of ambiguity give rise to four potential belief regimes characterized by differentials between the government's and the private sector's inter-temporal rates of substitutions, with implications for the prices of carbon and capital, framed in terms of distorted Arrow–Debreu pricing theory that establishes an equivalence between the optimal carbon tax and the permit price of an underlying asset—the government-imposed limit on emissions in economies with cap and trade. This paper shows that in most instances, skeptical beliefs and resulting ambiguities justify higher carbon taxes and lower capital taxes to offset the private sector's increased myopia compared with rational expectations. Conversely, ambiguities created by worst-case fears in either the private sector or in government tend produce forces in the opposite direction.

Keywords: Knightian uncertainty; multiplier preferences; Ramsey planner; social planner; carbon tax; capital tax; ambiguity premium; dynamic stochastic integrated general equilibrium (DSIGE); robust Arrow–Debreu asset prices

JEL Classification: C61; C73; D80; E62; H21; H23; H63; Q32; Q38; Q54

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1. Introduction

"The trouble with the world is that the stupid are cocksure and the intelligent are full of doubt." Bertrand Russell

That global warming poses an existential threat to the future of humanity and the planet is now a universally accepted scientific fact. Yet significant segments of society still harbor doubt and skepticism about what scientists tell us, with some, including a previous US Administration, either minimizing the threat or denying it altogether, and others fearing the worst.[1]

In the minds of many, doubt or skepticism is justified by uncertainties surrounding climate science, including the extent of terrestrial carbon uptake, mankind's role in this, the relationship between carbon and temperature, and, ultimately any claimed economic damages.[2] The nature and causes of resistance to accepting bad news and environmental threats have been much discussed in the literature (see Meyer and Kunreuther (2017), Kunreuther et al. (1978), Kaufmann et al. (2017)). At the simplest level, disbelief may be motivated by myopic economic self interest, such as a refusal to contemplate the possibly enormous costs of carbon abatement, and an unwillingness to accept significant taxation on fossil energy (see Lifton (2017)). Alternatively, the much publicized threats of pending climate-related disasters have likely contributed to a fear in some sectors of society that the worst of climate catastrophes will ensue. For the analyst, some humility may be in order, because, as Millner et al. (2010) have noted, "our knowledge of the climate system is not of sufficiently high quality to be described with unique probability distributions, and that formal frameworks that account for aversion to ambiguity are normatively legitimate". In a recent paper, Pindyck (2017) warned that excessive reliance on Integrated Assessment Models (IAMs), first introduced by Nordhaus (1993), "create a perception of knowledge and precision that is illusory, possibly fooling policy-makers into thinking that the forecasts that models generate have some kind of scientific legitimacy. The same goes for any illusion that the probability distributions underling uncertainty can even be known". It is this last thought that mostly motivates the present paper.

For purposes of exposition, I shall give specific meaning to the terms *skepticism* and *doubt*. Skeptics are said to be individuals who distort their probability assessments in favor of different outcomes than predicted by science. Because such beliefs are arbitrary, they are also, in principle, unknown to the government, thereby creating an ambiguity for the policy maker (see Hansen and Sargent (2012)). By doubt I shall mean the kind of pessimism regarding the trustworthiness of a given model that Hansen and Sargent (2012) have described as emanating from a belief that nature is likely to produce worst-case outcomes. Although the literature has focused solely on ambiguities inhabiting the policy authority, both, government and private agents, may be pessimistic regarding the climate model. Pessimists in this paper make decisions by playing a game against an imagined malevolent opponent. A powerful and rational motive for pessimism is the prospect of tipping points that Dietz et al. (2021) have described as the most important source of uncertainty, one capable of throwing off all modeling descriptions and justifying preparedness for the worst.[3] In this paper, all pessimistic players use Hansen and Sargent's (2008) min-max strategies to compute their policies, meaning they are revealed to the authority, assumed to be a Ramsey planner, in the form of additional implementability constraints under which such a planner must operate.

The preceding sources of ambiguity give rise to four potential belief regimes:

1. Unknown private beliefs
   (a) Political planner: The government strategically assumes private beliefs to be true. Its ignorance of private beliefs produces ambiguity.
   (b) Paternalistic planner: The government trusts and adheres to the true model. However, the presence of unknown private beliefs creates ambiguity for policy.
   (c) Pessimistic planner: Having doubts about the model and facing unknown private beliefs, the planner confronts two sources of ambiguity.
2. Known pessimistic private beliefs
   (a) Pessimistic planner: The planner's own doubts doubts about the model and the private sector's pessimistic doubts that constrain the Ramsey planner's policy produce two sources of ambiguity.

The framework for studying policy with ambiguity is based on work by Hansen and Sargent (2005, 2007, 2008), a general rationale for pursuing robust climate policy under deep uncertainty having been provided by Workman et al. (2021). In their theory, deviations of private-sector beliefs from some approximating or reference distribution—the true scientific distribution—are represented as martingale multiplier distortions M, later defined as likelihood ratios having well defined properties. A justification for this approach to modeling ambiguity is based on a theorem by Strzalecki (2011), who axiomatized the robust control criterion of multiplier preferences introduced by Hansen and Sargent (2001), relating them to other classes of preferences studied in decision theory, in particular, the variational preferences introduced by Maccheroni et al. (2006), and proving them to be equivalent to multiplier distortions of probabilities.

As indicated earlier, the study of ambiguity in the context of climate policy is not without precedent, but discussions have generally been limited to cases when only the planner has doubts about the approximating climate-economic model. Such doubts include concerns about potential mis-specification of alternative models and ambiguity over how much weight to assign to each of these models, while agents themselves are usually assumed to have rational beliefs (see Millner et al. (2012), Brock and Durlauf (2015), Cai et al. (2013), Cai and Lontzek (2019), Anderson et al. (2013), Berger et al. (2016), Li et al. (2016), Lemoine and Traeger (2016), Rezai and van der Ploeg (2017), and Barnett et al. (2020)). Hennlock's (2009) is no exception in that, although he attributes deep uncertainty to the consumer, the government, being a direct extension of the consumer, remains the consumer's sole agent, so that, in effect, it is the planner who is modeled as having doubts about the model. In a bit of a twist to the approaches taken by other researchers, Rezai and van der Ploeg (2017) studied the implications of adopting max-min, max-max, and min-max regret policies when the planner faces alternative models ranging from science-based paradigms to denialist imaginings, concluding that max-min or min-max regret climate policies that rely on a non-skeptic view of global warming lead to a substantial and moderate amount of caution, respectively, while max-max policies produce policies that do not match the beliefs of climate skeptics. Later, Rezai and van der Ploeg (2019) applied a version of *Pascal's wager* and asked: what would an agnostic but rational planner—one who does not know or care which model is correct—do when faced with some probability that the approximating model, adhered to by so-called deniers, is false? Their conclusions are briefly described in Section 13.

The literature on optimal climate policy has generally followed the tradition of welfare analysis based on expected utility maximization within the framework of an integrated climate assessment model, with government defined as a social planner (vid. Golosov et al. 2014) seeking to maximize the expected welfare of society unconstrained by private decisions or market outcomes. An alternative is to assume that the government is a Ramsey planner, likewise seeking to maximize consumer welfare, akin to the authority introduced by Chari et al. (1994) to study optimal dynamic capital taxation, but under constraints imposed by market equilibrium.[4] This paper studies both versions of government, in which each type of planner must acknowledge the possibly distorted beliefs held by the private sector.

In its essentials, the description of the economy here follows the recent literature on optimal carbon taxation, foremost among them Nordhaus (2008), Acemoglu et al. (2012), von Below (2012), Golosov et al.'s (2014), van der Ploeg and Withagen (2014), and Belfiori (2017, 2018). The analytical framework is a by now familiar dynamic stochastic integrated general equilibrium (DSIGE) model similar to those in Anderson et al. (2013) and Golosov et al. (2014), which in turn are based on RICE—Regional Dynamic Integrated Model of Climate and the Economy—developed by Nordhaus (1993, 2008, 2007).

The government's fiscal policy tools include bond finance and taxes on carbon and capital. I include a tax on capital because within a Ramsey planning framework, capital and Pigouvian carbon taxation are tightly linked: (1) the government's stochastic discount factor for the return to capital and for the expected social cost of carbon (SCC) is the same, and (2) the SCC and taxes on capital and carbon are influenced by the same shadow price.[5]

In essence, this paper will show how the planner's ignorance of the model or of skeptical private beliefs creates an endogenous gap between the government's and the household's discount factors, leading to a gap in their respective Arrow–Debreu pricing of carbon and capital that contributes to an *ambiguity premium* over the standard certainty-equivalent formulation of the expected social cost of carbon.[6] This finding is related to recent papers by von Below (2012), Barrage (2018), and Belfiori (2017), who proved in different contexts that the optimal tax on capital is negative, and the optimal tax on carbon is higher than the standard Pigou rate, if the government's subjective discount rate is *exogenously* lower than the public sector's.[7] The underlying reason is that climate change decreases the returns to capital, so that individuals, who are too impatient from a social point of view, i.e., skeptical, do not save enough without a capital subsidy and, at the same time, burn too much fossil fuel unless the latter is taxed sufficiently. The optimal policy response is therefore to tax capital less and to tax carbon more.[8] In the reverse case, a pessimistic public motivated to over-invest and to under-utilize carbon may justify lower carbon and higher capital taxation, unless the government is also pessimistic. In this paper, any disparity in discount factors between the government and the private sector is endogenously driven, in this instance by heterogeneity in beliefs, fear of mis-specification, and ambiguity.

The next section provides formal definitions of pessimism and skepticism as understood in this paper, using multiplier preferences introduced by Hansen and Sargent (2001, 2005, 2008). Section 3 derives the Euler conditions for a consumer who may be skeptical (Section 3.5) or pessimistic (Section 3.4), as defined in Section 2. Section 4 presents a three-factor production function subject to damaging climate-related total productivity shocks in a model of a firm renting capital and purchasing energy from the household. Section 5 uses results from Sections 3.3, 3.6, and 4 to derive two versions of Hotelling's rule. Subject to constraints derived in Section 6, Section 7 presents the Ramsey planner's Euler conditions for the three belief regimes under study. Section 8 derives the possibly distorted equilibrium prices of carbon damage and capital. In anticipation of the main results, Section 9 comments on this paper's methodology and approach, which contain some innovations. Section 10 derives the expected social cost of carbon, including an ambiguity premium that governments in all four policy/belief regimes will implement. Section 11 establishes the conditions under which the planner may or may not impose an additional ambiguity-related carbon tax premium over the social cost of carbon. Section 12 presents a number of conditions under which a planner may or may not raise the subsidy rate on capital, where it will become apparent that such conditions mirror those that drive results for the carbon tax premium. Finally, before the paper's conclusion, Section 14 describes a reverse feedback from taxation to beliefs whereby a planner facing an economy with pessimistic agents is able to manipulate debt and taxes to affect pessimistic beliefs.

Throughout, references to state-conditioned distorted (including robust) Arrow–Debreu prices reflect the basic theme in this paper: that a Ramsey planner's Pigouvian tax policy under ambiguity is able to implement allocations via equilibrium pricing of an underlying asset with unknown returns that have an equivalence in a cap and trade economy. In this respect, this paper is most closely related to Barnett et al. (2020), who use asset pricing methods not only to impute market valuations but also to ascertain social valuations, as this paper intends. As in Barnett et al. (2020), the asset prices in this paper can be viewed as equivalent to shadow prices of the expected discounted values of stochastic processes impinging on the economy.

Climate policy that is motivated by a planner's own deep or Knightian uncertainty is an interesting and important topic and has been fairly exhaustively treated in the literature. Furthermore, as has also been shown elsewhere in the literature, the effects of ambiguities originating from the planner turn out not always to be clear-cut, depending on specific features related to preferences and returns in the economy. More importantly, as maintained in this paper, an equally or possibly more urgent issue for policy must be the role of beliefs held in the private sector, because they affect consumption and investment decisions, where heterogeneity between private and government beliefs will surely impact policy. That public acceptance of climate science and its policy prescriptions are not unanimous is uncontroversial and has been well documented. However, little attention has been paid to its implications, particularly its effects on ambiguity in optimal climate policy. By altering inter-temporal rates of substitution or pricing kernels that determine consumption decisions and household wealth, belief distortions in the private sector impact a planner's implementability constraints and become more salient by the addition of ambiguities that arise when those beliefs are unknowable to the government. Whatever deep uncertainty may or may not already inhabit the mind of a planner, a welfare maximizing policy authority would be remiss in ignoring the effects of private-sector belief heterogeneity and associated ambiguities on its own policy decisions.

This paper then is the first systematic attempt to analyze the policy implications of ambiguities arising from belief heterogeneity in the private sector regarding the nature of anthropomorphic climate change. Significantly, this paper distinguishes between mere skepticism producing myopic behavior in the economy and true doubt as manifested by worst-case fears leading to increased foresight. The analytic choice to investigate the implications of belief heterogeneity and associated ambiguities by evaluating the planner's Euler conditions follows the example of Anderson et al. (2013). However, the particular approach leading to a role for second-order moments as factors in optimal policy is an innovation of this paper. The covariances between multiplier distortions and variables in the economy that arise as relevant to policy will be shown to allow more detailed descriptions of the effects of ambiguity on the social cost of carbon and on carbon and capital taxation. As will become clear later, ambiguities resulting from belief heterogeneity and distortions in the private sector produce ambiguity-related premiums on the social cost of carbon and, separately, on the carbon tax and on capital subsidies.

2. Multiplier Preferences

The representative consumer and the government share a reference probability model, given a joint density $\pi(x^t)$ of the history of shocks $x^t = x_0, \cdots, x_t$, where, as detailed later in Section 4 (see Equation (21)), x_t is a climate-induced damage shock to the production economy at time t. The consumer and the government do not necessarily agree that π is the true probability. The government may doubt the model fearing the worst, and the private sector may either have similar doubts or be skeptical in some arbitrary way. Either may then choose an alternative model via some distortion of π in a manner described by Hansen and Sargent (2001, 2005, 2008), who invoke the Radon-Nikodym theorem to express any alternative model as as a non-negative measurable mapping $\hat{\pi}(x^t) = M_t(x^t)\pi(x^t)$, with $\mathbb{E}_t M_t = 1$, where, since uncertainty is realized in $t = 0$, $M_0 = 1$.[9] Being the unconditional likelihood ratio $M_t(x^t) = \frac{\hat{\pi}_t(x^t)}{\pi_t(x^t)}$ of an alternative density to $\hat{\pi}_t(x^t)$, M_t, is a martingale with respect to the reference model π, $\mathbb{E}_t M_{t+1} = M_t$, with the interpretation of a change in measure. The *distorted expectation* of x_{t+1}, given history x^t, is[10]

$$\tilde{\mathbb{E}}_t[x_{t+1}|x^t] = \mathbb{E}_t[\frac{M_{t+1}}{M_t}x_{t+1}].$$

As described in more detail below, disbelief may take two forms, either as skepticism or as pessimism, the latter being a manifestation of worst-case fears. Importantly, throughout, any alternative model $\hat{\pi}(x^t)$ is assumed to be absolutely continuous with respect to the reference model $\pi(x^t)$.[11]

Later, it will be convenient to decompose M_t by defining the conditional likelihood ratio $m_t(x^{t+1}) \equiv \frac{M(x^{t+1})}{M(x^t)}$, such that $\mathbb{E}_t m_{t+1} = 1$.

2.1. Skepticism (Random Belief Distortion)

As indicated in the introduction, for the purpose of this paper, skepticism refers to an arbitrary rejection of the extant approximating model $\pi(x^t)$ in favor of some other model $M_t\pi(x^t)$, where M_t is a random variable with properties previously set out, including the assumption of absolute continuity with respect to the true distribution $\pi_t(x^t)$, which means that households can be skeptics but not outright climate change deniers.

2.2. Pessimism (Ambiguity Aversion)

Pessimism or model doubt refers to a worst-case belief distortion $\hat{\pi}_t(x^t) = M_t\pi_t(x^t)$ derived from the consumer's having solved a min-max problem shown later.

Following Hansen and Sargent (2008), define discounted *relative entropy* conditional on date zero information as the distance $v_0(\hat{\pi}_t, \pi_t)$ between $\hat{\pi}_t$ and π_t associated with M_t over time-t information and over an infinite horizon as

$$\begin{aligned} v_0(\hat{\pi}_t, \pi_t) &= (1-\beta)\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t M_t \log M_t \\ &= \beta \mathbb{E}_0 [\sum_{t=0}^{\infty} \beta^t M_t \mathbb{E}_t \frac{M_{t+1}}{M_t} (\log M_{t+1} - \log M_t)], \\ &= \beta \mathbb{E}_0 [\sum_{t=0}^{\infty} \beta^t M_t \mathbb{E}_t m_{t+1} \log m_{t+1}], \end{aligned}$$

where β is the subjective discount factor. With this definition, an agent's ambiguity about π is represented by a set of joint densities $\{M_t\}_{t=0}^{\infty}$ satisfying the constraint,

$$\beta \mathbb{E}_0 M_t \mathbb{E}_t [m_{t+1} \log m_{t+1}] \leq \eta, \tag{1}$$

where $\eta > 0$. In the following, a pessimistic consumer will choose a consumption plan subject to the constraint in (1).[12]

3. Households

3.1. CRRA Preferences

Consumers derive utility from consumption c_t, given a constant-elasticity preference function $u(c_t)$,[13]

$$\begin{aligned} u(c_t) &= \frac{c_t^{1-\gamma}}{1-\gamma}, \; 0 < \gamma < 1, \\ &= \log c_t; \;\; \gamma \to 1, \end{aligned}$$

with elasticities $\epsilon_{cc} = -u_{cc}c/u_c = \gamma$ (implying constant relative risk aversion).

If $\gamma = 1$ (logarithmic preferences), $\epsilon_{cc} = 1$. For the record, the constant intertemporal elasticity of substitution for consumption is $1/\gamma$. I will assume that $\gamma \leq 1$, which accords with much of the literature on long-run risk—the kind this paper is most concerned with.[14]

3.2. The Household's Budget Constraint

The household owns three assets: (1) the stock of depreciating capital with a net yield $[(1-\tau_t^k(x^t))(r_t(x^t)-\delta)]k_t(x^{t-1})$, where r_t is the real rental rate on capital $k_t(x^{t-1})$ left over from last period, and τ_t^k is the tax on capital; (2) a government bond $b(x^t)$, defined as an Arrow–Debreu security promising one unit of consumption in period $t+1$, if the state is x_{t+1} and zero otherwise, and (3) the resource Q_t of fossil fuels from which it draws E_t units every period, according to the law of motion[15]

$$Q_{t+1} = Q_t - E_t, \tag{2}$$

which, by the assumption of exhaustibility, implies

$$\sum_{t=0}^{\infty} E_t \leq Q_0. \tag{3}$$

The household sells E_t to the firm at a price p_t^e with after-tax revenue $(p_t^e(x^t) - \tau_t^e(x^t))E_t(x^t)$, where τ_t^e is an excise (carbon) tax per unit of energy.

The household receives income from (1) inelastically supplied labor H_t at the competitive wage $w_t = 1$, (2) rent from capital, (3) revenues from the sale of fossil energy, and (4) a lump-sum transfer from the government g_t. It spends its resources on consumption c_t, new capital k_{t+1}, and the purchase of a new Arrow security b_{t+1}, trading at the state-contingent price $\hat{p}_{t+1}(x_{t+1}|x^t)$ to be defined later. Summarizing, the household's one-period budget constraint is

$$\begin{aligned} 0 &\leq H_t(x^t) + b_t(x^t) + g_t(x^t) + R_t^k(x^t)k_t(x^{t-1}) \\ &+ (p_t^e - \tau_t^e)[Q_t(x^t) - Q_{t+1}(x^t)] - c_t(x^t) - k_{t+1}(x^t) - \mathbb{E}_t\hat{p}_{t+1}b_{t+1}(x^{t+1}) \\ &\equiv \mathcal{L}_t(x^t), \end{aligned} \tag{4}$$

where $R_t^k(x^t) = 1 + (1 - \tau_t^k)(r_t(x^t) - \delta)$ is the after-tax gross return to capital.

3.3. The Consumer's Maximization Problem

This section studies the two types of consumer introduced earlier: those who are skeptical of the model and form some arbitrary belief distortion M to the true distribution π according to Section 2.1, and those who are pessimistic and play a game against a malevolent force to determine a worst-case value for M following Section 2.2. It is convenient to set this problem up for the latter and then show the former to be a special case.

3.4. Pessimistic Consumer

The general framework for the consumer's problem is a game against some malevolent force representing extreme uncertainty about the model. Given the resource constraint in (2) and the budget constraint (4), the representative consumer solves the Lagrangian

$$\max_{\{c_t, E_t, b_{t+1}, k_{t+1}, Q_{t+1}\}} \min_{\{M_t, m_{t+1}\}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t M_t \left(u(c_t) + \lambda_t^c \mathcal{L}_t - \frac{\beta}{\sigma^c} m_{t+1} \log m_{t+1} \right),$$

subject to $M_{t+1} = m_{t+1}M_t$, $M_0 = 1$, $\mathbb{E}_t m_{t+1} = 1$, where λ_t^c is a time-varying Lagrangian multiplier associated with the household budget constraint (4), and $0 \geq -\frac{1}{\sigma^c} < \infty$ denotes a Lagrangian shadow cost for the penalty of deviating from rational expectations represented by the approximating distribution π, also known as the Kullback-Leibler distance (see Kullback and Leibler 1951) between the two probability measures π and $\hat{\pi}$; so $0 > \sigma^c > -\infty$ may be considered the consumer's *parameter of ambiguity aversion*.[16] As in all of this paper, expectations $\mathbb{E}_t$ are taken over the measure π.

The preceding criterion has the Bellman recursion

$$\begin{aligned} \mathcal{U}_t(k_t, b_t, Q_t) &= \max_{c_t, E_t, k_{t+1}, b_{t+1}, Q_{t+1}} \min_{m_{t+1}} u(c_t) + \lambda_t^c \mathcal{L}_t(x^t) \\ &+ \beta \mathbb{E}_t \left(m_{t+1} \mathcal{U}_{t+1}(k_{t+1}, b_{t+1}, Q_{t+1}) - \frac{\beta}{\sigma^c} m_{t+1} \log m_{t+1} \right) \\ &+ \varphi_t^c (Q_t - E_t - Q_{t+1}), \end{aligned} \tag{5}$$

where φ_t^c is the Lagrangian shadow price associated with the resource constrains (2).

3.4.1. Inner Minimization

The optimal conditional likelihood ratio that minimizes (5) has the familiar exponentially twisting form[17]

$$m_{t+1}^c = \frac{e^{\sigma^c \mathcal{U}_{t+1}}}{\mathbb{E}_t e^{\sigma^c \mathcal{U}_{t+1}}},$$

or, equivalently,

$$M_{t+1}^c = \frac{e^{\sigma^c \mathcal{U}_{t+1}}}{\mathbb{E}_t e^{\sigma^c \mathcal{U}_{t+1}}} M_t^c. \tag{6}$$

The worst-case martingale distortion in (6) is *pessimistic* in that it attaches higher probabilities to histories with low continuation utilities and lower probabilities to histories with high continuation utilities. Notably, because of its dependence on continuation values, it is also a function of future distortions m_{t+j}^c and future decisions by both the consumer and the government that, as shown later, determine equilibrium prices in the economy.

3.4.2. Outer Maximization with Implied Risk-Sensitive Recursion

Substituting m^c_{t+1} from (6) into (5) produces the *risk-sensitive* recursion[18]

$$\mathcal{U}(k_t, b_t, Q_t) = \max_{c_t, E_t, k_{t+1}, b_{t+1}, Q_{t+1}} u(c_t) + \frac{\beta}{\sigma^c}\mathbb{E}_t \log e^{\sigma^c \mathcal{U}(k_{t+1}, b_{t+1}, Q_{t+1})} + \lambda^c_t \mathcal{L}_t. \tag{7}$$

The first-order conditions (FONCs) for the flow variables $\{c_t, E_t\}$ are

$$\lambda^c_t = u_{c_t}, \tag{8}$$

$$(p^e_t - \tau^e_t)\lambda^c_t = \varphi^c_t. \tag{9}$$

The envelope conditions are,

$$\mathcal{U}_{k_t} = \lambda^c_t R^k_t = u_{c_t} R^k_t, \tag{10}$$

$$\mathcal{U}_{b_t} = \lambda^c_t = u_{c_t}, \tag{11}$$

$$\mathcal{U}_{Q_t} = \varphi^c_t. \tag{12}$$

In conjunction with these envelope conditions, the first-order conditions for the stock variables b_{t+1}, k_{t+1}, and Q_{t+1} are, respectively,

$$\hat{p}_{t+1} = \beta m^c_{t+1} \frac{u_{c_{t+1}}}{u_{c_t}}, \tag{13}$$

$$1 = \beta \mathbb{E}_t m^c_{t+1} \frac{u_{c_{t+1}}}{u_{c_t}} R^k_{t+1} = \mathbb{E}_t \hat{p}_{t+1} R^k_{t+1}, \tag{14}$$

$$\varphi^c_t = \beta \mathbb{E}_t m^c_{t+1} \varphi^c_{t+1}. \tag{15}$$

Note that $\hat{p}_{t+1}$ is the one-period worst-case equilibrium price of a state-contingent claim as shown later in (18). According to the preceding conditions, the price of such a claim is determined by continuation utilities and the random climate cost shock x_{t+1}. This is information that the government can exploit to determine optimal fiscal policy.

3.5. Skeptical Consumer

Let M^s denote a skeptical consumer's belief distortion. As stated earlier, skepticism represents an arbitrary random distortion $\hat{\pi}_t \equiv M^s_t \pi_t$ of the true distribution π_t. From the point of view of society (or the government), M^s_{t+1} is an unknowable exogenous variable. However, this household is sure of its beliefs and evaluates (5) by disabling the penalty on belief distortions and letting $\sigma^c \to 0$.

The Euler equations are the same as before, except that $m = m^s$ is random and unknown to the authorities, and the equilibrium price $\hat{p}$ of a state-contingent claim based on skepticism as defined here is unrelated to the consumer's continuation utility. Additionally, since m^s_{t+1} is random, it is unrelated to x_{t+1}.

3.6. Arrow–Debreu Prices under Belief Distortions

The belief distortions of probabilities, ranging from skepticism to deep uncertainty, treated in this paper, add an important dimension to stochastic discounting. This section shows how to construct their corresponding asset-price measures.

For preferences $u(c_t)$ distorted by a martingale process M_{t+j} (either M^s_{t+j} or M^c_{t+j}), the j-period-ahead stochastic discount factor (MSDF) is,

$$\hat{\varrho}_{t+j,t} = \beta^j \frac{M_{t+j}}{M_t} \frac{u_{c_{t+j}}}{u_{c_t}} \equiv \frac{M_{t+j}}{M_t} \varrho_{t+j,t}, \tag{16}$$

where $\varrho_{t+j,t}$ is the pricing kernel under rational expectations. When $j = 1$, this becomes the familiar one-period-ahead stochastic discount factor SDF.

Let $\hat{q}_{t+j}(x^{t+j})$ be the j-period numeraire

$$\hat{q}_{t+j}(x^{t+j}) \equiv \beta^j M_{t+j}\pi_{t+j}(x^{t+j})\frac{u_{c_{t+j}}(x^{t+j})}{u_{c_0}(x^0)}, \quad \hat{q}_0 = 1, M_0 = 1, \tag{17}$$

and define

$$\begin{aligned}\hat{p}_{t+j,t}(x_{t+j}|x^t) &\equiv \frac{\hat{q}_{t+j}(x^{t+j})}{\hat{q}_t(x^t)} = \beta^j\left(\prod_{i=1}^{j} m_{t+i}\frac{u_c(x^{t+i})}{u_c(x^t)}\right)\pi_{t+j}(x^{t+j}) \\ &= \frac{M_{t+j}}{M_t}\varrho_{t+j,t}\pi_{t+j}(x^{t+j})\end{aligned}$$

as the market's distorted $t+j$ equilibrium price in (13) of an Arrow–Debreu security in terms of consumption at history x^t, or equivalently,

$$\hat{p}_{t+j,t}(x_{t+j}|x^t) = \hat{\varrho}_{t+j,t}\pi_{t+j}(x^t), \tag{18}$$

which, for $j=1$, also corresponds to the first-order condition for capital in (14). For future reference, denote the undistorted rational expectations price by

$$p_{t+j,t}(x_{t+j}|x^t) = \varrho_{t+j,t}\pi_{t+j}(x^t). \tag{19}$$

4. Firms

The economy's output is produced by a continuum of atomistic firms with a Cobb-Douglas production technology:

$$Y_t = (1 - D_t(T_t - T_0))F(k_t, H_t, E_t, Q_t) = (1 - D_t(Q_t))k_t{}^{\alpha}E_t^{\nu}H_t^{1-\alpha-\nu}, \tag{20}$$

where k_t is the stock of capital, H_t is hours of labor, E_t is the flow of fossil energy, and Q_t is the remaining stock of carbon energy in the ground. As explained presently, $D_t(Q_t)$ is a damage function measuring the proportion of GDP lost due to the change in average global temperatures $T_t - T_0$ since the beginning of the Industrial Revolution at $t=0$.

Dietz and Venmans (2019) observe that "climate has delivered two important and related insights. First, global warming appears to be approximately linearly proportional to cumulative emissions of carbon dioxide. Second, the temperature response to an emission of CO_2 appears to be approximately instantaneous and then constant as a function of time". This conclusion accords with Matthews et al. (2012), and Collins et al. (2013), who earlier defined the *Transient Climate Response to Cumulative Carbon Emissions* (TCRE)[19]

$$\lambda_t^{TC} = \frac{T_t - T_0}{Q_0 - Q_t},$$

where $0 \leq Q_0 - Q_t = \sum_{i=0}^{t-1} E_i$ is accumulated carbon emissions since the beginning of the Industrial Revolution.[20] The TCRE parameter is generally assumed to be a stochastic variable, due to uncertainties surrounding climate modeling.

The preceding motivates a damage function having the following exponential form

$$D_t(Q_t) = e^{\zeta_t(T_t - T_0)} = e^{\zeta_t\lambda_t^{TC}(Q_0 - Q_t)} \equiv e^{x_t(Q_0 - Q_t)}, \quad \frac{\partial D_t}{\partial Q_t} = -x_t e^{x_t(Q_0 - Q_t)} < 0, \tag{21}$$

where ζ_t is a stochastic parameter that translates the damaging effects of temperature changes into units of GDP, and $x_t = \zeta_t\lambda_t^{TC}$ combines the damage parameter with the TCRE parameter λ_t^{TC}.[21] As posited in Section 2, I shall consider x_t to be a random variable with either known distribution $\pi_t(x_t)$ or unknown distribution giving rise to ambiguities described earlier. Confining the source of economic damage to a single catch-all variable follows the practice of a number of authors, including Li et al. (2016), who create a single source of model uncertainty with a stochastic variable γ that reduces their end-of-period capital stock. Others who have used similar exponential formulations include Golosov et al.'s (2014), and Anderson et al. (2013).[22] To the extent that there exist possibilities of catastrophic tipping points that would likely upend all calculations and planning, the underlying distribution of x_t may be taken as Knightian and unknowable by either the private sector or the government or both. Tipping points, analyzed by Lemoine and Traeger (2016), and Cai et al. (2013), are abrupt nonlinear climate disruptions that pose a potentially existential threat to humanity in ways that may override concerns with belief and skepticism. A further source of extreme uncertainty is *polar amplification* analyzed by Brock and Xepapadeas (2017).

The typical firm rents capital k_t from consumers and buys energy E_t from households in order to maximize an expected infinite stream of profits. Hours of labor H_t are supplied inelastically at the going wage 1—a standard assumption in this literature (see Golosov et al. (2014) and Li et al. (2016)).

$$\begin{aligned} & \max_{k_t, H_t, E_t} \mathbb{E}_o \sum_{t=0}^{\infty} \hat{q}(x_t)[Y_t - r_t k_t - H_t - p_t^e E_t], \\ = & \max_{k_t, H_t, E_t} \mathbb{E}_o \sum_{t=0}^{\infty} \hat{q}_t(x_t)\Big((1 - e^{-x_t(Q_t - Q_0)}) k_t{}^{\alpha} E_t^{\nu} H_t^{1-\alpha-\nu} - r_t k_t - H_t - p_t^e E_t\Big), \end{aligned}$$

where $\hat{q}(x_t|x_0)$ is the belief-distorted and possibly robust numeraire defined earlier in (17). The first-order conditions with respect to $\{k_t, H_t, E_t\}$ are

$$r_t = Y_{k_t} = \alpha \frac{Y_t}{k_t}, \tag{22}$$

$$1 = Y_{H_t} = (1 - \alpha - \nu) \frac{Y_t}{H_t}, \tag{23}$$

$$p_t^e = Y_{E_t} = \nu \frac{Y_t}{E_t}. \tag{24}$$

5. Two Versions of Hotelling's Rule

Denote by R_{t+1}^e the rate of return to energy, net of the carbon tax τ_t^c,

$$R_{t+1}^e = \frac{p_{t+1}^e - \tau_{t+1}^e}{p_t^e - \tau_t^e}. \tag{25}$$

Additionally, since, for either $m = m^s$ or $m = m^c$, (9) and (15) imply

$$p_t^e - \tau_t^e = \beta \mathbb{E}_t m_{t+1} \frac{u_{c_{t+1}}}{u_{c_t}} (p_{t+1}^e - \tau_{t+1}^e), \tag{26}$$

it follows that

$$\mathbb{E}_t \hat{p}_{t+1} R_{t+1}^e = 1, \tag{27}$$

where expectations are taken with respect to $\pi(x^t)$, as before. Substituting (24) for period t and $t+1$ in (26) and using the result $p^e = Y_E$ from (24), yields the firm's dynamic rule for optimal energy use, given taxes,

$$Y_{E_t} - \tau_t^e = \mathbb{E}_t \hat{\varrho}_{t+1} (Y_{E_{t+1}} - \tau_{t+1}^e), \tag{28}$$

which is a version of Hotelling's rule that will become relevant later for determining the social cost of carbon.

If $m_{t+1} = 1$, the preceding equation confirms Hotelling's original formula that with zero or constant taxes, energy consumption falls over time at the subjective rate of discount. If consumers are climate-skeptic, a multiplier $m_{t+1}^s < 1$ effectively lowers the private-sector's discount factor for future benefits of fossil fuel in favor of current returns, leading to increased current consumption relative to rational expectations. If the consumer is pessimistic with $m_{t+1} = m^c_{t+1}$, energy expenditures are delayed, following the same argument.

Hotelling's (1931) original rule states that the price of an exhaustible resource net of extraction costs should rise at the rate of interest, which, on average, is above the rate of real GDP growth. By Equation (14), the t-period Arrow–Debreu price of a claim on a unit of capital is

$$P^k_t \quad = \quad \mathbb{E}_t \sum_{j=0}^{\infty} \hat{p}_{t+j,t} R^k_{t+j}, \tag{29}$$

while Equation (27) implies that the t-period Arrow–Debreu price of a claim on a unit of carbon energy is

$$P^e_t \quad = \quad \mathbb{E}_t \sum_{j=0}^{\infty} \hat{p}_{t+j,t} R^e_{t+j}. \tag{30}$$

From Equations (14) and (27) follows a version of Hotelling's rule in terms of returns:

$$\textit{Version } 1: \qquad R^e_{t+1} = R^k_{t+1}, \ \forall t, \tag{31}$$

while Equations (29) and (30) restate Hotelling's rule in terms of Arrow–Debreu contingent prices:

$$\textit{Version } 2: \qquad P^e_{t+1} = P^k_{t+1}, \ \forall t. \tag{32}$$

Note that in each case, returns and prices are distorted by the consumer's beliefs, be they skeptical m^s or pessimistic m^c.

These two versions of Hotelling's rule (31) and (32) constitute binding no-arbitrage conditions that require prices and returns to capital and fossil fuel to be equal: in equilibrium, the return to fossil fuel left in the ground for one more period equals the return to the next unit of capital. This rule illuminates two important features of a competitive market for exhaustible energy when there is uncertainty and skepticism regarding the underlying model: (i) the pricing of energy resources continues to obey the laws of asset markets requiring equality of returns to all activities, including capital, bonds, and energy stores, but (ii) the market now uses a martingale-distorted and possibly robust stochastic discount factor to evaluate the expected future returns to all assets.

Remark 1. *Hotelling's rule assures efficient allocation but not necessarily socially optimal outcomes if it fails to internalize costs to society created by private economic activity. Later, Section 11 shows how the government can remedy this failure with a public version of Hotelling's rule that includes a social accounting of all costs.*

The next few sections discuss how a Ramsey planner implements competitive equilibrium depending on assumptions about heterogeneity in beliefs, the degree of doubt about the model by either the government or the public, and by how ignorant the planner is about private beliefs.

6. The Ramsey Planner's Constraints

6.1. National Income Identity and the Government Budget Constraint

In all belief regimes considered here, a Ramsey planner commits to policy in period 0 by choosing a competitive equilibrium that maximizes the consumer's expected utility over time. This means the government chooses allocations that satisfy the natural resource constraint (2) and (3) and the national income resource constraint that output exhausts consumption plus investment plus government spending, which, to keep things simple, is assumed to be entirely devoted to a lump-sum transfer g_t,

$$Y_t = c_t + k_{t+1} - (1-\delta)k_t + g_t. \tag{33}$$

The government's budget constraint requires that spending and the redemption of bonds from the preceding period be covered by tax receipts, lump-sum transfers, and new issuance of bonds:

$$b_t \quad = \quad \mathbb{E}_t \hat{p}_{t+1} b_{t+1} - g_t + \tau_t^e E_t + \tau_t^k (r_t - \delta) k_t.$$

When $b_t < 0$, the government is a lender. Solved forward, the preceding equation becomes the government's dynamic budget constraint,

$$b_t \leq \mathbb{E}_t \sum_{j=0}^{\infty} \hat{q}_{t+j} [\tau_{t+j}^k (r_{t+j} - \delta) k_{t+j} - \tau_{t+j}^e (Q_{t+j+1} - Q_{t+j}) - g_{t+j}]. \tag{34}$$

Note the added term involving receipts from the carbon tax.

6.2. The Ramsey Planner's Implementability Constraints

In solving for optimal taxation in the case of a Ramsey planner, I use a so-called primal approach due to Chamley (1986) that searches directly for allocations by solving the government's problem subject to an *implementability* constraint. A starting point is the household's dynamic budget constraint (4), which, when solved forward for b_t, utilizing the no-arbitrage condition (14) and a no-Ponzi game condition, yields the intertemporal budget constraint[23]

$$b_t \quad \geq \quad \mathbb{E}_t \lim_{T\to\infty} \sum_{j=0}^{T-1} \hat{p}_{t+j} [c_{t+j} - H_{t+j} - g_{t+j}]. \tag{35}$$

Let $\mathcal{W}_t$ denote household wealth in period t, composed of government bonds, the after-tax equity value of physical capital, and fossil fuels still in the ground, valued at current after-tax energy prices:

$$\mathcal{W}_t \quad \equiv \quad b_t + R_t^k k_t + (p_t^e - \tau_t^e) Q_t. \tag{36}$$

Appendix B shows that

$$\begin{aligned} \mathcal{W}_t \quad &\geq \quad c_t - H_t - g_t + \beta \mathbb{E}_t m_{t+1} \frac{u_{c_{t+1}}}{u_{c_t}} \mathcal{W}_{t+1} \\ &= \quad c_t - H_t - g_t + \mathbb{E}_t \hat{p}_{t+1} \mathcal{W}_{t+1}, \end{aligned} \tag{37}$$

where $\hat{p}_{t+j,t}$ is the market's previously defined distorted $t+j$ equilibrium price of an Arrow–Debreu security in terms of consumption at history x^t. Solved forward, household wealth is

$$\mathcal{W}_t \quad \geq \quad \mathbb{E}_t \sum_{j=0}^{\infty} \hat{p}_{t+j,t} [c_{t+j} - H_{t+j} - g_{t+j}]. \tag{38}$$

For later use, it is convenient to define the marginal-utility-of-consumption-scaled market value of wealth $\Upsilon_t = u_{c_t} \mathcal{W}_t$, so that the Ramsey planner's implementability constraint (37) becomes, equivalently,

$$\Upsilon_t \quad \geq \quad \Omega_t + \mathbb{E}_t m_{t+1} \Upsilon_{t+1}, \tag{39}$$

where

$$\Omega_t = u_{c_t}[c_t - H_t - g_t], \tag{40}$$

with derivative,

$$\begin{aligned}\Omega_{c_t} &= \frac{u_{cc_t}}{u_{c_t}}(c_t - H_t - g_t)u_{c_t} + u_{c_t} \\ &= [1 - \gamma c_t^{-1}(c_t - H_t - g_t)]u_{c_t} = [1 - \gamma + \gamma \frac{H_t + g_t}{c_t}]u_{c_t} \geq 0,\end{aligned} \tag{41}$$

since, from Section 3.1, $\frac{u_{cc_t}}{u_{c_t}} = -\gamma/c_t$. Note that if the constraint (39) is nonbinding, the government is the social planner widely treated in the literature.

Solved forward,

$$\Upsilon_t = \mathbb{E}_t u_{c,t} \sum_{j=0}^{\infty} \hat{p}_{t+j}(c_{t+j} - H_{t+j} - g_{t+j}) = u_{ct} b_t, \tag{42}$$

is the government's surplus valued in terms of the marginal utility of consumption.

Finally, by its very definition, a Ramsey planner heeds all equilibrium constraints imposed by competitive markets. In particular, when consumers are pessimistic with belief distortions defined in terms of their continuation values, where $m_t = m^c_{t+1} = \frac{e^{\sigma^c \mathcal{U}_{t+1}}}{\mathbb{E}_t e^{\sigma^c \mathcal{U}_{t+1}}}$, the planner faces two additional implementability constraints that come from (6) and (7):

$$M^c_{t+1} = \frac{e^{\sigma^c \mathcal{U}_{t+1}}}{\mathbb{E}_t e^{\sigma^c \mathcal{U}_{t+1}}} M^c_t, \tag{43}$$

$$\mathcal{U}_t = u(c_t) + \frac{\beta}{\sigma^c} \log \mathbb{E}_t e^{\sigma^c \mathcal{U}_{t+1}}. \tag{44}$$

7. Ramsey Planning in Four Belief Regimes

Subject to the fossil resource constraint (2) and (3), the national income identity (33), and the implementability constraint (39), the taxing Ramsey authority chooses $\{c, E, k, Q, \Upsilon\}$ to maximize society's expected welfare, and $\{N, n\}$ to minimize discounted *relative entropy* defined in (1),

$$\max_{c,E,k,Q,\Upsilon} \min_{N,n,} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t N_t(x^t) \left[u(c_t) - \frac{\beta}{\sigma}(\mathbb{E}_t n_{t+1} \log n_{t+1}) \right], \tag{45}$$

subject to $N_{t+1} = n_{t+1} N_t$, $N_0 = 1$, $\mathbb{E}_t n_{t+1} = 1$, and $0 > \sigma > -\infty$, where N_t is the government's martingale multiplier, equivalent to M_t defined earlier for the consumer, and $\sigma < 0$ is the planner's *parameter of ambiguity aversion*.

Exploiting a linear homogeneity property such that $V(\Upsilon_t, k_t, Q_t, N_t) = N_t \mathcal{V}_t(\Upsilon_t, k_t, Q_t)$, the problem may be cast as the recursive (Bellman) Lagrangian

$$\begin{aligned}\mathcal{V}_t(\Upsilon_t, k_t, Q_t) &= \max_{c_t, E_t, k_{t+1}, Q_{t+1} \Upsilon_{t+1}} \min_{n_{t+1}} u(c_t) - \frac{\beta}{\sigma} \mathbb{E}_t n_{t+1} \log n_{t+1} \\ &+ \beta \mathbb{E}_t n_{t+1} \mathcal{V}_{t+1}(\Upsilon_{t+1}, k_{t+1}, Q_{t+1}) \\ &+ \Phi_t[\Omega_t + \mathbb{E}_t m_{t+1} \Upsilon_{t+1} - \Upsilon_t] + \mathcal{M}_t,\end{aligned} \tag{46}$$

where

$$\mathcal{M}_t = \begin{pmatrix} \lambda_t[(1 - e^{x_t(Q_0 - Q_t)})F(k_t, H_t, E_t) + (1-\delta)k_t - c_t - g_t - k_{t+1}] \\ \varphi_t[Q_t - E_t - Q_{t+1}] \\ v[Q_0 - \sum_{i=0}^{\infty} E_i] \end{pmatrix},$$

is a group of Lagrangian constraints and λ_t, Φ_t, φ_t are non-negative Lagrangian co-state variables, and v is constant.[24]

The preceding problem constitutes a Stackelberg game between the government as the leader and the private sector as the follower. Embedded in this game is another game with a minimizing opponent to represent worst-case outcomes. This latter game, framed as *inner minimization*, may be played by either government or consumers, as in Section 3.4, or even both, depending on respective attitudes toward extreme risk. The solution to this subgame implies a plan for *outer maximization* of an indirect value function, typically a *risk sensitivity recursion*, such as the one introduced in Section 3.4.2.

7.1. Political Planner

In this example, consumers are assumed to have arbitrarily distorted—skeptical—beliefs about the approximating scientific model π, as described in Section 2.1. Importantly, the government is ignorant of these beliefs. The reason for its ignorance is fundamental: as noted by Hansen and Sargent (2012), the government's ambiguity arises because the possible space of models that are unknown to the fiscal planner but known to the consumer is so vast that it is impossible to infer the private sector's probability model from finite data, providing a motive to construct robust climate and fiscal policies by solving a so-called multiplier problem that protects against worst-case belief distortions. As Hansen and Sargent (2012) put it, the government's ignorance of private beliefs is akin to a set or cloud of probability distributions over events x centered on the reference or approximating density π constrained by a discounted relative entropy set of probability distributions reflecting the unknown beliefs of the private sector.[25] Its ignorance notwithstanding, this political planner acts under the assumption that the unknown beliefs are true.

With this in mind, the recursion (46) becomes

$$\begin{aligned} \mathcal{V}_t(\mathrm{Y}_t, k_t, Q_t) \quad &= \max_{c_t, E_t, k_{t+1}, Q_{t+1} \mathrm{Y}_{t+1}} \min_{n_{t+1}} u(c_t) - \frac{\beta}{\sigma}\mathbb{E}_t n_{t+1} \log n_{t+1} \\ &+ \beta \mathbb{E}_t n_{t+1} \mathcal{V}_{t+1}(\mathrm{Y}_{t+1}, k_{t+1}, Q_{t+1}) \\ &+ \Phi_t[\Omega_t + \mathbb{E}_t n_{t+1} \mathrm{Y}_{t+1} - \mathrm{Y}_t] + \mathcal{M}_t. \end{aligned} \tag{47}$$

The political government's acceptance of private-sector beliefs M_t as true means that m_{t+1}, which multiplies Y_{t+1} in the implementability constraint, is set equal to n_{t+1}—the authority adopts the consumer's distortion as its own.[26] Ambiguity is activated with the entropy constraint penalizing deviations of distorted beliefs from true beliefs (1), where $-\frac{1}{\sigma}$ is a Lagrangian multiplier, and $\sigma < 0$ measures the planner's ambiguity aversion. As σ moves toward $-\infty$, the government's preference for robustness rises. As σ approaches a break-down $\underline{\sigma} < 0$ from above, the government's concern about distortions to expectations is maximal. Conversely, as σ approaches zero, the government's preference for robustness diminishes, until, in the limit the government fully adopts the approximating reference model as true and not subject to doubt.

7.1.1. Inner Game with Nature

Ambiguity for this planner produces an *ex post* worst-case probability model with distorted *ex post homogeneity* in beliefs between the government and the private sector.

Minimization of (47) with respect to n_{t+1} leads to

$$n_{t+1} \quad = \quad \frac{e^{\sigma[\mathcal{V}_{t+1} + \Phi_t \mathrm{Y}_{t+1}]}}{\mathbb{E}_t e^{\sigma[\mathcal{V}_{t+1} + \Phi_t \mathrm{Y}_{t+1}]}} \equiv n_{t+1}^{PO}, \tag{48}$$

with limits $\lim_{\sigma \downarrow -\infty} n_{t+1}^{PO} \to 0$, and $\lim_{\sigma \uparrow 0} n_{t+1}^{PO} \to 1$, indicating that the conditional likelihood ratio n^{PO} is inversely related to the intensity of the planner's doubts about private beliefs, approaching 1 as doubt ceases.

7.1.2. Outer Maximization with Implied Risk-Sensitive Recursion

The continuation value $\mathcal{V}_{t+1}$ in the exponent of the formula for n^{PO} comes from the planner's distrust of the reference model itself, while continuation wealth Υ_{t+1} points to the planner's ignorance of private-sector beliefs. Formula (48), indicates that, given $\sigma < 0$, ambiguity leads this robust planner to put more probability weight on histories with low continuation values $\mathcal{V}_{t+1}$ and Υ_{t+1}, and lower probabilities on histories with high continuation values.

Substituting (48) for both n_{t+1} and m_{t+1} in (47) produces a variation on Hansen and Sargent's (1995) discounted risk-sensitive recursion, one that is augmented with household wealth Υ, a forward-looking entity valued at the shadow price Φ, in the exponent,[27]

$$\begin{aligned}\mathcal{V}(\Upsilon_t, k_t, Q_t) &= \max_{c_t, E_t, k_{t+1}, Q_{t+1}, \Upsilon_{t+1}} u(c_t) + \Phi_t[\Omega_t - \Upsilon_t] + \mathcal{M}_t \\ &+ \frac{\beta}{\sigma} \log \mathbb{E}_t e^{\sigma(\mathcal{V}_{t+1} + \Phi_t \Upsilon_{t+1})}. \qquad (49)\end{aligned}$$

The distortion n^{PO} that attains the minimum of the right side of (49) tilts the x_{t+1} distribution exponentially toward lower continuation values via multiplication of $\pi(x_{t+1})$ by n_{t+1}^{PO} in (48).

The first-order conditions (FONCs) for $\{c_t, E_t\}$ for an interior maximum are: $\forall t \geq 1$,

Flows

$$c_t: \quad \lambda_t = u_c + \Omega_{c_t}\Phi_t, \qquad (50)$$

$$E_t: \quad \Upsilon_{E_t}\lambda_t = \varphi_t + v, \qquad (51)$$

where $\Upsilon_{E_t} = \nu \frac{\Upsilon_t}{E_t}$, is the marginal product of fossil energy based on the Cobb-Douglas assumption.

Envelope conditions

$$k_t: \quad \mathcal{V}_{k_t} = (1 - \delta + \Upsilon_{k_t})\lambda_t, \qquad (52)$$

$$Q_t: \quad \mathcal{V}_{Q_t} = \varphi_t + \lambda_t x_t D_t F_t, \qquad (53)$$

$$\Upsilon_t: \quad \mathcal{V}_{\Upsilon_t} = -\Phi_t. \qquad (54)$$

Note that $\lambda \geq 0$ and $\varphi \geq 0$ imply $V_k \geq 0$ and $\mathcal{V}_{Q_t} \geq 0$, respectively. Further, $\Phi \geq 0$ implies $V_\Upsilon \leq 0$.

Stocks

$$\begin{aligned} k_{t+1}: & \\ 1 &= \frac{\beta}{\lambda_t}\mathbb{E}_t n_{t+1}^{PO}\mathcal{V}_{k_{t+1}} = \beta \mathbb{E}_t n_{t+1}^{PO}\frac{\lambda_{t+1}}{\lambda_t}(1 - \delta + \Upsilon_{k_{t+1}}), \qquad (55) \\ Q_{t+1}: & \\ \varphi_t &= \beta \mathbb{E}_t n_{t+1}^{PO}\mathcal{V}_{Q_{t+1}} = \beta \mathbb{E}_t n_{t+1}^{PO}(\varphi_{t+1} + \lambda_{t+1}x_{t+1}D_{t+1}F_{t+1}), \qquad (56) \\ \Upsilon_{t+1}: & \\ \Phi_t &= -\mathcal{V}_{\Upsilon_{t+1}} = \Phi_{t+1} = \bar{\Phi}, \qquad (57) \end{aligned}$$

where the second equality in each equation follows from the envelope conditions (52)–(54), and (56), giving a marginal utility valuation of the benefit of fossil energy net of climate damages. Given (57), the Lagrangian multiplier Φ_t is a constant and, by the Kuhn-Tucker condition, is zero if the wealth constraint is nonbinding, indicating that the planner is a social and not a Ramsey planner.

Note that because both the private sector's and the planner's probabilities are twisted by a martingale multiplier, the ambiguity for this planner effectively delivers *ex post* a model of endogenously distorted *homogeneous* beliefs. Hansen and Sargent (2012) emphasize that n^{PO} is not intended to "solve" an impossible inference problem, and being the planner's *cautious* inference about unknown private beliefs, should be viewed as merely a device to construct a robust Ramsey policy. If the planner were to solve the private sector's Euler equations using the minimizing n^{PO} in order to derive its *ex post* decision rules for consumption, labor, and energy, it would not necessarily end up reproducing their observed values, meaning that private beliefs cannot be reverse-engineered from such observations.

7.1.3. Paternalistic Planner

Woodford (2010) originally introduced a monetary authority facing a type of ambiguity described here: while trusting the reference model π, the government is ignorant about the private sector's distorted beliefs m^s. It expresses its ambiguity by setting $\sigma < 0$. Trust in its own model means that the planner sets n_{t+1} multiplying continuation value $\mathcal{V}_{t+1}$ equal to unity.[28]

7.1.4. Inner Game with Nature

The minimizing value of n_{t+1} is

$$n_{t+1} = \frac{e^{\sigma[\Phi_t Y_{t+1}]}}{\mathbb{E}_t e^{\sigma[\Phi_t Y_{t+1}]}} \equiv n^{PA}_{t+1}, \tag{58}$$

with limits $\lim_{\sigma\downarrow -\infty} n^{PA}_{t+1} \to 0$, and $\lim_{\sigma\uparrow 0} n^{PA}_{t+1} \to 1$, indicating that the conditional likelihood ratio n^{PA} is inversely related to the intensity of the planner's doubt about private beliefs, approaching 1 as doubt ceases.

An important distinction between the political planner and the paternalistic planner with ambiguity is that here, the planner's worst-case distortion of beliefs n^{PA} is solely determined by continuation values of wealth, since the only sources of ambiguity are private-sector beliefs that distort the consumer's expectation of future household wealth. Formula (58) instructs us that, via n^{PA}, a robust paternalistic planner assigns greater probability weights to histories with low continuation values of wealth, weighted with marginal utility of consumption in (39).

7.1.5. Outer Maximization with Implied Risk-Sensitive Recursion

Substitution of the formula for n^{PA}_{t+1} in (46) implies another variation on Hansen and Sargent's (1995) discounted risk-sensitive recursion,

$$\begin{aligned}\mathcal{V}(Y_t, k_t, Q_t) &= \max_{c_t, E_t, k_{t+1}, Q_{t+1}, Y_{t+1}} \{u(c_t) + \Phi_t[\Omega_t - Y_t] + \frac{\beta}{\sigma}\mathbb{E}_t \log e^{\sigma\Phi_t Y_{t+1}} + \mathcal{M}_t \\ &+ \beta\mathbb{E}_t \mathcal{V}_{t+1}(Y_{t+1}, k_{t+1}, Q_{t+1})\},\end{aligned}$$

which differs from the recursion (49) for a political planner in that the exponent does not include the continuation value $\mathcal{V}_{t+1}$, since, here, risk sensitivity does not apply to the planner's own trusted model.

The FONCs for $\{c, E\}$ are previously given by (50) and (51). The envelope conditions are also the same as before. However, for choosing $\{k_{t+1}, Q_{t+1}, Y_{t+1}\}$, the government must solve

$$1 = \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t}(1 - \delta + Y_{k_{t+1}}), \tag{59}$$

$$\varphi_t = \beta\mathbb{E}_t \mathcal{V}_{Q_{t+1}} = \beta\mathbb{E}_t(\varphi_{t+1} + \lambda_{t+1}x_{t+1}D_{t+1}F_{t+1}), \tag{60}$$

$$\Phi_t = -\mathcal{V}_{Y_{t+1}} = \Phi_{t+1} = n^{PA}_{t+1}\Phi_t. \tag{61}$$

From (61) follows that Φ_t is a martingale: $\mathbb{E}_t\Phi_{t+1} = \mathbb{E}_t n^{PA}_{t+1}\Phi_t = \Phi_t$, unless the wealth constraint $0 \leq [\Omega_t + \mathbb{E}_t n^{PA}_{t+1} Y_{t+1} - Y_t]$ is non-binding, in which case $\Phi_t = 0$, and the government reverts to a social planner.

7.2. Pessimistic Planner, Skeptical Consumer

This section treats a variation of the paternalistic planner in Section 7.1.3, where, instead of trusting the approximating model π, the authority has pessimistic doubts about it, meaning it now faces two kinds of ambiguity: one that derives from its ignorance of private beliefs, indexed by M, and the other stemming from its own doubts about the model, indexed by N. Accordingly, the planner minimizes with respect to both m and n, given two Lagrange penalty functions. For the sake of simplicity, I will assume that in the following recursion, a single risk sensitivity σ applies to both kinds of ambiguities:

$$\begin{aligned} \mathcal{V}_t(\Upsilon_t, k_t, Q_t) &= \max_{c_t, E_t, k_{t+1}, Q_{t+1} \Upsilon_{t+1}} \min_{n_{t+1}, m_{t+1}} u(c_t) \\ &- \frac{\beta}{\sigma}\mathbb{E}_t(n_{t+1}\log n_{t+1} + m_{t+1}\log m_{t+1}) \\ &+ \beta\mathbb{E}_t n_{t+1}\mathcal{V}_{t+1}(\Upsilon_{t+1}, k_{t+1}, Q_{t+1}) \\ &+ \Phi_t[\Omega_t + \mathbb{E}_t m_{t+1}\Upsilon_{t+1} - \Upsilon_t] + \mathcal{M}_t. \end{aligned}$$

7.2.1. Inner Minimization

Minimization with respect to n and m produces the following worst-case multipliers

$$n^p_{t+1} = \frac{e^{\sigma\mathcal{V}_{t+1}}}{\mathbb{E}_t e^{\sigma\mathcal{V}_{t+1}}}, \tag{62}$$

$$m^p_{t+1} = \frac{e^{\sigma\Phi_t\Upsilon_{t+1}}}{\mathbb{E}_t e^{\sigma\Phi_t\Upsilon_{t+1}}}. \tag{63}$$

7.2.2. Outer Maximization with Implied Risk-Sensitive Recursion

Substitution as before yields the risk-sensitive recursion

$$\begin{aligned} \mathcal{V}(\Upsilon_t, k_t, Q_t) &= \max_{c_t, H_t, E_t, k_{t+1}, Q_{t+1}, \Upsilon_{t+1}} u(c_t, H_t) + \Phi_t[\Omega_t - \Upsilon_t] + \mathcal{M}_t \\ &+ \frac{\beta}{\sigma}(\log\mathbb{E}_t e^{\sigma\mathcal{V}_{t+1}} + \log\mathbb{E}_t e^{\sigma\Phi_t\Upsilon_{t+1}}). \end{aligned} \tag{64}$$

The first-order conditions for $\{k_{t+1}, Q_{t+1}, \Upsilon_{t+1}\}$ are

$$1 = \frac{\beta}{\lambda_t}\mathbb{E}_t n^p_{t+1}\mathcal{V}_{k_{t+1}} = \beta\mathbb{E}_t n^p_{t+1}\frac{\lambda_{t+1}}{\lambda_t}(1 - \delta + Y_{k_{t+1}}), \tag{65}$$

$$\varphi_t = \beta\mathbb{E}_t n^p_{t+1}\mathcal{V}_{Q_{t+1}} = \beta\mathbb{E}_t n^p_{t+1}(\varphi_{t+1} + \lambda_{t+1}x_{t+1}D_{t+1}F_{t+1}) \tag{66}$$

$$\Phi_t = -\mathcal{V}_{\Upsilon_{t+1}} = \Phi_{t+1} = \frac{m^p_{t+1}}{n^p_{t+1}}\Phi_t. \tag{67}$$

Those for $\{c, H, E\}$ remain the same as before.

A special case:

1. The consumer has rational expectations ($m^p = 1$). As noted earlier, this case has been widely treated in papers on robust climate policy.

7.3. Pessimistic Planner, Pessimistic Consumer

This section treats a variation on the preceding belief regime by replacing its skeptical consumers with the pessimistic consumers from Section 3.4. Doing so will require adding two more implementability constraints: (1) the law of motion for the households' worst-case beliefs M^c_t in (43), because the authority needs to keep track of its evolution, and (2) the consumer's risk-sensitive utility recursion (44), because increments to the worst-case likelihood ratio M_t^c are determined by that household's utility $\mathcal{U}_t$.[29] For this policy maker, the Bellman recursion is,

$$\begin{aligned}\mathcal{V}_t(\Upsilon_t, k_t, Q_t) &= \max_{c_t, E_t, k_{t+1}, Q_{t+1}\Upsilon_{t+1}, M_t^c, \mathcal{U}_t} \min_{n_{t+1},} u(c_t) - \frac{\beta}{\sigma}\mathbb{E}_t(n_{t+1}\log n_{t+1}) \\ &+ \beta\mathbb{E}_t n_{t+1}\mathcal{V}_{t+1}(\Upsilon_{t+1}, k_{t+1}, Q_{t+1}) \\ &+ \Phi_t[\Omega_t + \beta\mathbb{E}_t\frac{M_{t+1}^c}{M_t^c}\Upsilon_{t+1} - \Upsilon_t] + \mathcal{M}_t \\ &+ \beta\mathbb{E}_t\mu_{t+1}\left[\frac{e^{\sigma^c\mathcal{U}_{t+1}}}{\mathbb{E}_t e^{\sigma^c\mathcal{U}_{t+1}}}M_t^c - M_{t+1}^c\right] \\ &+ \varepsilon_t\left[u(c_t, H_t) + \frac{\beta}{\sigma^c}\log\mathbb{E}_t e^{\sigma^c\mathcal{U}_{t+1}} - \mathcal{U}_t\right] + \mathcal{M}_t,\end{aligned}$$

where μ_{t+1} and ε_t are the Lagrangian shadow prices for the law of motion for M_{t+1}^c and the consumer's worst-case utility $\mathcal{U}_t$, respectively.

Outer Maximization with Implied Risk-Sensitive Recursion

With m^c computed by the consumer in Section 3.4,

$$m_{t+1}^c = \frac{M_{t+1}^c}{M_t^c} = \frac{e^{\sigma_n\mathcal{U}_{t+1}}}{\mathbb{E}_t e^{\sigma_n\mathcal{U}_{t+1}}},$$

and n^p the worst-case multiplier chosen by this planner,

$$n_{t+1}^p = \frac{e^{\sigma_n\mathcal{V}_{t+1}}}{\mathbb{E}_t e^{\sigma_n\mathcal{V}_{t+1}}},$$

the risk-sensitive recursion to be solved under dual ambiguities is,

$$\begin{aligned}\mathcal{V}(\Upsilon_t, k_t, Q_t) &= \max_{c_t, E_t, M_t^c, \mathcal{U}_t k_{t+1}, Q_{t+1}, \Upsilon_{t+1}} u(c_t) + \Phi_t[\Omega_t + \beta\mathbb{E}_t\frac{M_{t+1}^c}{M_t^c}\Upsilon_{t+1} - \Upsilon_t] \\ &+ \frac{\beta}{\sigma}\log\mathbb{E}_t e^{\sigma\mathcal{V}_{t+1}} + \beta\mathbb{E}_t\mu_{t+1}\left[\frac{e^{\sigma^c\mathcal{U}_{t+1}}}{\mathbb{E}_t e^{\sigma^c\mathcal{U}_{t+1}}}M_t^c - M_{t+1}^c\right] \\ &+ \varepsilon_t\left[u(c_t, H_t) + \frac{\beta}{\sigma^c}\log\mathbb{E}_t e^{\sigma^c\mathcal{U}_{t+1}} - \mathcal{U}_t\right] + \mathcal{M}_t.\end{aligned}$$

The first-order condition for c changes a little from before and becomes

$$c_t: \quad \lambda_t = [1 + \varepsilon_t]u_c + \Omega_{c_t}\Phi_t, \tag{68}$$

while the condition for E remains (51). The first-order conditions for $\{k_{t+1}, Q_{t+1}, \Upsilon_{t+1}\}$ are

$$\begin{aligned}1 &= \beta\mathbb{E}_t n_{t+1}^p\frac{\lambda_{t+1}}{\lambda_t}(1 - \delta + \Upsilon_{k_{t+1}}), &(69)\\ \varphi_t &= \beta\mathbb{E}_t n_{t+1}^p\mathcal{V}_{Q_{t+1}} = \beta\mathbb{E}_t n_{t+1}^p(\varphi_{t+1} + \lambda_{t+1}x_{t+1}D_{t+1}F_{t+1}), &(70)\\ \Phi_{t+1} &= \frac{m_{t+1}^c}{n_{t+1}^p}\Phi_t, &(71)\\ 0 &\leq \Phi_t[\Omega_t + \mathbb{E}_t m_{t+1}^c\Upsilon_{t+1} - \Upsilon_t]. &(72)\end{aligned}$$

From (71), Φ_t is the submartingale:

$$\mathbb{E}_t\Phi_{t+1} \geq \frac{\mathbb{E}_t m_{t+1}^c}{\mathbb{E}_t n_{t+1}^p}\Phi_t = \Phi_t, \tag{73}$$

unless the wealth constraint is nonbinding, when, by the Kuhn-Tucker condition (72), $\Phi_t = 0$. Finally, given endogenous belief distortions M^c in the private sector, the first-order conditions with respect to M_t^c and $\mathcal{U}_t$ are

$$M_t^c: \quad \mu_t - \frac{\Phi_{t-1}}{M_{t-1}^c}Y_t = \beta\mathbb{E}_t m_{t+1}^c\left(\mu_{t+1} - \frac{\Phi_t}{M_t^c}Y_{t+1}\right), \tag{74}$$

$$\mathcal{U}_t: \quad \varepsilon_t = \sigma^c m_t^c M_{t-1}^c(\mu_t - \mathbb{E}_{t-1}m_t^c\mu_t) + m_t^c\varepsilon_{t-1}. \tag{75}$$

The forward solution of (74) implies that the shadow value of increasing m_t^c is proportional to the value of debt (to the consumer) in units of the marginal utility of consumption,

$$\mu_t = \frac{\Phi_{t-1}}{M_{t-1}^c}Y_t = \frac{\Phi_{t-1}}{M_{t-1}^c}u_{c,t}b_t, \tag{76}$$

obtained by using (42). Substituting this in (75) yields

$$\varepsilon_t = \sigma^c\Phi_{t-1}m_t^c u_{ct}(b_t - \mathbb{E}_{t-1}m_t^c b_t) + m_t^c\varepsilon_{t-1}, \tag{77}$$

where the term in parentheses is the innovation in government debt, with positive surprises producing a negative shock to the pessimistic likelihood ratio.

8. The Equilibrium Price of Capital

The consumption Euler condition (68) implies the discount factor,

$$\begin{aligned} \varrho_{t+j,t}^* &\equiv \beta\tfrac{\lambda_{t+j}}{\lambda_t} \\ &= \beta\tfrac{(1+\varepsilon_{t+j})u_{c_{t+j}}+\Omega_{c_{t+j}}\Phi_{t+j}}{(1+\varepsilon_t)u_{c_t}+\Omega_{c_t}\Phi_t} \\ &= \beta\tfrac{u_{c_{t+j}}}{u_{c_t}}\tfrac{1+\varepsilon_{t+j}+[1-\gamma+\gamma\vartheta_{t+1}]\Phi_{t+j}}{1+\varepsilon_t+[1-\gamma+\gamma\vartheta_t]\Phi_t} \\ &\equiv \Psi(\varepsilon_{t+j}, n_{t+j}^*)\varrho_{t+j,t}, \end{aligned} \tag{78}$$

where

$$\Psi(\varepsilon_{t+j}, n_{t+j}^*) = \frac{1+\varepsilon_{t+j}+[1-\gamma+\gamma\vartheta_{t+j}]n_{t+j}^*\Phi_{t+j-1}}{1+\varepsilon_t+[1-\gamma+\gamma\vartheta_t]\Phi_t}, \tag{79}$$

$\vartheta_t = \frac{H_t+g_t}{c_t}$ is the inverse of the average propensity to consume, namely the ratio of wage income plus the lump-sum rebate to consumption, and n_{t+j}^*, associated with the planner's implementability constraint on household wealth, varies according to policy regime as shown in Table 1. The shadow prices μ and ε are zero, except when the private sector has pessimistic beliefs $m = m^c$.

For the Ramsey plans derived previously, the conditions for capital k imply the distorted discount factor,

$$\varrho_{t+j,t}^{**} \equiv n_{t+j}^{**}\beta\frac{\lambda_{t+j}}{\lambda_t} = n_{t+j}^{**}\Psi(\varepsilon_{t+j}, n_{t+j}^*)\varrho_{t+j,t}, \tag{80}$$

where, like n_{t+j}^*, n_{t+j}^{**} varies by belief regime as shown in Table 1.[30] Note that from (78) and (80), the discount factor $\varrho_{t+j,t}^{**}$ is an n^{**}-distorted version of the previous consumption discount factor $\varrho_{t+j,t}^*$. The corresponding $t+j$ distorted equilibrium price of capital is

$$\hat{p}_{t+j,t}^{**} = n_{t+j}^{**}\hat{p}_{t+j,t}^*. \tag{81}$$

Table 1. Equilibrium Arrow–Debreu prices in alternative belief regimes.

	m	n^*	n^{**}	$\hat{p}$	$\hat{p}^*$	$\hat{p}^{**}$
Skeptical consumers						
Political planner	m^s	1	n^{PO}	$m^s p$	$\Psi(0,1)p$	$n^{PO}\Psi(0,1)p$
Paternalistic planner	m^s	n^{PA}	1	$m^s p$	$\Psi(0,n^{PA})p$	$\Psi(0,n^{PA})p$
Pessimistic planner	m^s	$\frac{m^p}{n^p}$	n^p	$m^s p$	$\Psi(0,\frac{m^p}{n^p})p$	$n^p\Psi(0,\frac{m^p}{n^p})p$
Pessimistic consumers						
Pessimistic planner	m^c	$\frac{m^c}{n^p}$	n^p	$m^c p$	$\Psi(\varepsilon,\frac{m^c}{n^p})p$	$n^p\Psi(\varepsilon,\frac{m^c}{n^p})p$

9. A Comment about Methodology

In the extant literature, the typical approach to finding analytically tractable solutions to the kind of maximin dynamic programming problems posed in Section 7 is to form their Isaacs-Bellman-Flemming equations that involve guessing and verifying functional forms as well as specifying detailed assumptions regarding preferences and probability distributions. See for example Hennlock (2009) and Li et al. (2016). As a bit of an exception, Anderson et al. (2013) do provide some analytic insights based on their problem's first-order conditions, especially regarding the importance of the role of deep uncertainty in measuring the TCRE parameter defined in Section 4. However, they leave more detailed conclusions to an evaluation of numerical solutions of their stochastic finite-horizon robust optimization problems.

An innovation of this paper is to derive specific and detailed formulas for the social cost of carbon and both carbon and capital taxes through evaluations of the Euler conditions derived from optimization. As will become apparent in the next three sections, each case will reveal important roles for certain second-order moments in the economy, specifically for a number of covariances between measures of the martingale belief distortions and a variety of economic variables, including net returns to fossil energy and capital that arise as a consequence of evaluating the expectations of products of random variables.

Given some key assumptions and a number of results stated as lemmas in Section 13.1, it becomes possible to sign these covariances, allowing us to determine with fair accuracy the likely signs of ambiguity premiums that must be added to the social cost of carbon, the carbon tax, and any capital subsidy in the various belief regimes analyzed in this paper.

10. The Social Cost of Carbon

The generic form of the first-order condition for carbon stores Q_t, based (alternatively) on (56), (61), (66) and (70), is

$$\varphi_t = \beta\mathbb{E}_t n^{**}_{t+1}(\varphi_{t+1} + \lambda_{t+1}x_{t+1}D_{t+1}F_{t+1}). \tag{82}$$

Defining the marginal-utility scaled shadow price $\varpi_t = \varphi_t/\lambda_t$, this becomes

$$\begin{aligned}\varpi_t &= \mathbb{E}_t n^{**}_{t+1}\beta\frac{\lambda_{t+1}}{\lambda_t}(\varpi_{t+1} + x_{t+1}D_{t+1}F_{t+1}) \\ &\equiv \mathbb{E}_t\varrho^{**}_{t+1}[\varpi_{t+1} + x_{t+1}D_{t+1}F_{t+1}].\end{aligned} \tag{83}$$

Formula (83) gives a recursion for worst-case climate-caused damages from the point of view of a planner who may or may not be facing ambiguity, depending on n^{**}. The *social cost of carbon* is its forward solution,

$$\varpi_t = \mathbb{E}_t \lim_{T\to\infty}\sum_{j=0}^{T}(\prod_{i=0}^{t} p^{**}_{t+i,t})x_{t+j}D_{t+j}F_{t+j}. \tag{84}$$

Formula (84) reveals that damages are priced at $p^{**}_{t+j,t} = \varrho^{**}_{t+j,t}\pi_{t+j}$, i.e., the current t-period robust Arrow–Debreu price of capital in Formula (81), demonstrating an equivalence between capital and climate damages. In essence, accumulated carbon emissions constitute a negative asset that, in a competitive economy, is optimally priced like any other asset.[31]

It is convenient to define the instantaneous undistorted social cost of carbon,

$$\Lambda_{t+1} \equiv \varrho_{t+1}[\varpi_{t+1} + x_{t+1}D_{t+1}F_{t+1}], \tag{85}$$

and also

$$f(\Phi_t, \varepsilon_t) \equiv \frac{1}{1+\varepsilon_t + \Phi_t[1-\gamma+\gamma\vartheta_t]} \leq f(\Phi_t, 0) < f(0,0) = 1, \tag{86}$$

where $\vartheta_t = \frac{H_t+g_t}{c_t}$ $\forall t$. Then use (78) and (79) to write the worst-case social cost of carbon (83) as a distortion of Λ_{t+1}

$$\begin{aligned} \varpi_t &= \mathbb{E}_t n^{**}_{t+1}\Psi(\varepsilon_{t+1}, n^*_{t+1})\Lambda_{t+1} \\ &= f(\Phi_t,\varepsilon_t)\mathbb{E}_t\left[n^{**}_{t+1}\Lambda_{t+1}\left(1+\varepsilon_{t+1} + [1-\gamma+\gamma\vartheta_{t+1}]n^*_{t+1}\Phi_t\right)\right]. \end{aligned} \tag{87}$$

Notably, the SCC is the sum of two parts.

The first part is

$$I: \ \ f(\Phi_t,\varepsilon_t)(1+\varepsilon_{t+1})\mathbb{E}_t\left[n^{**}_{t+1}\Lambda_{t+1}\right],$$

which discounts future damages valued at the planner's n^{**}-distorted price of damages. Based on Table 1, n^{**} is n^{PO} for the political planner, 1 for the paternalistic planner, and n^p for the pessimistic planner facing either climate-skeptic or climate-pessimistic consumers.

The second part is

$$II: \ \ f(\Phi_t,\varepsilon_t)\mathbb{E}_t\left[n^{**}_{t+1}n^*_{t+1}\Lambda_{t+1}(1-\gamma+\gamma\vartheta_{t+1})\Phi_t\right],$$

which augments damages in part I with the value of the planner's implementability constraint (if binding) at the n^*-distorted price of the net benefits of fossil energy. Based on Table 1, the combined ambiguity distortion $n^* \times n^{**}$ is n^{PO} for the political planner, n^{PA} for the paternalistic planner, m^p for the pessimistic planner facing climate-skeptical consumers, and m^c for the pessimistic planner facing climate-pessimistic consumers.

In the special instance when beliefs are rational and homogeneous, and the Ramsey planner's implementability constraint (39) is not binding ($\Phi_t = 0$), the social cost of carbon is the expected value of Λ_{t+1} (see for example Golosov et al. (2014)),

$$\varpi^{SP}_t = \mathbb{E}_t\varrho_{t+1}[\varpi_{t+1} + x_{t+1}D_{t+1}F_{t+1}] = \mathbb{E}_t\Lambda_{t+1} \equiv \Lambda^s_t. \tag{88}$$

11. The Carbon Tax

The conventional formula for the optimal carbon tax, as derived in Golosov et al. (2014), is

$$Social \ \ Planner's \ \ Carbon Tax \ : \qquad \tau^{e-SP}_t = \varpi^{SP}_t = \Lambda^s_t. \tag{89}$$

This section shows that when beliefs are heterogeneous, the preceding formula for the carbon tax is inadequate.

The Ramsey planner's first-order condition (51) with respect to energy E_t is

$$Y_{E_t}\lambda_t - \varphi_t \quad = \quad v,$$

where Y is defined in (20). Advancing one period and taking discounted expectations, the preceding expression implies,

$$\mathbb{E}_t\beta\left(Y_{E_{t+1}}\lambda_{t+1} - \varphi_{t+1}\right) \quad = \quad v.$$

Together, these two expressions imply

$$\mathbb{E}_t \frac{\beta \frac{\lambda_{t+1}}{\lambda_t}(Y_{E_{t+1}} - \varphi_{t+1}/\lambda_{t+1})}{Y_{E_t} - \varphi_t/\lambda_t} \quad = \quad \mathbb{E}_t \frac{\varrho^*_{t+1}(Y_{E_{t+1}} - \varpi_{t+1})}{Y_{E_t} - \varpi_t} = 1. \tag{90}$$

In Formula (90), $\varrho^*_{t+1,t}$ is the discount factor a Ramsey planner applies to future expected excess returns to energy over the social cost of carbon—which I shall call the *Net Social Benefit of Carbon*. The corresponding $t+j$ equilibrium price of fossil energy is

$$\hat{p}^*_{t+j,t} = \varrho^*_{t+j,t}\pi_{t+j}(x_{t+j}|x^t). \tag{91}$$

Market equilibrium requires that returns to all assets and activities be equal:

$$\mathbb{E}_t \left[\frac{\varrho^*_{t+1}(Y_{E_{t+1}} - \varpi_{t+1})}{Y_{E_t} - \varpi_t} \right] \quad = \quad \mathbb{E}_t \varrho^{**}_{t+1}(1 - \delta + Y_{k_{t+1}}) = 1, \tag{92}$$

where the right-hand side comes from Euler Equation (55) (or (60), (65), (69)).

Formula (92) is a version of Hotelling's welfare-optimal rule and posits that, in equilibrium, the expected socially optimal pre-tax gross return on the extant stock of fossil fuel remaining in the ground, which society discounts with stochastic discount factor ϱ^*, is equal to the expected pre-tax gross return to capital in place, discounted using the government's SDF ϱ^{**}. By comparison, Hotelling's rule, when derived for laissez-faire in (31), posits an equivalence between the private after-tax return to fossil fuel in the ground and the private after-tax gross return to the extant stock of capital, discounted using the private sector's distorted SDF $\hat{\varrho}_{t+1}$. Since both ϱ^*_{t+1} and $\hat{\varrho}_{t+1}$ may reflect ambiguous beliefs, formula (92) is potentially a robust version of Hotelling's rule. Note that the expected sign of the numerator is positive (or negative) in period $t+1$ if its sign is positive (or negative) in period t.

Market equilibrium for the price of fossil energy $p^e = Y_E$ was previously derived as obeying the difference Equation (28),

$$Y_{E_t} - \tau^e_t \quad = \quad \mathbb{E}_t \hat{\varrho}_{t+1}(Y_{E_{t+1}} - \tau^e_{t+1}), \tag{93}$$

while, from (90), a socially optimal energy price must obey the rule

$$Y_{E_t} - \varpi_t \quad = \quad \mathbb{E}_t \varrho^*_{t+1}(Y_{E_{t+1}} - \varpi_{t+1}).$$

Since, in equilibrium, both expressions must hold, subtract the first from the second equation to eliminate Y_{E_t}:

$$\begin{aligned} \tau^e_t - \varpi_t \quad &= \quad \mathbb{E}_t \varrho^*_{t+1}[Y_{E_{t+1}} - \varpi_{t+1}] - \mathbb{E}_t[\hat{\varrho}_{t+1}(Y_{E_{t+1}} - \tau^e_{t+1})] \\ &+ \quad E_t\hat{\varrho}_{t+1}\varpi_{t+1} - E_t\hat{\varrho}_{t+1}\varpi_{t+1} \\ &= \quad \mathbb{E}_t\hat{\varrho}_{t+1}[\tau^e_{t+1} - \varpi_{t+1}] + \mathbb{E}_t\varrho^*_{t+1}[Y_{E_{t+1}} - \varpi_{t+1}] - \mathbb{E}_t\hat{\varrho}_{t+1}(Y_{E_{t+1}} - \varpi_{t+1}). \end{aligned}$$

Adding and subtracting $E_t\hat{\varrho}_{t+1}\varpi_{t+1}$ yields

$$\tau^e_t - \varpi_t \quad = \quad \mathbb{E}_t\hat{\varrho}_{t+1}[\tau^e_{t+1} - \varpi_{t+1}] + \mathbb{E}_t Z_{t+1}, \tag{94}$$

where

$$Z_{t+1} \quad = \quad (\varrho^*_{t+1} - \hat{\varrho}_{t+1})(Y_{E_{t+1}} - \varpi_{t+1}),$$

is the difference between the government's discounted *Net Social Benefit* of fossil fuel and the private sector's discounted *Net Social Benefit*, where, from (90), $Y_{E_{t+j}} \geq \varpi_{t+j}$. Intuitively, the carbon tax exceeds the social cost of carbon ϖ in every period if $\forall j > 0$, the social discount factor is higher than the private discount factor, $\varrho^*_{t+j} \geq \hat{\varrho}_{t+j}$. So if, in every period, the private sector is more myopic, i.e., less patient with respect to returns to fossil fuels than the planner, the government will add a premium to the carbon tax above the expected social cost of carbon ϖ. Exactly how and by how much, is determined as follows.

Lemma 1 (The carbon tax). *The carbon tax is the sum of two terms: (1) the expected social cost of carbon ϖ_t and (2) a premium χ_t:*

$$\hat{\tau}_t^e \quad = \quad \varpi_t + \chi_t, \tag{95}$$

where, utilizing (17),

$$\begin{aligned} \chi_t &\equiv \mathbb{E}_t \sum_{j=0}^{\infty} \prod_{i=0}^{j} \hat{\varrho}_{t+i} Z_{t+j} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(\prod_{i=1}^{j} m_{t+i} \frac{u_c(x^{t+i})}{u_c(x^t)} \right) Z_{t+j} \\ &= \mathbb{E}_t \sum_{j=0}^{\infty} \frac{M_{t+j}}{M_t} \varrho_{t+j,t} Z_{t+j} = \mathbb{E}_t \sum_{j=0}^{\infty} \hat{p}_{t+j,t} (\varrho^*_{t+j} - \hat{\varrho}_{t+j})(Y_{E_{t+j}} - \varpi_{t+j}) \\ &= \mathbb{E}_t \sum_{j=0}^{\infty} \hat{p}_{t+j,t} Z_{t+j}. \end{aligned} \tag{96}$$

Proof. Solve the difference Equation (94). □

The first component of $\hat{\tau}^e$ is the Social Cost of Carbon previously derived that may or may not already contain an ambiguity-related premium. The second component adds a further ambiguity premium depending on belief regime. In essence, a government uses this formula to impose a premium on (or grant a discount toward) the carbon tax if cumulative *expected* differentially discounted *net private benefits* of fossil fuel are positive (or negative), where the sign and size of the premium (or discount) χ_t depends on the signs and sizes of all future Z_{t+j} priced at $\hat{p}_{t+j,t}$ that need to be determined. Note that in Formula (96), $\hat{p}_{t+j,t}(Y_{E_{t+j}} - \varpi_{t+j})$ is the (possibly belief distorted) market value of the excess of private returns of fossil fuels over their social cost in period $t+j$. The premium χ_t is the expected sum of all such terms, each multiplied by the difference between the government's and the private sector's discount factor. In general, the premium is positive if the public is myopic compared with the planner: $\varrho^*_{t+j} - \hat{\varrho}_{t+j} > 0$ in all periods. Intuitively, a skeptical consumer who disbelieves the seriousness of climate change, will tend to use more energy than warranted from society's point of view because it myopically undervalues future climate costs. By contrast, a pessimistic private sector would opt to use less. As will become apparent later, this calculus is modified, if the authority itself has ambiguity about private beliefs.

An asset-pricing interpretation of (95) is that the optimal carbon tax is the sum of two possibly robust asset prices: (1) expected cumulative fossil fuel damages per unit of carbon valued at prices p^{**}_{t+j} in (84), and (2) cumulative net private benefits per unit of carbon over the social cost of carbon, valued at $\hat{p}_{t+j}$ and weighted by the difference in discount factors $\varrho^*_{t+j} - \hat{\varrho}_{t+j}$. The interpretation of $\hat{\tau}_t^e$ as the potentially robust price of an underlying asset—the government-imposed cap on emissions—extends a result by Belfiori (2017), who also derived an equivalence between the optimal carbon tax and the optimal price of traded carbon permits in an economy with a cap and trade.

The case of a social planner under rational expectations, in which $\Phi = 0$ and beliefs are undistorted and homogeneous, is a suitable benchmark for comparison. However, it is exceptional in that in all other policy/belief regimes, the government may either impose a premium or give a concession. The premium (or concession) in Formula (96) is the expected value of the sum of products of random variables over all $t+j$, so the sign is not obvious from inspection, although it can be deciphered via decomposition into covariance components derived in Appendix D, which distinguishes between two cases for each of the possible policy regimes when either (1) the wealth constraint in (39) is binding ($\Phi_t > 0$), implying Ramsey planning, or (2) it is not ($\Phi_t = 0$), implying social planning.

To pursue this, define a new variable Ξ^e_{t+j} as the weighted proportional difference between the two stochastic discount factors ϱ^*_{t+j} and $\hat{\varrho}_{t+j}$:

$$\Xi^e_{t+j} \equiv \varsigma^e_{t+j} \frac{\varrho^*_{t+j} - \hat{\varrho}_{t+j}}{\hat{\varrho}_{t+j}} = \varsigma^e_{t+j}\left(\frac{\varrho^*_{t+j}}{\hat{\varrho}_{t+j}} - 1\right), \tag{97}$$

where

$$\varsigma^e_{t+j} \equiv \frac{\hat{\varrho}_{t+j}(Y_{E_{t+j}} - \varpi_{t+j}),}{\mathbb{E}_t\hat{\varrho}_{t+j}(Y_{E_{t+j}} - \varpi_{t+j}),} \Longrightarrow \mathbb{E}_t\varsigma^e_{t+j} = 1, \tag{98}$$

is the j-th period's normalized discounted excess return to fossil energy over the social cost of carbon. Utilizing (78), Ξ^e_{t+j} is

$$\Xi^e_{t+j} = \varsigma^e_{t+j}(\frac{\Psi(\varepsilon_{t+j}, n^*_{t+j})}{m_{t+j}} - 1), \tag{99}$$

where $\Psi(\varepsilon_{t+j}, n^*_{t+j})$ is defined in (79). Notice that Ξ^e depends on the government's belief multiplier n^* which varies according to policy regime, as shown in Table 1, and on the private sector's belief distortion $m_{t+1} = m^c_{t+1}$ when consumers are pessimistic and $m_{t+1} = m^s_{t+1}$ when they are skeptical. Importantly, whereas m^s is an exogenous martingale process, m^c is the pessimistic consumer's worst-case multiplier that depends on continuation utility as derived in (6). From the preceding, using (78) and (79), the expected value of Ξ^e_{t+j} is

$$\begin{aligned} \mathbb{E}_t\Xi^e_{t+j} &= \mathbb{E}_t\frac{\varrho^*_{t+j} - \hat{\varrho}_{t+j}}{\hat{\varrho}_{t+j}}\varsigma^e_{t+j} \\ &= \mathbb{E}_t\left[\left(\frac{1}{m_{t+j}}\frac{1 + \varepsilon_{t+j} + [1 - \gamma + \gamma\vartheta_{t+j}]n^*_{t+j}\Phi_{t+j-1}}{1 + \varepsilon_t + [1 - \gamma + \gamma\vartheta_t]\Phi_t} - 1\right)\varsigma^e_{t+j}\right]. \end{aligned} \tag{100}$$

Note that (100) gives the difference between two non-centered covariances with the normalized discounted excess return to fossil energy, one involving the government's and the other the private sector's stochastic discount factor.

Lemma 2 is key to determining the sign of the carbon tax premium. Based on (95),

Lemma 2.

$$sign\ \chi_t \ = \ sign[\hat{\tau}^e_t - \varpi_t] \ = sign\ \mathbb{E}_t\Xi^e_{t+j}\ \forall j. \tag{101}$$

Proof. First,

$$\mathbb{E}_t Z_{t+j} \gtreqless 0, \quad if\ \ \mathbb{E}_t\Xi^e_{t+j} \gtreqless 0,\ j = 1, \cdots \infty.$$

To show this, multiply and divide Z_{t+j} by $\hat{\varrho}_{t+j}\mathbb{E}_t\hat{\varrho}_{t+j}(Y_{E_{t+j}} - \varpi_{t+j})$:

$$\begin{aligned} Z_{t+j} &= Z_{t+j}\frac{\hat{\varrho}_{t+j}}{\hat{\varrho}_{t+j}}\frac{\mathbb{E}_t\hat{\varrho}_{t+j}(Y_{E_{t+j}} - \varpi_{t+j})}{\mathbb{E}_t\hat{\varrho}_{t+j}(Y_{E_{t+j}} - \varpi_{t+j})} \\ &= \left[\frac{\hat{\varrho}_{t+j}(Y_{E_{t+j}} - \varpi_{t+j})}{\mathbb{E}_t\hat{\varrho}_{t+j}(Y_{E_{t+j}} - \varpi_{t+j})}\frac{\varrho^*_{t+j} - \hat{\varrho}_{t+j}}{\hat{\varrho}_{t+j}}\right]\mathbb{E}_t\hat{\varrho}_{t+j}(Y_{E_{t+j}} - \varpi_{t+j}) \\ &= \left[\varsigma^e_{t+j}\frac{\varrho^*_{t+j} - \hat{\varrho}_{t+j}}{\hat{\varrho}_{t+j}}\right]\mathbb{E}_t\hat{\varrho}_{t+j}(Y_{E_{t+j}} - \varpi_{t+j}) \\ &= \Xi^e_{t+j}\mathbb{E}_t\hat{\varrho}_{t+j}(Y_{E_{t+j}} - \varpi_{t+j}), \qquad (102) \\ \Longrightarrow \mathbb{E}_t Z_{t+j} &= \mathbb{E}_t\Xi^e_{t+j}\mathbb{E}_t\hat{\varrho}_{t+j}(Y_{E_{t+j}} - \varpi_{t+j}), \end{aligned}$$

where, from (90), $Y_{E_{t+j}} \geq \varpi_{t+j}$. It follows that $\mathbb{E}_t Z_{t+j}$ and $\mathbb{E}_t \Xi^e_{t+j}$ have the same sign. Finally, with ϖ_t defined in (84), the lemma follows from Formulas (96) and (102). □

In the absence of any known time-dependent anomalies, there is no reason to believe that the sign of $\mathbb{E}_t \Xi^e_{t+j}$ is different at different times, justifying

Assumption 1. *sign* $\mathbb{E}_t \sum_{j=0}^{\infty} \Xi^e_{t+j} =$ *sign* $\mathbb{E}_t \Xi^e_{t+j} \quad \forall j$

Since in (96), the sign of χ_t depends on the sign of all its future expected elements, $\mathbb{E}_t \hat{p}_{t,t+j} Z_{t+j}$, we require

Lemma 3. *If* $\mathbb{E}_t \Xi^e_{t+j} \gtreqless 0$ *for any* $j \geq 0$, *then* $\chi_t \gtreqless 0$ *for any* $j \geq 0$.

Proof. First, in (96),

$$\mathbb{E}_t \hat{p}_{t+j,t} Z_{t+j} \gtreqless 0, \quad if \quad \forall j \geq 1, \quad \mathbb{E}_t \hat{p}_{t+j,t} \Xi^e_{t+j} \gtreqless 0.$$

The lemma follows by applying Assumption 1, and (96) and (102). □

A suitable formula for the carbon tax in terms of the Ξ^e_{t+j} is found by substituting (96) and (102) into (95):

$$\hat{\tau}^e_t = \varpi_t + \mathbb{E}_t \sum_{j=0}^{\infty} \hat{p}_{t+j,t} \Xi^e_{t+j} \hat{\varrho}_{t+j} (Y_{E_{t+j}} - \varpi_{t+j}). \tag{103}$$

The essence of the preceding is that if we know the sign of Ξ^e_{t+j} for any j, then we shall know the sign of any premium χ_t over the standard carbon tax. The implications of this for the different belief regimes are stated in several propositions in Section 13.

12. An *Ex Ante* Tax on Capital

While $p^e_t(x^t)$ and $\tau^e_t(x^t)$ are uniquely determined in (24) and (95), respectively, it is easy to demonstrate, following Chamley (1986), Zhu (1992), and Chari et al. (1994), that a state-by-state capital income tax is not uniquely determined because an implementable allocation $\{b, k, Q\}$ that uniquely determines household wealth $\mathcal{W}$ in (36) can be obtained by a multiplicity of capital tax and bond policies $\{\tau^k, b\}$ at prices $\{\hat{p}, r\}$.[32] However, across states of nature, the history-dependent value of tax payments is fully determined. So with this in mind, define the *effective* or *ex ante* tax rate on capital as the ratio of the prices of two "assets", one yielding a stream of tax revenues and the other yielding a stream of gross before-tax capital returns, conditional on history x^t, where the latter is defined by

$$\bar{\tau}^k_{t+1}(x^t) \quad \equiv \quad \frac{\mathbb{E}_t \hat{p}_{t+1,t}(x_{t+1}, x^t) \tau^k_{t+1}(x^{t+1})(Y_{k_{t+1}}(x^{t+1}) - \delta)}{\mathbb{E}_t \hat{p}_{t+1,t}(x_{t+1}, x^t)(Y_{k_{t+1}}(x^{t+1}) - \delta))}. \tag{104}$$

In this formula, the *ex ante* tax rate $\bar{\tau}^k_{t+1}(x^t)$ is the ratio of expected tax revenues to expected gross returns from capital, conditional on history x^t and valued at possibly distorted Arrow–Debreu prices $\hat{p}$.

With the use of (14), (104) is re-written as

$$\begin{aligned} \bar{\tau}^k_{t+1}(x^t) \quad &\equiv \quad \frac{\mathbb{E}_t \hat{p}_{t+1,t}(x_{t+1}, x^t)[1 - \delta + Y_{k_{t+1}}(x^{t+1})] - 1}{\mathbb{E}_t \hat{p}_{t+1,t}(x_{t+1}, x^t)(Y_{k_{t+1}}(x^{t+1}) - \delta)}, \\ &= \quad \frac{\mathbb{E}_t \hat{\varrho}_{t+1}[1 - \delta + Y_{k_{t+1}}] - 1}{\mathbb{E}_t \hat{\varrho}_{t+1}(Y_{k_{t+1}} - \delta)}. \end{aligned} \tag{105}$$

Based on consumption-Euler Equations (50) or (68),

$$\begin{aligned} 1 &= \mathbb{E}_t n^{**}_{t+1}\beta\frac{\lambda_{t+1}}{\lambda_t}(1+Y_{k_{t+1}}-\delta) \\ &= \mathbb{E}_t n^{**}_{t+1}\beta\frac{[1+\varepsilon_{t+1}]u_{c_{t+1}}+\Omega_{c_{t+1}}\Phi_{t+1}}{[1+\varepsilon_t]u_{c_t}+\Omega_{c_t}\Phi_t}(1+Y_{k_{t+1}}-\delta) \\ &= \mathbb{E}_t n^{**}_{t+1}\beta\frac{1+\varepsilon_{t+1}+[1-\gamma+\gamma\vartheta_{t+1}]n^*_{t+1}\Phi_{t+j-1}}{1+\varepsilon_t+[1-\gamma+\gamma\vartheta_t]\Phi_t}(1+Y_{k_{t+1}}-\delta), \end{aligned} \tag{106}$$

the government discounts *gross* returns to capital in period $t+1$ using ϱ^{**}, derived in (79), which is also used to discount future damages resulting from fossil use in (84), as shown earlier, where the distorting factor n^{**}_{t+1}varies according to belief regime as detailed in Table 1.

Substituting (107) into (105), an alternative expression for $\bar{\tau}^k_t$ is

$$\bar{\tau}^k_{t+1} = \frac{\mathbb{E}_t[(\hat{\varrho}_{t+1}-\varrho^{**}_{t+1})(1-\delta+Y_{k_{t+1}}]}{\mathbb{E}_t[\hat{\varrho}_{t+1}(Y_{k_{t+1}}-\delta)]}. \tag{107}$$

Noteworthy in this expression is that it echoes the expression in (96) determining the sign of the carbon tax premium, likewise depending on the difference between the private sector's discount factor and that of the government, $\hat{\varrho}_{t+1}-\varrho^{**}_{t+1}$. Simply put, the numerator in (107) is the difference between two non-centered covariances between the stochastic pre-tax return $1-\delta+Y_{k_{t+1}}$ and the private sector's and the government's SDF, respectively. In general terms, the *ex ante* tax on capital is negative if the private sector is more myopic, i.e., less patient with respect to returns to capital than the government, specifically, if the covariance of private returns with the private stochastic discount factor is less than the corresponding covariance with the planner's stochastic discount factor. Conversely, if private returns are better correlated with the government's stochastic discount factor, or are equal to it, then the average tax is positive or zero. Earlier literature by Chamley (1986), Judd (1985), and Atkeson et al. (1999), concludes that in a deterministic economy with time-additive preferences, the optimal *ex ante* tax on capital is zero, except possibly at time 0. Subsequently, Chari et al. (1994) and Zhu (1992) found that in a stochastic economy with homogeneous beliefs, the tax is positive (or negative) if a weighted average of the change in period elasticities of the utility function is positive (or negative). Below, I shall show how a divergence in beliefs between the private sector and government produces a gap between the government's and the private sector's discount factors that contribute additional motives to subsidize or penalize capital, depending on the source of ambiguity.

Now mimic expression (100) and define the weighted proportional difference between the private and the government's stochastic discount factors,

$$\Xi^k_{t+1} \equiv \varsigma^k_{t+1}\frac{\hat{\varrho}_{t+1}-\varrho^{**}_{t+1}}{\hat{\varrho}_{t+1}} = \varsigma^k_{t+1}(1-\frac{\varrho^{**}_{t+1}}{\hat{\varrho}_{t+1}}) = \varsigma^k_{t+1}(1-\frac{n^{**}_{t+1}}{m_{t+1}}\Psi(\varepsilon_{t+1},n^*_{t+1})), \tag{108}$$

where

$$\varsigma^k_{t+1} \equiv \frac{\hat{\varrho}_{t+1}[1-\delta+Y_{k_{t+1}}]}{\mathbb{E}_t[\hat{\varrho}_{t+1}(1-\delta+Y_{k_{t+1}})]}, \tag{109}$$

is the normalized discounted gross return to capital, such that $\mathbb{E}_t\varsigma^k_{t+1}=1$, and where $\Psi(\varepsilon_{t+1},n^*_{t+1})$ is defined in (79). The next lemma proves that the sign of the capital tax equals the sign of $E_t\Xi^k_{t+1}$:

Lemma 4.

$$\bar{\tau}^k_{t+1} \lesseqqgtr 0, \;\; if \;\; \mathbb{E}_t\Xi^k_{t+1} \lesseqqgtr 0. \tag{110}$$

Proof. Use (109) in (108) and write,

$$\begin{aligned} \mathbb{E}_t\Xi^k_{t+1} &\equiv \mathbb{E}_t[\varsigma^k_{t+1}\frac{\hat{\varrho}_{t+1}-\varrho^{**}_{t+1}}{\hat{\varrho}_{t+1}}] \\ &= \mathbb{E}_t[\frac{\hat{\varrho}_{t+1}[1-\delta+Y_{k_{t+1}}]}{E_t[\hat{\varrho}_{t+1}(1-\delta+Y_{k_{t+1}})]}\frac{\hat{\varrho}_{t+1}-\varrho^{**}_{t+1}}{\hat{\varrho}_{t+1}}] \\ &= \frac{\mathbb{E}_t[\hat{\varrho}_{t+1}(Y_{k_{t+1}}-\delta)]}{\mathbb{E}_t[\hat{\varrho}_{t+1}(1-\delta+Y_{k_{t+1}})]}\bar{\tau}^k_{t+1}, \end{aligned} \tag{111}$$

where the last term is obtained by multiplying and dividing by $\mathbb{E}_t[\hat{\varrho}_{t+1}(Y_{k_{t+1}}-\delta)]$ and using (107). The proof follows by observing that the term multiplying $\bar{\tau}^k_{t+1}$ is positive. □

For later use, re-write (111) as

$$\begin{aligned} \mathbb{E}_t\Xi^k_{t+1} &= \mathbb{E}_t\varsigma^k_{t+1}\left(1-\frac{n^{**}_{t+1}}{m_{t+1}}\frac{1+\varepsilon_{t+1}+[1-\gamma+\gamma\vartheta_{t+1}]n^*_{t+1}\Phi_t}{1+\varepsilon_t+[1-\gamma+\gamma\vartheta_t]\Phi_t}\right), \\ &= \mathbb{E}_t\varsigma^k_{t+1}\left(1-f(\Phi_t,\varepsilon_t)\frac{n^{**}_{t+1}}{m_{t+1}}(1+\varepsilon_{t+1}+[1-\gamma+\gamma\vartheta_{t+1}]n^*_{t+1}\Phi_t\right). \end{aligned} \tag{112}$$

13. Policy Implications of Belief Heterogeneity and Ambiguity

13.1. Preliminary Results

Having derived formulas for the expected social cost of carbon, and carbon as well as capital taxation, we are now prepared to consider the policy implications for the various belief regimes, with particular focus on any premia or surcharges that may arise as a consequence of belief distortions and ambiguities. The results are summarized in six propositions. As will become apparent, the sign and size of surcharges on the carbon tax and those of the *ex ante* tax on capital will depend on the signs of several key second-order moments, namely covariances involving measures related to the state of the economy, ς^k and ς^e, the average propensity to consume out of disposable income ϑ, and measures of the intensity of robustness concerns reflected in the endogenous worst-case belief distortions m^c, m^p, n^p, n^{POL}, n^{PAT} that arise from evaluating Formulas (87), (100), and (112) containing the expectations of products and ratios of random variables. The resulting expressions involve sums of expectations of their individual components and their covariances, known to be non-trivial because, as shown later, the assumption of endogeneity means that all involved variables are functions of the fundamental climate shock process x_t and are therefore related to each other. The manner of deconstructing optimal tax policy applied in this paper is novel in the literature, which has mostly had to rely on simulations to quantify policy effects, and should be considered a unique contribution of this paper.

The signs of the covariances needed to evaluate expectations and to prove later propositions are not always clear from intuition and need to be formally derived and stated in the form of several lemmas.

Assumption 2. $\frac{dc_{t+1}}{dx_{t+1}} \leq 0$.

It is generally assumed that consumption declines with global warming (see Frankhouser and Tol 2005; Weitzman 2009).

Lemma 5.

$$\frac{d\hat{\varrho}_{t+j}}{dx_{t+j}} > 0.$$

Proof.

$$\begin{aligned}\frac{d\hat{Q}_{t+j}}{dx_{t+j}} &= \beta M_{t+j}\frac{d\frac{u_{c_{t+j}}}{u_{c_t}}}{dx_{t+j}} + \beta\frac{u_{c_{t+j}}}{u_{c_t}}\frac{dM_{t+j}}{dx_{t+j}} \\ &= -\beta M_{t+j}\left(\frac{c_t}{c_{t+j}}\right)^{\gamma}\left(\gamma c_{t+j}^{-1}\frac{dc_{t+j}}{dx_{t+j}}\right) \\ &+ \beta\left(\frac{c_t}{c_{t+j}}\right)^{\gamma}\frac{dM_{t+j}}{dx_{t+j}} > 0,\end{aligned}$$

where first terms in each line are positive by Assumption 2, and the second lines are also positive since $\frac{dM_{t+j}}{dx_{t+j}} > 0$ if $M = M^c$, and zero otherwise. □

Assumption 3. $\frac{dk_{t+1}}{dx_{t+1}} < 0$.

Li et al. (2016) assume that capital utilization declines upon a climate cost shock. Frankhouser and Tol (2005, p. 5) observe that "the overall effect of climate change on the accumulation of capital is in principle ambiguous", but that "it seems safe to speculate that the capital accumulation effect will probably be negative". More recent evidence that a significant portion (50%) of total GDP losses can be attributed to disincentives to invest capital is less ambiguous (see Willner et al. 2021).

Assumption 4. $\frac{dH_{t+1}}{dx_{t+1}} \leq 0$.

Labor input likely declines as a result of a climate cost shock since, as documented in Dasgupta et al. (2021); Kjellstrom (2014); Kjellstrom et al. (2009); Somanathan et al. (2018), productive labor is lost as a consequence of warming, justifying

Assumption 5. $\frac{dE_{t+1}}{dx_{t+1}} \simeq 0$.

Wilbanks et al. (2008) find that climate warming reduces energy use and production, as verified by the BEA (BEA 2019), although, as an Environmental Protection Agency web page, deleted by the Trump Administration in 2017 but saved and still available on (EPA 2017), reports, climate warming may lead to partially offsetting heating and cooling demands.

Lemma 6.

$$\begin{aligned}\frac{dY_{z_{i,t}}}{dx_t} &= -(Q_0 - Q_t)D_tF_{z_{i,t}} + \sum_{j=1}^{n}\frac{\partial Y_{z_{i,t}}}{\partial z_{j,t}}\frac{dz_{j,t}}{dx_t}, \\ &\approx -(Q_0 - Q_t)D_tF_{z_{i,t}} < 0, \end{aligned} \tag{113}$$

for all factors of production, k, E, and H.

Proof. The signs of $\frac{dz_{j,t}}{dx_t}$ are given in Assumptions 3–5, and $\frac{\partial Y_{z_{i,t}}}{\partial z_{j,t}} > 0$. Recent literature (Dasgupta et al. 2021; Njuki et al. 2020; Zhang et al. 2017) suggests that the indicated sum is likely negative but small, leaving as main driver of climate cost shocks their effects on total factor productivity, here basically represented by the damage function. □

Assumption 6. $\frac{db_{t+1}}{dx_{t+1}} > 0$.

The effect of a climate shock on debt b_t can plausibly be said to be positive. In their study of Columbia and Peru, Maldonado and Gallagher (2022) provide some evidence that climate shocks significantly affect public debt trajectories towards significantly higher levels and, in some cases, raise probabilities of increasing debt during climate stress. Nor do developed economies in Europe seem to be immune from this effect. For example, Zenios (2022) combined projections from the IMF World Economic Outlook with simulations of versions of an IAM model obtained from Emmerling et al. (2016) and Gazzotti et al. (2021) to show that climate shocks will raise the sovereign debt-to-GDP ratio in Italy and Cyprus over time.

Lemma 7.

$$\frac{dm^c_{t+1}}{dx_{t+1}} \geq 0.$$

Proof. From (6),

$$\frac{dm^c_{t+1}}{dx_{t+1}} = \frac{\partial m^c_{t+1}}{\partial \mathcal{U}_{t+1}} \frac{d\mathcal{U}_{t+1}}{dx_{t+1}} \quad = \quad \sigma^c m^c_{t+1}(1 - m^c_{t+1}) \frac{d\mathcal{U}_{t+1}}{dx_{t+1}}.$$

The term multiplying $\frac{d\mathcal{U}_{t+1}}{dx_{t+1}}$ is negative, indicating that decreases in $\mathcal{U}_{t+1}$ raise m^p_{t+1} toward 1. The envelope conditions (10)–(12) imply

$$\begin{aligned} \frac{d\mathcal{U}_{t+1}}{dx_{t+1}} &= \mathcal{U}_{k_{t+1}} \frac{dk_{t+1}}{dx_{t+1}} + \mathcal{U}_{b_{t+1}} \frac{db_{t+1}}{dx_{t+1}} + \mathcal{U}_{Q_{t+1}} \frac{dQ_{t+1}}{dx_{t+1}} \\ &= u_{c_{t+1}} [R^k_{t+1} \frac{dk_{t+1}}{dx_{t+1}} + \frac{db_{t+1}}{dx_{t+1}}] \leq 0, \end{aligned}$$

if the negative effect on capital outweighs the presumably positive effect on debt. Otherwise, the result follows from $\sigma^c < 0$ and $m^c \leq 1$.[33] □

Lemma 8.

$$\frac{dn^{PO}_{t+1}}{dx_{t+1}} \quad > \quad 0.$$

Proof. Evaluate

$$\begin{aligned} \frac{dn^{PO}_{t+1}}{dx_{t+1}} &= \frac{\partial n^{PO}_{t+1}}{\partial \mathcal{V}_{t+1}} \frac{d\mathcal{V}_{t+1}}{dx_{t+1}} + \frac{\partial n^{PO}_{t+1}}{\partial \mathcal{V}_{t+1}} \frac{d\Upsilon_{t+1}}{dx_{t+1}} \\ &= \sigma n^{PO}_{t+1}(1 - n^{PO}_{t+1}) \left(\mathcal{V}_{k_{t+1}} \frac{dk_{t+1}}{dx_{t+1}} + \mathcal{V}_{\Upsilon_{t+1}} \frac{d\Upsilon_{t+1}}{dx_{t+1}} + \bar{\Phi} \frac{d\Upsilon_{t+1}}{dx_{t+1}} \right) \\ &= \sigma n^{PO}_{t+1}(1 - n^{PO}_{t+1}) \left((1 - \delta + \Upsilon_{k_{t+1}})(u_{c_{t+1}} + \Omega_{c_{t+1}} \bar{\Phi}) \frac{dk_{t+1}}{dx_{t+1}} \right) \\ &+ \sigma n^{PO}_{t+1}(1 - n^{PO}_{t+1}) \left(\Phi_t \frac{d\Upsilon_{t+1}}{dx_{t+1}} - \Phi_{t+1} \frac{d\Upsilon_{t+1}}{dx_{t+1}} \right) \\ &= \sigma n^{PO}_{t+1}(1 - n^{PO}_{t+1}) \left((1 - \delta + \Upsilon_{k_{t+1}})(u_{c_{t+1}} + \Omega_{c_{t+1}} \bar{\Phi}) \frac{dk_{t+1}}{dx_{t+1}} \right) > 0, \end{aligned}$$

which follows from $\sigma < 0$, $n^{PO} \leq 1$, envelope conditions (52) and (54), Euler condition (50) with $\xi = 0$, from Assumption 3.[34], and from the previous result that $\Omega_{c_{t+1}} > 0$.[35] □

The preceding result accords with the properties $0 < n^{PO}_{t+1} \leq 1$, $\lim_{\sigma \uparrow 0} n^{PO}_{t+1} \to 1$, $\lim_{\sigma \downarrow -\infty} n^{PO}_{t+1} \to 0$, and the partial derivative

$$\frac{\partial n^{PO}_{t+1}}{\partial \sigma} = (\mathcal{V}_{t+1} + \Phi_t \Upsilon_{t+1})(1 - n^{PO}_{t+1}) n^{PO}_{t+1} \geq 0. \tag{114}$$

As one might expect intuitively, increased ambiguity aversion ($\sigma \downarrow$) reduces n^{PO} and drives it towards zero, a result that echoes those in Gilboa and Schmeidler (1989) and Millner et al. (2012).

Lemma 9.

$$\frac{dn^{PA}_{t+1}}{dx_{t+1}} > 0.$$

Proof. From (39) and results (41),

$$\begin{aligned} \frac{dn^{PA}_{t+1}}{dx_{t+1}} &= \frac{\partial n^{PA}_{t+1}}{\partial \Upsilon_{t+1}} \frac{d\Upsilon_{t+1}}{dx_{t+1}} = \sigma \Phi_t n^{PA}_{t+1}(1 - n^{PA}_{t+1}) \frac{d\Upsilon_{t+1}}{dx_{t+1}} \\ &= \sigma \Phi_t n^{PA}_{t+1}(1 - n^{PA}_{t+1}) \left(\Omega_{c_{t+1}} \frac{dc_{t+1}}{dx_{t+1}} \right) \geq 0. \end{aligned}$$

Given $\sigma < 0$ and Assumption 2, this implies a positive correlation between n^{PA} and x. □

By the preceding arguments,

Lemma 10.

$$\frac{\partial n^{p}_{t+1}}{\partial x_{t+1}} \geq 0.$$

Proof. The proof follows argument similar to the proof of Lemma 10. □

The preceding discussion leads up to the following two lemmas:

Lemma 11. *Given definition (109),*

$$\frac{d\varsigma^{k}_{t+1}}{dx_{t+1}} \gtreqless 0.$$

Proof. Based on Lemma 5 and the result in (113),

$$\begin{aligned} \frac{d\varsigma^{k}_{t+1}}{dx_{t+1}} &= \frac{d\hat{\varrho}_{t+1}(1 - \delta + \Upsilon_{k_{t+1}})}{dx_{t+1}} \\ &= (1 - \delta + \Upsilon_{k_{t+1}}) \frac{d\hat{\varrho}_{t+1}}{dx_{t+1}} + \hat{\varrho}_{t+1} \frac{d\Upsilon_{k_{t+1}}}{dx_{t+1}} > 0. \end{aligned}$$

□

Lemma 12.

$$\frac{d\varsigma^{e}_{t+j}}{dx_{t+j}} > 0 \;\; \forall j > 0.$$

Proof. Use Lemma 5 and the result in (113) and note that, from Formula (98), the sign of $\frac{d\varsigma^e_{t+j}}{dx_{t+j}}$ is the same as the sign of

$$\frac{d\hat{\varrho}_{t+j}(Y_{E_{t+j}} - \varpi_{t+j})}{dx_{t+j}} = (Y_{E_{t+j}} - \varpi_{t+j})\frac{d\hat{\varrho}_{t+j}}{dx_{t+j}} + \hat{\varrho}_{t+j}\frac{dY_{E_{t+j}}}{dx_{t+j}} > 0.$$

□

Lemma 13. $\frac{d\Lambda_{t+j}}{dx_{t+j}} \geq 0.$

Proof. This follows from its definition in (85). □

Assumption 7.

$$\frac{d\vartheta_{t+j}}{dx_{t+j}} = \frac{d\frac{H_t+g_t}{c_t}}{dx_{t+j}} \approx 0.$$

Since climate lowers both income and consumption, the net effect on ϑ is likely negligible, justifying an assumption that ϑ is not strongly negatively correlated with Λ, ς^k, ς^e, m^c, n^{PO}, m^c, n^{PA}, and n^p, all of which are positively correlated with x.

Under the assumption that climate shocks are the sole stochastic process x driving the economy, the next proposition reflects the fact that variables that are functions of x must be correlated with each other. The signs of the covariances listed therein follow from the preceding lemmas.

Lemma 14. *Signs of key covariances*

(i) The signs of $cov(\Lambda,\vartheta)$, $cov(\varsigma^e,\vartheta)$, $cov(\varsigma^k,\vartheta)$, $cov(n^{PO},\Lambda)$, $cov(n^{PO},\Lambda\vartheta)$, $cov(n^{PA},\Lambda)$, $cov(n^{PA},\Lambda\vartheta)$, $cov(n^{PA},\vartheta\varsigma^e)$, $cov(n^{PA},\varsigma^k)$, $cov(n^{PA},\vartheta\varsigma^k)$, $cov(m^{PA},\Lambda)$, $cov(m^{PA},\vartheta\Lambda)$, $cov(n^p,\Lambda)$, $cov(n^p,\varsigma^k)$, $cov(n^p,m^c)$, $cov(m^p,\varsigma^k)$, $cov(m^p,\Lambda)$, $cov(m^p,\vartheta\Lambda)$, $cov(m^p\varsigma^k,\vartheta)$, $cov(m^c m^p,\Lambda)$, $cov(m^c,\Lambda)$, *and* $cov(m^c,\vartheta\Lambda)$ *are positive.*

(ii) The signs of $cov(\varsigma^e,\frac{1}{m^c})$, $cov(n^p,\frac{1}{m^c})$, $cov(\varsigma^e,\frac{1}{n^p})$, *and* $cov(\vartheta\varsigma^e,\frac{1}{n^p})$ *are negative.*

(iii) The signs of $cov(\varsigma^k,\frac{n^p}{m^c})$, $cov(\varsigma^e,\frac{n^p}{m^c})$, $cov(\varsigma^e,\frac{m^p}{n^p})$, *and* $cov(\varsigma^e\vartheta,\frac{m^p}{n^p})$ *are indeterminate.*

The following propositions distinguish two situations: (i) when the implementability (the marginal-utility-of-consumption value of household wealth) constraint is binding, ($\Phi > 0$), and (ii) when it is not, ($\Phi = 0$). In the latter case, all policies revert to those of an unconstrained social planner.[36] To set a baseline for comparison, the first proposition establishes results that obtain with homogeneous beliefs under rational expectations RE.

13.2. Homogeneous Beliefs

Proposition 1 (Benchmark rational expectations). *If (i) beliefs are homogeneous and rational, and (ii) labor income plus tax rebates as a proportion of consumption do not decrease over time in all periods, then*

1. *The social cost of carbon exceeds the standard formulation* Λ^s_t *derived in (88) for a social planner, unless* $\bar{\Phi} = 0$*,*

$$\begin{aligned}\varpi^{RE}_t &= \Lambda^s_t + \gamma\bar{\Phi}f(\bar{\Phi},0)[(\mathbb{E}_t\vartheta_{t+1} - \vartheta_t)\Lambda^s_t + cov(\Lambda,\vartheta)] \geq \Lambda^s_t, \;\; \bar{\Phi} > 0, \; \forall t,\\ &= \Lambda^s_t, \;\; \Phi_t = 0 \; \forall t.\end{aligned}$$

2. *The carbon tax premium is positive, unless* $\Phi = 0$*,*

$$\begin{aligned}\mathbb{E}_t\Xi^{e-RE}_{t+j} &= \gamma\bar{\Phi}f(\bar{\Phi},0)\left[\mathbb{E}_t\vartheta_{t+j} - \vartheta_t + cov(\varsigma^e,\vartheta)\right] > 0, \;\; \bar{\Phi} > 0, \; \forall t\\ &= 0, \;\; \bar{\Phi} = 0 \; \forall t.\end{aligned}$$

3. *The ex ante rate on capital is negative, unless* $\Phi = 0$,

$$\begin{aligned} \mathbb{E}_t \Xi_{t+1}^{k-RE} &= -\gamma \bar{\Phi} f(\bar{\Phi}, 0)\left[\mathbb{E}_t \vartheta_{t+1} - \vartheta_t + cov(\varsigma^k, \vartheta)\right] < 0, \;\; \bar{\Phi} > 0, \forall t \\ &= 0, \;\; \bar{\Phi} = 0 \; \forall t. \end{aligned}$$

This proposition sets up a background and baseline for comparison with the conclusions for the remaining belief regimes. It also establishes an important distinction between social and Ramsey planning. For a social planner, defined as one for whom the implementability constraint (39) is not binding ($\Phi = 0$)—nor invoked in most of the literature—the policy settings coincide with the formulas derived in Golosov et al. (2014) for the optimal carbon tax and the social cost of carbon, and with the optimal tax on capital derived by Zhu (1992), Chari et al. (1994), and Atkeson et al. (1999). By contrast, a true Ramsey planner, being mindful of preferences and household budgets, adjusts all calculations by factors involving the inter-temporal rate of substitution and the average propensity to consume.

13.3. Heterogeneous Beliefs: Skeptical Consumers

The next proposition establishes that even without ambiguity, skepticism alters optimal policy.

Proposition 2 (**No ambiguity**). *By producing incentives to spend more on fossil energy and less on capital than is socially optimal, the mere presence of skepticism is an inducement to the Ramsey planner but not a social planner to raise the carbon tax and to subsidize capital, where*

1. *Social cost of carbon is higher than under RE, unless* $\Phi = 0$*:*

$$\begin{aligned} \varpi_t &= \varpi_t^{RE} > \Lambda_t^s \;\; \Phi_t > 0, \;\; \forall t, \\ &= \Lambda_t^s \;\; \Phi_t = 0; \end{aligned}$$

2. *The carbon tax is higher than under RE, unless* $\Phi = 0$*:*

$$\begin{aligned} \mathbb{E}_t \Xi_{t+j}^{e} &\geq \mathbb{E}_t \Xi_{t+j}^{e-RE} \geq 0, \;\; \Phi_t > 0 \;\; \forall t \\ &= 0, \;\; \Phi_t = 0; \end{aligned}$$

3. *Ex ante capital tax rate is less than under RE, unless* $\Phi = 0$*:*

$$\begin{aligned} \mathbb{E}_t \Xi_{t+1}^{k} &\leq \mathbb{E}_t \Xi_{t+1}^{k-RE} \leq 0, \;\; \Phi_t > 0 \;\; \forall t \\ &= 0, \;\; \Phi_t = 0. \end{aligned}$$

This proposition highlights an intriguing point: having little faith in climate science, climate skeptics might naturally want to pay a lower carbon tax. Yet, with manifestly poetic justice, the very consequence of skepticism by itself is an increase in both the social cost of carbon and the carbon tax, and a decrease in the tax on capital.

The next two propositions provide results for two regimes in which the Ramsey planner faces a climate-skeptical public whose beliefs are not known and are, indeed unknowable. The first regime is political in that the planner believes the unknown beliefs of the private sector to be true. In the second regime, the planner is paternalistic in that it believes the science model to be true.

Proposition 3 (**Political planner**). *Ignorance of private beliefs believed to be true leads to the following policy alterations:*

1. *The social cost of carbon contains an ambiguity premium for both Ramsey and social plans:*

$$\begin{aligned}\varpi_t &= \varpi_t^{RE} + \bar{\Phi} f(\bar{\Phi},0)\Big[(1-\gamma)cov(n^{PO},\Lambda) + \gamma cov(n^{PO},\Lambda\vartheta)\Big] \\ &+ f(\bar{\Phi},0)cov(n^{PO},\Lambda) \geq \varpi_t^{RE}, \ \forall t, \\ &= \Lambda_t^s + cov(n^{PO},\Lambda) \geq \Lambda_t^s, \ \bar{\Phi} = 0;\end{aligned}$$

2. *The carbon tax is higher than under RE, unless* $\Phi = 0$*:*

$$\begin{aligned}\mathbb{E}_t \Xi_{t+j}^{e-PO} &\geq \mathbb{E}_t \Xi_{t+j}^{e-RE} \geq 0, \ \ \Phi_t > 0 \ \forall t, \\ &= 0, \ \ \Phi_t = 0;\end{aligned}$$

3. *Ex ante capital tax is lower than under RE, unless* $\Phi = 0$*:*

$$\begin{aligned}\mathbb{E}_t \Xi_{t+1}^{k-PO} &\leq \mathbb{E}_t \Xi_{t+1}^{k-RE} \leq 0, \ \bar{\Phi} > 0 \ \ \forall t \\ &= 0, \ \ \bar{\Phi} = 0.\end{aligned}$$

The preceding proposition establishes that a political planner's ignorance about private beliefs, even if held to be correct, justifies a positive ambiguity premium for the social cost of carbon, activated by correlations between the planner's worst-case belief multiplier n^{PO} and the social cost of carbon that would apply if rational expectations prevailed. In addition, unless the government is a social planner, ambiguity raises the carbon tax and lowers the capital tax.

Proposition 4 (Paternalistic planner). *Ignorance of private beliefs that the planner also believes to be false leads to the following policy alterations:*

1. *The social cost of carbon contains an ambiguity premium, unless* $\Phi = 0$*:*

$$\begin{aligned}\varpi_t &= \varpi_t^{RE} + \Phi_t f(\Phi_t,0)\Big[(1-\gamma)cov(n^{PA},\Lambda) + \gamma cov(n^{PA},\Lambda\vartheta)\Big] \\ &\geq \varpi_t^{RE} \ \ \Phi_t > 0,;\forall t, \\ &= \Lambda_t^s, \ \Phi_t = 0;\end{aligned}$$

2. *The carbon tax contains an ambiguity premium, unless* $\Phi = 0$*:*

$$\begin{aligned}\mathbb{E}_t \Xi_{t+j}^{e-PA} &\geq \mathbb{E}_t \Xi_{t+j}^{e-RE} + \gamma \Phi_t f(\Phi_t,0)cov(n^{PA},\varsigma^e\vartheta) \geq \mathbb{E}_t \Xi_{t+1}^{e-RE} \\ &= \geq 0, \ \ \Phi_t > 0 \ \ \forall t, \\ &= 0, \ \ \Phi_t = 0;\end{aligned}$$

3. *The ex ante capital tax contains an ambiguity subsidy, unless* $\Phi = 0$*:*

$$\begin{aligned}\mathbb{E}_t \Xi_{t+1}^{k-PA} &\leq \mathbb{E}_t \Xi_{t+1}^{k-RE} - \Phi_t f(\Phi_t,0)\Big[1 + \gamma cov(n^{PA},\varsigma^k) + \gamma cov(n^{PA},\varsigma^k\vartheta)\Big] \\ &\leq \mathbb{E}_t \Xi_{t+1}^{k-RE} \leq 0, \ \ \Phi_t > 0 \ \ \forall t, \\ &= 0, \ \ \Phi_t = 0.\end{aligned}$$

The main distinction between a political and a paternalistic planner, as defined in this paper, is that for the latter, an ambiguity premium for the social cost of carbon applies only if the government is a Ramsey planner and not a social planner. For both types of Ramsey planner, a positive ambiguity premium on the social cost of carbon is optimal because the planner's worst-case martingale belief distortion correlates with the certainty-equivalent version of the social cost of carbon.

The next proposition introduces the possibility of the government itself having doubts about the model.

Proposition 5 (Pessimistic planner—skeptical consumer). *Ignorance of private beliefs that the government believes to be false, in combination with a planner's doubts about the model, leads to the following policy alterations:*

1. *The social cost of carbon contains an ambiguity premium in both Ramsey and social plans:*

$$\begin{aligned} \varpi_t &\geq \varpi_t^{RE} + f(\Phi_t, 0)cov(n^p, \Lambda) \\ &+ f(\Phi_t, 0)\Phi_t[(1-\gamma)cov(m^p, \Lambda) + \gamma cov(m^p, \vartheta\Lambda)] \geq \varpi_t^{RE},\ \Phi_t > 0 \\ &\geq \varpi_t^{RE} + cov(n^p, \Lambda) \geq \varpi_t^{RE},\ \ \Phi_t = 0\ \ \forall t. \end{aligned}$$

2. *The carbon tax contains a positive or negative ambiguity premium, unless $\Phi = 0$:*

$$\begin{aligned} \mathbb{E}_t \Xi_{t+j}^e &\geq \mathbb{E}_t \Xi_{t+j}^{e-RE} + (1-\gamma)\Phi_t f(\Phi_t, 0)\left[cov(m^p, \frac{1}{n^p}) + cov(\varsigma^e, \frac{m^p}{n^p})\right] \\ &+ \gamma\Phi_t f(\Phi_t, 0)\left[cov(\vartheta, \varsigma^e) + cov(m^p, \frac{1}{n^p}) + cov(\varsigma^e\vartheta, \frac{m^p}{n^p})\right] \\ &\gtreqless \Xi_{t+j}^{e-RE},\ \ \Phi_t > 0\ \ \forall t \\ &= 0,\ \ \Phi_t = 0. \end{aligned}$$

3. *The ex ante capital tax is a subsidy in both Ramsey and social plans:*

$$\begin{aligned} \mathbb{E}_t \Xi_{t+1}^{k-R-s} &\leq \mathbb{E}_t \Xi_{t+1}^{k-RE} - f(\Phi_t, 0)[cov(\varsigma^k, n^p) + (1-\gamma)cov(\varsigma^k, m^p)] \\ &- \gamma\Phi_t f(\Phi_t, 0)cov(\vartheta, \varsigma^k m^p)] \leq \mathbb{E}_t \Xi_{t+1}^{k-RE} \leq 0,\ \ \Phi_t \geq 0, \\ &= -cov(\varsigma^k, n^p) - (1-\gamma)cov(\varsigma^k, m^p) \leq 0,\ \ \Phi_t = 0. \end{aligned}$$

Consistent with Propositions 3 and 4, the marginal contribution of an increased correlation between the government's ignorance of skeptical private beliefs $m^p\pi$ with the normalized discounted excess return to fossil energy over the social cost of carbon ς^e or the discounted gross return to capital ς^k, is to raise both the social cost of carbon and the carbon tax, and to lower the *ex ante tax* on capital. The marginal contribution of correlations involving the planner's own model doubts is to raise the social cost of carbon and to lower both the carbon tax and the *ex ante* tax on capital.

In the next and final proposition, the government remains pessimistic, but consumers are pessimistic rather than skeptical.

13.4. Heterogeneous Beliefs: Pessimistic Consumers

Proposition 6 (Pessimistic planner—pessimistic consumer). *The combination of pessimism in the private sector and the government's pessimism has the following implications:*

1. *The social cost of carbon contains an ambiguity premium in both Ramsey and social plans:*

$$\begin{aligned} \varpi_t &\geq [1 - f(\Phi_t, \varepsilon_t)\varepsilon_t]\varpi_t^{RE} + f(\Phi_t, \varepsilon_t)[\varepsilon_t + cov(n^p, \Lambda)] \\ &+ f(\Phi_t, \varepsilon_t)\Phi_t[(1-\gamma)cov(m^c, \Lambda) + \gamma cov(m^c, \vartheta\Lambda)] \\ &\geq \varpi_t^{RE},\ \ \Phi_t > 0, \\ &\geq \frac{1}{1+\varepsilon_t}[\varpi_t^{RE} + \varepsilon_t + cov(n^p, \Lambda)] > \Lambda_t^s\ \ \Phi_t = 0,\ \ \forall t. \end{aligned}$$

2. *If* $\Phi > 0$*, the premium on the carbon tax may be positive or negative, and is negative otherwise:*

$$\begin{aligned}
\mathbb{E}_t\Xi^e_{t+j} &\geq \mathbb{E}_t\Xi^{e-RE}_{t+j} \\
&- f(\Phi_t,\varepsilon_t)f(\Phi_t,0)[1+\Phi_t(1-\gamma+\gamma\mathbb{E}_t\vartheta_{t+1}+\gamma cov(\varsigma^e,\vartheta))]\varepsilon_t \\
&+ f(\Phi_t,\varepsilon_t)\left[\varepsilon_t + cov(\varsigma^e,\frac{1}{m^c}) + \Phi_t[(1-\gamma)cov(\varsigma^e,\frac{1}{n^p}) + \gamma cov(\vartheta\varsigma^e,\frac{1}{n^p})]\right] \\
&\lesseqgtr \mathbb{E}_t\Xi^{e-RE}_{t+j},\ \ \Phi_t > 0,\ \ \forall t \\
&= \frac{1}{1+\varepsilon_t}cov(\varsigma^e,\frac{1}{m^c}) < 0,\ \ \Phi_t = 0.\ \ \forall t
\end{aligned}$$

3. *The ex ante capital tax rate may be positive or negative:*

$$\begin{aligned}
\mathbb{E}_t\Xi^{k-R-p}_{t+1} &\leq \mathbb{E}_t\Xi^{k-RE}_{t+1} \\
&- f(\Phi_t,\varepsilon_t)f(\Phi_t,0)\left[1+\Phi_t\Big(1-\gamma+\gamma[\mathbb{E}_t\vartheta_{t+1}+cov(\varsigma^k,\vartheta)]\Big)\right]\varepsilon_t \\
&- f(\Phi_t,\varepsilon_t)\left[cov(\varsigma^k,\frac{n^p}{m^c}) + cov(\frac{1}{m^c},n^p) + (1+\varepsilon_t)cov(\varsigma^k,n^p)\right] \\
&\lesseqgtr \mathbb{E}_t\Xi^{k-RE}_{t+1},\ \ \Phi_t > 0 \\
&= -\frac{1}{1+\varepsilon_t}\left[cov(\varsigma^k,\frac{n^p}{m^c}) + cov(\frac{1}{m^c},n^p)\right] \\
&- cov(\varsigma^k,n^p) \lesseqgtr 0,\ \ \Phi_t = 0.
\end{aligned}$$

As in Proposition 5, which concerned skeptical beliefs, in this belief regime, the marginal effect of private-sector pessimism m^c is to raise the social cost of carbon. However, the effect on carbon and capital taxation is the opposite, producing a reduction in the carbon tax and an increase in the tax on capital. The intuition is that increased consumer doubts about the climate model tend to motivate carbon consumption below the socially optimal level and to increase capital spending above its socially optimal level. The net effect of an increase in ϖ_t and a decrease in $\mathbb{E}_t\Xi^e_{t+j}$ may or may not in the end produce a lower carbon tax itself, because from (95) is the sum of two terms: $\varpi_t + \chi_t$, where χ is the sum of terms that contain expected future values of Ξ^e_{t+j}.

Proposition 6 echoes Proposition 5 in that the marginal contribution of any correlation between the planner's own model doubts n^p and asset returns represented by ς^k and ς^e is to likewise raise the social cost of carbon and to lower both the carbon tax and the *ex ante* tax on capital. This becomes apparent if the role of private beliefs is de-activated by setting $m^p \equiv 1$ or $m^c \equiv 1$, respectively, leading to

$$\varpi_t = \varpi^{RE}_t + f(\Phi_t,0)cov(n^p,\Lambda), \tag{115}$$

$$\begin{aligned}
\mathbb{E}_t\Xi^e_{t+j} &\geq \mathbb{E}_t\Xi^{e-RE}_{t+j} \\
&+ f(\Phi_t,0)\Phi_t\left[(1-\gamma)cov(\varsigma^e,\frac{1}{n^p}) + \gamma cov(\vartheta\varsigma^e,\frac{1}{n^p})\right],
\end{aligned} \tag{116}$$

$$\mathbb{E}_t\Xi^{k-R-s}_{t+1} \leq \mathbb{E}_t\Xi^{k-RE}_{t+1} - f(\Phi_t,0)cov(\varsigma^k,n^p). \tag{117}$$

The belief regimes in the three preceding formulas, representing policies stripped of any effects due to belief distortions in the private sector—skeptical or pessimistic—are most closely related to the extant literature on robust climate policy and so serve best for comparisons, to which I now turn.

In a policy regime most similar to that treated in Proposition 5, with m^p set equal to 1 but differing in some details, Hennlock (2009) attributes doubts about the model not to a policy authority *per se* but to a utility-maximizing representative consumer representing society who computes robust feedback rules that, as in (115), generate an ambiguity premium on the expected social cost of carbon, so that with nonlinear damage, policy becomes more responsive to changes in climate. Li et al. (2016) study a dynamic optimization problem that is nearly identical to the planning models in Section 7, but under the assumption that the government is a social planner ($\Phi = 0$ in the present paper) and not a Ramsey planner. They find that even a relatively small increase in the concern about model uncertainty can cause a significant drop in optimal energy extraction and a rise in the socially optimal carbon tax, which in this paper would also correspond to the result in (115). Likewise, Cai and Lontzek (2019) find that with empirically plausible parameterizations of Epstein-Zin preferences to represent attitudes towards risk, the uncertainty associated with anthropogenic climate change implies carbon taxes much higher than associated with deterministic models, while Lemoine and Traeger (2016) conclude that a government's aversion to Knightian uncertainty in the face of an ambiguous tipping point increases the optimal tax on carbon dioxide emissions, but only by a small amount. As with Li et al. (2016), since the derived tax effects come via changes in the SCC, those two conclusions also best correspond to formula (115). Heal and Millner (2013) do not consider taxation *per se*, but find that the value of abatement (that would presumably include carbon taxation) rises as ambiguity aversion increases. Rezai and van der Ploeg (2017) take a somewhat different approach to modeling ambiguity and consider a so-called agnostic policy authority—essentially a government having ambiguity about the approximating climate model—as facing potential models ranging from denialist to scientific. If such a government pursues max-max policies, it imposes higher carbon taxes as a precaution. Later, with the help of Nordhaus's (1993) DICE model to simulate carbon taxation, Rezai and van der Ploeg (2019) broaden their earlier results and consider an agnostic planner who adopts Pascal's Wager (Pascal's (1670))[37] with the question: what would such an agnostic but rational planner—one who does not know or care which model is correct but who wishes to avoid the worst—do when faced with some probability that the approximating model, adhered to by so-called deniers, is false? Their conclusion is that the *hedge-your-bet* optimal carbon tax is quite close to the optimal tax derived in a non-denialist scientific setting, even if the probability of the model being false is a mere ten percent. Further, when ambiguity about whether scientists or deniers are correct rises, as represented by a parameter of constant relative ambiguity aversion, the optimal carbon pricing policy moves ever closer to the science-based policy.

Finally, Anderson et al. (2013), who study a model similar in spirit to the models in this paper, except for an additional robustness channel capable of affecting growth, find that a planner's increased deep uncertainty about the model can result in either a decrease or an increase in the optimal carbon tax, depending on other factors, such as market features and social preferences. In the present paper, the conclusions are driven by similar forces involving preferences and market features, but in the form of second-order moments represented by the covariances between worst-case martingale belief multipliers m and relevant market features, such as net asset-returns to fossil energy and capital.

Table 2 summarizes the preceding propositions. A principal leitmotif of this paper is belief heterogeneity, a main driver of ambiguities in all regimes studied here. Even if (as a mental, albeit unrealistic exercise in Proposition 2) one were to assume away ambiguity, the very presence of heterogeneity in beliefs between the private sector and the government, leading to a positive spread between their respective discount factors, is sufficient to alter the policies of a Ramsey planner, though not those of a social planner. As a consequence, the government increases its estimate of the social cost of carbon and adds a premium to the carbon tax, while raising the capital subsidy rate.

A government with its own doubts about the model is compelled to raise the social cost of carbon and acquires further motives to either raise or lower taxes, depending on the case. However, in all instances, the planner will raise the social cost of carbon. The political and paternalistic planners of Propositions 3 and 4 face ambiguity because of their ignorance of arbitrary private beliefs held by consumers who regard the scientific model with skepticism and discount the future at relatively higher rates. They therefore use relatively more fossil energy and invest in relatively less capital than is socially optimal. To encourage socially optimal choices, the government taxes carbon and subsidizes capital.

The pessimistic planners in Propositions 5 and 6 are identical except with respect to the kind of beliefs they face. The former operates under ambiguities that result from doubts about the model itself and from its ignorance of private beliefs. The latter government confronts a single ambiguity caused by its own doubts about the model, but being constrained by consumers who have known pessimistic beliefs, this government effectively manages two kinds of ambiguities, its own and those of the public. In both cases, the government's doubts about the model, indexed by n^p, provide motives to raise the social cost of carbon and the capital subsidy but to lower the carbon tax. A pessimistic government's ignorance of the beliefs of skeptical consumers in Proposition 5 motivates policies that mimic those of the political and paternalistic planners in Propositions 3 and 4.

Pessimistic consumers discount the future less than do skeptical or rational consumers and therefore use less carbon energy and invest more in capital relative to socially optimal rates and quantities. So the presence of private-sector pessimism is an inducement for the government to lower carbon taxes and to raise the capital tax. Given that the pessimistic government's own doubts produce motives in the opposite direction, the net effect of private and government doubts can be ambiguous.

Table 2. Propositions 1–6.

	Φ	ϖ_t	χ_t	$\bar{\tau}_t^k$
I. Homogeneous Beliefs (RE)				
Ramsey	>0	$\geq\Lambda^s$	$\chi^{RE} \geq 0$	$\tau^{k-RE} \leq 0$
Social	0	Λ^s	0	0
II. Heterogeneous Beliefs				
1. **Skeptical consumers**				
Ambiguity absent				
Ramsey	>0	$\geq\Lambda^s$	$\geq\chi^{RE}$	$\leq\tau^{k-RE}$
Social	0	Λ^s	≥ 0	≤ 0
Ambiguity present				
Political planner				
Ramsey	>0	$\geq\Lambda^s$	$\geq\chi^{RE}$	$\leq\tau^{k-RE}$
Social	0	$\geq\Lambda^s$	≥ 0	≤ 0
Paternalistic planner				
Ramsey	>0	$\geq\Lambda^s$	$\geq\chi_t^{RE}$	$\leq\tau^{k-RE}$
Social	0	$\geq\Lambda^s$	≥ 0	≤ 0
Pessimistic planner				
Ramsey	>0	$\geq\Lambda^s$	$\gtreqless\chi^{RE}$	$\lesseqgtr\tau^{k-RE}$
Social	0	$\geq\Lambda^s$	$\geq\chi^{RE}$	$\leq\tau^{k-RE}$
Effect of planner's ambiguity				
Ramsey	>0	$\uparrow$	$\uparrow$	$\downarrow$
Social	0	$\uparrow$	$\uparrow$	$\downarrow$
Effect of consumer's ambiguity				
Ramsey	>0	$\uparrow$	$\downarrow$	$\downarrow$
Social	0	$\uparrow$	$\downarrow$	$\downarrow$
2. **Pessimistic consumers**				
Pessimistic planner				
Ramsey	>0	$\geq\Lambda^s$	$\gtreqless\chi^{RE}$	$\lesseqgtr\tau^{k-RE}$
Social	0	$\geq\Lambda^s$	$\gtreqless\chi_t^{RE}$	$\lesseqgtr\tau^{k-RE}$
Effect of planner's ambiguity				
Ramsey	>0	$\uparrow$	$\uparrow\downarrow$	$\uparrow\downarrow$
Social	0	$\uparrow$	$\uparrow\downarrow$	$\uparrow\downarrow$
Effect of consumer's ambiguity				
Ramsey	>0	$\uparrow$	$\uparrow\downarrow$	$\uparrow\downarrow$
Social	0	NA	$\downarrow$	$\uparrow\downarrow$

14. A Feedback from Taxes to Consumers' Pessimistic Beliefs

When faced with pessimistic consumers, the policies of a Ramsey planner ($\Phi > 0$) present an instance of a possible two-way feedback between carbon tax policy and private-sector pessimism via the debt channel.[38] For example, consider a positive surprise in debt $b_t > \mathbb{E}_{t-1} m_t^c b_t$. From (77), the shadow value to the taxing authority of the consumer's utility ε_t must drop, implying a drop in the consumer's worst-case utility $\mathcal{U}_t$, hence an increased pessimism via (6).

The tax implications follow from formulas (79), (86), (87), (95), (100) and (112), which show that the social cost of carbon and the carbon tax are decreasing functions of ε_t, and that the *ex ante tax* on capital is an increasing function of ε_t, so that a t-period surprise increase in debt that causes a drop in ε_t and an increase in consumer pessimism should be associated with an increase in the SCC and the carbon tax, and a drop in the tax on capital.

The taxing authority is motivated by two considerations. On the one hand, for purely fiscal reasons, it wants to raise taxes in those states against which it is cheaper to issue debt. It therefore raises tax rates in high-debt states caused by climate shocks, and conversely lowers taxes in low-debt states when climate is calmer. The government also has a goal of setting the prices of carbon and capital in ways that are optimal for society. It turns out that these twin goals coincide: by manipulating debt and taxes to raise (or reduce) the household's utility and turning it more (or less) pessimistic and less (or more) willing to use fossil energy but more (or less) willing to spend on capital, the government manages both, an optimal allocation and a need to smooth debt and taxes over time.

15. Conclusions

This paper is about the climate policy implications of belief heterogeneity and ambiguity in a dynamic market economy governed by a benign welfare maximizing authority, here referred to as a Ramsey planner. To keep this paper focused, many important features relevant for practical policy, such as non-linearities in the mechanisms from the burning of fossil fuels to climate change, are set aside, as are some macroeconomic issues, such as the implications of either exogenous or directed endogenous technological change on growth.[39] Nor do I account for substitution among available types of fossil energy inputs, such as oil, gas, and coal, and green energy from sun, water, and even nuclear power. Finally, following many examples in the literature, to keep the presentation manageable, all sources of uncertainty have been combined into a single "climate-cost shock" variable representing all CO_2-related economic climate damages, including those related to the dynamics of CO_2, productivity shocks other than from climate, and cost shocks from alternative sources of energy, including renewable energy.

This paper, has, in the main, sought to adhere to the spirit of its antecedents, including Stern (2007), Nordhaus (2008), Acemoglu et al. (2012), von Below (2012), van der Ploeg and Withagen (2014), Golosov et al. (2014), Belfiori (2017), Rezai and van der Ploeg (2017), Barrage (2018), and Cai and Lontzek (2019). Golosov et al. (2014), whose model I consider a benchmark for comparisons, proved that the optimal carbon tax (expressed as a proportion of GDP) depends solely on the social cost of carbon, the carbon persistence parameter, and a discount factor. While this is true here as well when beliefs are homogeneous and rational, this paper's value added is an accounting of how belief distortions about the underlying model will alter that discount factor, depending on type of ambiguity.

The principal insight in this paper is that dissonance in beliefs, expressed as skepticism or doubt about forces governing economic outcomes arising from climate cost shocks, produce ambiguity and create wedges between the private sector's discount factor and the government's discount factor and therefore between the way in which consumers and society differentially price two assets with uncertain pay-offs: one from physical capital, whose returns are positive, and the other from atmospheric carbon accumulation, whose returns are negative.

Ambiguity that is due to the government's ignorance of optimistically distorted private beliefs raises the expected social cost of carbon and the carbon tax, which is offset by a lowering of capital taxes. This conclusion carries a certain political irony: a public that confidently denies or minimizes the fact of climate change may actually see its carbon taxes increased.

Ambiguity that arises as a consequence of neither government nor private sector trusting the scientific model has mixed effects depending on the relative degree of pessimism in either sector. So, while any doubts held by either the planner or the private sector produce an ambiguity premium to the social cost of carbon, their differential tax effects are mixed. A combination of government doubt about the model and consumer skepticism motivates lowering the tax on capital, but has ambiguous effects on any carbon tax premium. The combination of fear of model misspecification on the part of both the government and the private sector justifies an ambiguity premium for the social cost of carbon, but opposing effects of some key correlations in the economy leave the net effect on carbon tax premiums ambiguous. The source of this disparity of outcomes derives from two channels: (i) an indirect positive effect through the social cost of carbon and (ii) a direct positive *or* negative effect on the tax itself, depending on the source of ambiguity.

In all cases discussed in this paper, the perspective of Arrow–Debreu asset pricing theory illuminates an equivalence between Pigouvian carbon taxation and optimal pricing of carbon permits in a cap-and-trade economy. This correspondence extends to conditions of Knightian uncertainty, when derived asset prices must meet the test of robustness.

Being theoretic, this paper begs the question of just how applicable the findings herein might be to the real world. In the absence of counter-factual history, the best approach is stochastic simulations of a properly calibrated DSGIE climate-economy model governed by alternative Ramsey regimes discussed here. Alternatively, solving Isaacs-Bellman-Flemming equations associated with the belief regimes in Section 7 as Hennlock (2009) and Li et al. (2016) have done, also holds promise.

Finally, a word of caution. This paper is normative in that it posits not what is but what should be, based on the ideal of welfare maximization implemented by a benign authority heedful of consumers' preferences and budgets. In the reality, such a Ramsey planner may be mere fiction, even in nominal democracies, such as the United States, where an overriding authority, the Supreme Court, has recently ruled that the Executive has no authority to implement carbon policies without explicit and detailed instructions by a legislature that has shown little inclination to address the approaching climate catastrophe. In the real world, policy regimes may even turn rogue: in 2017, the Envronmental Protection Agency under the previous US Administration scrubbed its website of all references to climate. The EPA's website EPA (2017), cited earlier, is available only because it was copied and preserved on another website.

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Appendix A. The Household's Intertemporal Budget Constraint

Solve (4) forward for b_t, starting with $t = 0$:

$$\begin{aligned} b_t \geq & \sum_{t=0}^{T-1} \left(\Pi_{j=0}^{t-1} \sum_{x^{j+1}} \hat{p}_{j+1}(x_{j+1}) \right) [c_t(x^t) - H_t - g_t] \\ & + \left(\Pi_{j=0}^{T-1} \sum_{x^{j+1}} \hat{p}_{j+1} x_{j+1,0} \right) \left[\sum_{x^{T-1}} k_T + \sum_{x^T} \hat{p}_{T+1} x_{T+1} b_{T+1} \right] \\ & + \left(\Pi_{j=0}^{T-1} \sum_{x^{j+1}} \hat{p}_{j+1} x_{j+1} \right) \sum_{x^T} [p_T^e - \tau_T^e] Q_{T-1}(x^T) \\ & + \sum_{t=1}^{T-1} \left(\Pi_{j=0}^{t-1} \sum_{x^{j+1}} \hat{p}_{j+1} x_{j+1} \right) \left[1 - \sum_{x^t} \hat{p}_{t+1} x_{t+1} R_t^k \right] k_t \\ & + \sum_{t=1}^{T-1} \left(\Pi_{j=0}^{t-1} \sum_{x^{j+1}} \hat{p}_{j+1} x_{j+1} \right) \left[1 - \sum_{x^t} \hat{p}_{t+1} x_{t+1} R_t^k \right] [p_t^e - \tau_t^e] Q_t, \end{aligned} \tag{A1}$$

where $p_0 = 1$, and Hotelling's rule (31) ($\frac{p_{t+1}^e - \tau_{t+1}^e}{p_t^e - \tau_t^e} = R_{t+1}^k$) is used in the last term. By the no-arbitrage condition (14), the last two lines are zero. The third line vanishes under the assumption of resource exhaustibility, where $\lim_{T\to\infty} Q_{T-1} = 0$. The second line disappears as a consequence of the household's no-Ponzi game condition

$$\lim_{T\to\infty} \sum_{x^T} \hat{q}_T(x^T)[k_{T+1}(x^T) + \sum_{x_T} \hat{p}_{T+1}(x_{T+1}|x^T) b_{T+1}(x^T)] = 0. \tag{A2}$$

This leaves the first line to be evaluated. Note, from (17), the t-step ahead pricing kernel,

$$\begin{aligned} \hat{q}_t(x^t, x_0) &\equiv \Pi_{j=0}^{t-1} \hat{p}_{j+1,0}(x^j) \\ &= \Pi_{j=0}^{t-1} \beta m_{j+1}(x^{j+1}) \pi_j(x^j) \frac{u_{c_{j+1}}(x^{j+1})}{u_{c_j}(x^j)} \\ &= \beta^t M_t(x^t) \pi_t(x^t) \frac{u_{c_t}(x^t)}{u_{c_0}(x^0)}, \quad q_0 = 1, M_0 = 1, \end{aligned} \tag{A3}$$

is the price of an Arrow–Debreu contract written at $t = 0$. Substituting the last line in (A3) into the term in parentheses in the first line of (A1) then yields an expression for the household's intertemporal budget constraint shown in (35).

Appendix B. Derivation of Implementability Constraint (37)

From (36), the expectation of $\mathcal{W}_{t+1}$ with respect to the distorted measure $m_{t+1}\pi_{t+1}$, valued at the price of an Arrow–Debreu security b, is

$$\begin{aligned} & \sum_{x_{t+1}} \hat{p}_{t+1}(x_{t+1}|x^t) \mathcal{W}_{t+1}(x^{t+1}) = \\ & \sum_{x_{t+1}} \hat{p}_{t+1}(x_{t+1}|x^t)[b_{t+1}(x^{t+1}) + R_{t+1}^k(x^{t+1}) k_{t+1}(x^t)] \\ + & \sum_{x_{t+1}} \hat{p}_{t+1}(x_{t+1}|x^t)(p_{t+1}^e(x^{t+1}) - \tau_{t+1}^e(x^{t+1})) Q_{t+1}(x^t) \\ = & \sum_{x_{t+1}} \hat{p}_{t+1}(x_{t+1}|x^t)[b_{t+1}(x^{t+1}) + R_{t+1}^k(x^{t+1}) k_{t+1}(x^t)] \\ + & [p_t^e(x^t) - \tau_t^e(x^t)] \sum_{x_{t+1}} \hat{p}_{t+1}(x_{t+1}|x^t) R_{t+1}^k(x^{t+1}) Q_{t+1}(x^t), \end{aligned}$$

where (25) and Hotelling's rule (31) are used to get the last line. Application of no-arbitrage condition (14) further simplifies the expression for household wealth:

$$\begin{aligned}\sum_{x_{t+1}} \hat{p}_{t+1}(x_{t+1}|x^t)\mathcal{W}_{t+1}(x^{t+1}) &= \sum_{x_{t+1}}\Big[\hat{p}_{t+1}(x_{t+1}|x^t)b_{t+1}(x^{t+1}) + k_{t+1}(x^t)\Big] \\ &+ [p^e_t(x^t) - \tau^e_t(x^t)]Q_{t+1}(x^t), \end{aligned} \tag{A4}$$

indicating that the consumer's expected wealth in the following period includes the unused store of fossil fuel valued at its current after-tax price in addition to the capital stock and bonds carried over into the next period. Using the household's period budget constraint (4) and (A4), household wealth defined in (36) is, equivalently,

$$\begin{aligned}\mathcal{W}_t(x^t) &= b_t(x^t) + R^k_t k_t(x^{t-1}) + (p^e_t(x^t) - \tau^e_t(x^t))Q_t(x^{t-1}) \\ &= c_t(x^t) - H_t(x^t) - g_t(x^t) + k_{t+1}(x^t) \\ &+ \sum_{x^{t+1}}\Big[\hat{p}_{t+1}(x_{t+1}|x^t)b_{t+1}(x^{t+1}) + (p^e_t(x^t) - \tau^e_t(x^t))Q_{t+1}(x^t)\Big] \\ &\geq c_t(x^t) - H_t(x^t) - g_t(x^t) + \sum_{x_{t+1}} \hat{p}_{t+1}(x_{t+1}|x^t)\mathcal{W}_{t+1}(x^{t+1}) \\ &= c_t(x^t) - H_t(x^t) - g_t(x^t) + \sum_{x_{t+1}} \hat{\varrho}_{t+1}\pi_{t+1}\mathcal{W}_{t+1}(x^{t+1}) \\ &= c_t(x^t) - H_t(x^t) - g_t(x^t) + \beta\sum_{x_{t+1}} m_{t+1}\pi_{t+1}\frac{u_{c_{t+1}}}{u_{c_t}}\mathcal{W}_{t+1}(x^{t+1}). \end{aligned} \tag{A5}$$

Appendix C. The Social Cost of Carbon

This appendix evaluates the expectation (83) for the four belief regimes:

$$\varpi_t = f(\Phi_t, \varepsilon_t)\mathbb{E}_t\big[n^{**}_{t+1}\Lambda_{t+1}\big(1 + \varepsilon_{t+1} + [1 - \gamma + \gamma\vartheta_{t+1}]n^*_{t+1}\Phi_t\big)\big],$$

where $\Lambda_{t+1} = \varrho_{t+1}[(1-\rho)\varpi_{t+1} + x_{t+1}Y_{t+1}]$, $\mathbb{E}_t\Lambda_{t+1} = \Lambda^s_t$, $f(\Phi_t, \varepsilon_t) = \frac{1}{1+\varepsilon_t+[1-\gamma+\gamma\vartheta_t]\Phi_t}$ $\leq f(\Phi_t, 0) < f(0,0) = 1$, $\vartheta_t = \frac{H_t+g_t}{c_t}$ $\forall t$, and $\varepsilon_{t+1} > 0$ only if consumers are pessimistic.[40]

A. Homogeneous beliefs

RE solution ($n^* = 1; n^{**} = 1, \varepsilon = 0, \Phi_t = \bar{\Phi}$)

$$\begin{aligned}\varpi^{RE}_t &= f(\bar{\Phi}, 0)\mathbb{E}_t[\Lambda_{t+1}(1 + [1 - \gamma + \gamma\vartheta_{t+1}]\bar{\Phi})] \\ &= f(\bar{\Phi}, 0)\mathbb{E}_t[1 + (1-\gamma)\bar{\Phi}]\mathbb{E}_t\Lambda_{t+1} + \gamma\bar{\Phi}f(\bar{\Phi}, 0)\mathbb{E}_t\Lambda_{t+1}\vartheta_{t+1} \\ &= f(\bar{\Phi}, 0)[(1 + \bar{\Phi}[1 - \gamma + \gamma\mathbb{E}_t\vartheta_{t+1}])\Lambda^s_t + \bar{\Phi}\gamma cov(\Lambda, \vartheta)], \\ &= \Lambda^s_t + \gamma\bar{\Phi}f(\bar{\Phi}, 0)[(\mathbb{E}_t\vartheta_{t+1} - \vartheta_t)\Lambda^s_t + cov(\Lambda, \vartheta)], \quad \bar{\Phi} > 0, \ \forall t, \\ &= \Lambda^s_t, \quad \bar{\Phi} = 0 \ \ \forall t. \end{aligned} \tag{A6}$$

B. Heterogeneous beliefs

1. **Skeptical private sector**

   (a) **Planner has no ambiguity about distorted private beliefs** m^s ($n^* = m^s, n^{**} = 1, \varepsilon = 0$)

$$\begin{aligned}\varpi^o_t &= f(\Phi_t, 0)\mathbb{E}_t\big[\Lambda_{t+1}\big(1 + [(1-\gamma) + \gamma\vartheta_{t+1}]m^o_{t+1}\Phi_t\big)\big] \\ &= f(\Phi_t, 0)\mathbb{E}_t\Lambda_{t+1} + (1-\gamma)\Phi_t f(\Phi_t, 0)\mathbb{E}_t\Lambda_{t+1} + \gamma\Phi_t f(\Phi_t, 0)\mathbb{E}_t\Lambda_{t+1}\vartheta_{t+1} \\ &= f(\Phi_t, 0)[(1 + \Phi_t(1 - \gamma + \gamma\mathbb{E}_t\vartheta_{t+1})\Lambda^s_t + \gamma\Phi_t cov(\Lambda, \vartheta)] \\ &= \varpi^{RE}_t, \quad \Phi_t > 0 \\ &= \Lambda^s_t, \quad \Phi_t = 0 \ \forall t. \end{aligned} \tag{A7}$$

   (b) **Political planner** ($n^* = 1; n^{**} = n^{PO}, \varepsilon = 0$)

$$
\begin{aligned}
\varpi_t &= f(\bar{\Phi},0)\mathbb{E}_t\Big[n^{PO}_{t+1}\Lambda_{t+1}(1+[1-\gamma+\gamma\vartheta_{t+1}]\bar{\Phi})\Big] \\
&= f(\bar{\Phi},0)(1+(1-\gamma)\bar{\Phi})\mathbb{E}_t n^{PO}_{t+1}\Lambda_{t+1}+\gamma\bar{\Phi}f(\bar{\Phi},0)\mathbb{E}_t n^{PO}_{t+1}\Lambda_{t+1}\vartheta_{t+1} \\
&= f(\bar{\Phi},0)[(1+\bar{\Phi}[1-\gamma+\gamma\mathbb{E}_t\vartheta_{t+1}])\Lambda^s_t+\gamma\bar{\Phi}cov(\Lambda,\vartheta)]+f(\bar{\Phi},0)cov(n^{PO},\Lambda) \\
&+ \bar{\Phi}f(\bar{\Phi},0)\Big[(1-\gamma)cov(n^{PO},\Lambda)+\gamma cov(n^{PO},\Lambda\vartheta)\Big] \\
&= \varpi^{RE}_t+\bar{\Phi}f(\bar{\Phi},0)\Big[(1-\gamma)cov(n^{PO},\Lambda)+\gamma cov(n^{PO},\Lambda\vartheta)\Big] \\
&+ f(\bar{\Phi},0)cov(n^{PO},\Lambda)\geq\varpi^{RE}_t,\ \bar{\Phi}>0,\ \forall t \\
&= \Lambda^s_t+cov(n^{PO},\Lambda)\geq\Lambda^s_t,\ \bar{\Phi}=0.
\end{aligned} \tag{A8}
$$

(c) **Paternalistic planner** ($n^*=n^{PA}, n^{**}=1, \varepsilon=0$)

$$
\begin{aligned}
\varpi_t &= f(\Phi_t,0)\mathbb{E}_t\Big[\Lambda_{t+1}\Big(1+[1-\gamma+\gamma\vartheta_{t+1}]n^{PA}_{t+1}\Phi_t\Big)\Big] \\
&= f(\Phi_t,0)(\Lambda^s_t+(1-\gamma)\Phi_t\mathbb{E}_t n^{PA}_{t+1}\Lambda_{t+1}+\gamma\Phi_t f(\Phi_t,0)\mathbb{E}_t n^{PA}_{t+1}\Lambda_{t+1}\vartheta_{t+1} \\
&= f(\Phi_t,0)[(1+\Phi_t[1-\gamma+\gamma\mathbb{E}_t\vartheta_{t+1}])\Lambda^s_t+\gamma\Phi_t cov(\Lambda,\vartheta)]+f(\Phi_t,0)cov(n^{PA},\Lambda) \\
&+ \Phi_t f(\Phi_t,0)\Big[(1-\gamma)cov(n^{PA},\Lambda)+\gamma cov(n^{PA},\Lambda\vartheta)\Big] \\
&= \varpi^{RE}_t+\Phi_t f(\Phi_t,0)\Big[(1-\gamma)cov(n^{PA},\Lambda)+\gamma cov(n^{PA},\Lambda\vartheta)\Big]\geq\varpi^{RE}_t,\ \Phi_t>0,\ \forall t \\
&= \Lambda^s_t,\ \Phi_t=0\ \forall t.
\end{aligned} \tag{A9}
$$

(d) **Pessimistic planner** ($n^*=\frac{m^p}{n^p}, n^{**}=n^p, \varepsilon=0$.

$$
\begin{aligned}
\varpi_t &= f(\Phi_t,0)\mathbb{E}_t\left[n^p_{t+1}\Lambda_{t+1}\left(1+[1-\gamma+\gamma\vartheta_{t+1}]\frac{m^p_{t+1}}{n^p_{t+1}}\Phi_t\right)\right] \\
&= f(\Phi_t,0)[\Lambda^s_t+cov(n^p,\Lambda)]+(1-\gamma)\Phi_t f(\Phi_t,0)[\Lambda^s_t+cov(m^p,\Lambda)] \\
&+ \gamma\Phi_t f(\Phi_t,0)[\Lambda^s_t\mathbb{E}_t\vartheta_{t+1}+cov(\Lambda,\vartheta)+cov(m^p,\vartheta\Lambda)] \\
&= f(\Phi_t,0)[(1+\Phi_t[1-\gamma+\gamma\mathbb{E}_t\vartheta_{t+1})])\Lambda^s_t+\gamma\Phi_t cov(\Lambda,\vartheta))+f(\Phi_t,0)cov(n^p,\Lambda) \\
&+ f(\Phi_t,0)\Phi_t[(1-\gamma)cov(m^p,\Lambda)+\gamma cov(m^p,\vartheta\Lambda)]. \\
&= \varpi^{RE}_t+f(\Phi_t,0)[cov(n^p,\Lambda)+\Phi_t[(1-\gamma)cov(m^p,\Lambda)+\gamma cov(m^p,\vartheta\Lambda)],\ \Phi_t>0,\ \forall t, \\
&= \Lambda^s_t+cov(n^p,\Lambda)\geq\Lambda^s_t,\ \Phi_t=0,\ \ \forall t.
\end{aligned} \tag{A10}
$$

2. **Pessimistic consumers**

(a) **Pessimistic planner** ($n^*=\frac{m^c}{n^p}, n^{**}=n^p, \varepsilon\neq 0$)[41]

$$
\begin{aligned}
\varpi_t &= f(\Phi_t,\varepsilon_t)\mathbb{E}_t\left[n^p_{t+1}\Lambda_{t+1}\left(1+\varepsilon_{t+1}+[1-\gamma+\gamma\vartheta_{t+1}]\frac{m^c_{t+1}}{n^p_{t+1}}\Phi_t\right)\right] \\
&= f(\Phi_t,\varepsilon_t)[\mathbb{E}_t n^p_{t+1}\Lambda_{t+1}+\mathbb{E}_t n^p_{t+1}\Lambda_{t+1}\varepsilon_{t+1}] \\
&+ f(\Phi_t,\varepsilon_t)[(1-\gamma)\Phi_t\mathbb{E}_t m^c_{t+1}\Lambda_{t+1}+\gamma\Phi_t\mathbb{E}_t m^c_{t+1}\Lambda_{t+1}\vartheta_{t+1}] \\
&\geq f(\Phi_t,\varepsilon_t)[\Lambda^s_t+cov(n^p,\Lambda)+\varepsilon_t] \\
&+ (1-\gamma)\Phi_t f(\Phi_t,\varepsilon_t)[\Lambda^s_t+cov(m^c,\Lambda)] \\
&+ \gamma\Phi_t f(\Phi_t,\varepsilon_t)[\Lambda^s_t\mathbb{E}_t\vartheta_{t+1}+cov(\Lambda,\vartheta)+cov(m^c,\vartheta\Lambda)] \\
&= f(\Phi_t,\varepsilon_t)[(1+\Phi_t[1-\gamma+\gamma\mathbb{E}_t\vartheta_{t+1}])\Lambda^s_t+\gamma\Phi_t cov(\Lambda,\vartheta)] \\
&+ f(\Phi_t,\varepsilon)[\varepsilon_t+cov(n^p,\Lambda)] \\
&+ f(\Phi_t,\varepsilon_t)\Phi_t[(1-\gamma)cov(m^c,\Lambda)+\gamma cov(m^c,\vartheta\Lambda)].
\end{aligned}
$$

Adding and subtracting

$$\varpi_t^{RE} = f(\Phi_t, 0)[(1 + \Phi_t[1 - \gamma + \gamma\mathbb{E}_t\vartheta_{t+1}])\Lambda_t^s + \gamma\Phi_t cov(\Lambda, \vartheta)],$$

leads to,

$$\begin{aligned} \varpi_t &\geq [1 - f(\Phi_t, \varepsilon_t)\varepsilon_t]\varpi_t^{RE} + f(\Phi_t, \varepsilon_t)[\varepsilon_t + cov(n^p, \Lambda)] \\ &+ f(\Phi_t, \varepsilon_t)\Phi_t[(1-\gamma)cov(m^c, \Lambda) + \gamma cov(m^c, \vartheta\Lambda)] \\ &\geq \varpi_t^{RE}, \;\; \Phi_t > 0, \\ &\geq \Lambda_t^* + cov(n^p, \Lambda) > \Lambda_t^s \;\; \Phi_t = 0, \varepsilon_t = 0, \;\; \forall t. \end{aligned} \tag{A11}$$

Appendix D. Sign of Carbon Tax Premium χ

For the decompositions below, recall that the skeptic's belief multiplier m^s is assumed to be an arbitrary, independent random variable, uncorrelated with anything, where $\mathbb{E}_t m_{t+j}^s = 1$. This last property is shared by all martingale multipliers in this paper, so $\mathbb{E}_t n_{t+j}^{PO} = 1$ and $\mathbb{E}_t n_{t+j}^{PA} = 1$, as well. Where applicable, the derivations below apply Jensen's inequality to a convex function, such that, for all versions of m displayed in Table 1, $\mathbb{E}_t \frac{1}{m_{t+1}} \geq \frac{1}{\mathbb{E}_t m_{t+1}} = 1$. Also used, where necessary, is the law of iterated expectations.[42]

From (100),

$$\mathbb{E}_t\Xi_{t+j}^e = \mathbb{E}_t\left[\left(f(\Phi_t, \varepsilon_t)\frac{1}{m_{t+j}}(1 + \varepsilon_{t+j} + [1 - \gamma + \gamma\vartheta_{t+j}]n_{t+j}^*\Phi_{t+j-1}) - 1\right)\varsigma_{t+j}^e\right],$$

where $\vartheta_t = \frac{H_t + g_t}{c_t}$, $\Phi_{t+j} = n_{t+j}^*\Phi_{t+j-1}$, n^* is defined in Table 1, $f(\Phi_t, \varepsilon_t)$ is defined as before, and $m_{t+j} = m_{t+j}^s$ or $m_{t+j} = m_{t+j}^c$.[43]

A. Homogeneous beliefs

RE solution ($m = 1$, $n^* = 1$, $\varepsilon = 0$, $\Phi_t = \bar{\Phi}$, $\varepsilon = 0$)[44]

$$\begin{aligned} \mathbb{E}_t\Xi_{t+j}^{e-RE} &= \mathbb{E}_t\varsigma_{t+j}^e\left[f(\bar{\Phi}, 0)\left(1 + [1 - \gamma + \gamma\vartheta_{t+j}]\bar{\Phi}\right) - 1\right] \\ &= f(\bar{\Phi}, 0)\mathbb{E}_t\varsigma_{t+j}^e[1 + (1-\gamma)\bar{\Phi}] + \gamma\bar{\Phi}f(\bar{\Phi}, 0)\mathbb{E}_t\varsigma_{t+j}^e\vartheta_{t+j} - \mathbb{E}_t\varsigma_{t+j}^e \\ &= f(\bar{\Phi}, 0)[1 + \bar{\Phi}(1 - \gamma + \gamma\mathbb{E}_t\vartheta_{t+1} + \gamma cov(\varsigma^e, \vartheta))] - 1 \\ &= \bar{\Phi}f(\bar{\Phi}, 0)\gamma\left[\mathbb{E}_t\vartheta_{t+j} - \vartheta_t + cov(\varsigma^e, \vartheta)\right] > 0, \;\; \bar{\Phi} > 0, \\ &= 0, \;\; \bar{\Phi} = 0 \; \forall t. \end{aligned} \tag{A12}$$

The preceding utilizes $\mathbb{E}_t\varsigma_{t+1}^e = 1$, and

$$\begin{aligned} f(\Phi_t, 0)[1 + (1 - \gamma + \gamma\mathbb{E}_t\vartheta_{t+j})\Phi_t] - 1 &= \frac{1 + (1 - \gamma + \gamma\mathbb{E}_t\vartheta_{t+j})\Phi_t}{1 + (1 - \gamma + \gamma\vartheta_t]\Phi_t} - 1 \\ &= \gamma\Phi_t f(\Phi_t, 0)(\mathbb{E}_t\vartheta_{t+j} - \vartheta_t). \end{aligned}$$

B. Heterogeneous beliefs

1. **Skeptical private sector**

(a) **No ambiguity** ($m = m^s$; $n^* = 1 \rightarrow \Phi_t = \bar{\Phi}_t$, $\varepsilon = 0$)[45]

$$\begin{aligned}
\mathbb{E}_t \Xi^e_{t+j} &= \mathbb{E}_t \varsigma^e_{t+j} \left[\frac{f(\bar{\Phi},0)}{m^s_{t+j}} \left(1 + [1-\gamma+\gamma\vartheta_{t+j}]\bar{\Phi}\right) - 1 \right] \\
&= f(\bar{\Phi},0)\mathbb{E}_t \varsigma^e_{t+j} \frac{1}{m^s_{t+j}} [1 + (1-\gamma)\bar{\Phi}] \\
&+ \gamma f(\bar{\Phi},0)\mathbb{E}_t \varsigma^e_{t+j} \frac{1}{m^s_{t+j}} \vartheta_{t+j}\bar{\Phi} - \mathbb{E}_t \varsigma^e_{t+j} \\
&\geq f(\bar{\Phi},0) \frac{1}{\mathbb{E}_t m^s_{t+j}} \mathbb{E}_t[\varsigma^e_{t+j}(1 + (1-\gamma)\bar{\Phi})] \\
&+ \gamma\bar{\Phi} f(\bar{\Phi},0) \frac{1}{\mathbb{E}_t m^s_{t+j}} \left(\mathbb{E}_t \varsigma^e_{t+j}\vartheta_{t+j} + cov(\varsigma^e, \vartheta) \right) - \mathbb{E}_t \varsigma^e \\
&= f(\bar{\Phi},0)\left[1 + \bar{\Phi}\left(1-\gamma+\gamma[\mathbb{E}_t\vartheta_{t+j} + cov(\varsigma^e,\vartheta)]\right)\right] - 1 \\
&= \gamma\bar{\Phi} f(\bar{\Phi},0)\left[\mathbb{E}_t\vartheta_{t+j} - \vartheta_t + cov(\varsigma^e,\vartheta)\right] > 0 \\
&= \mathbb{E}_t \Xi^{e-RE}_{t+j} > 0, \;\; \bar{\Phi} > 0, \\
&= 0, \;\; \bar{\Phi} = 0, \; \forall t.
\end{aligned} \tag{A13}$$

(b) **Political planner** ($m = m^s$, $n^* = 1$, $\varepsilon = 0$).

$$\begin{aligned}
\mathbb{E}_t \Xi^{e-PO}_{t+j} &= \mathbb{E}_t \varsigma^e_{t+j} \left[\frac{f(\bar{\Phi},0)}{m^s_{t+j}} \left(1 + [1-\gamma+\gamma\vartheta_{t+j}]\bar{\Phi}\right) - 1 \right] \\
&= f(\bar{\Phi},0)\mathbb{E}_t \frac{1}{m^s_{t+j}} \mathbb{E}_t \varsigma^e_{t+j} [1 + (1-\gamma)\bar{\Phi}] \\
&+ \gamma f(\bar{\Phi},0)\mathbb{E}_t \frac{1}{m^s_{t+j}} \varsigma^e_{t+j}\vartheta_{t+j}\bar{\Phi} - \mathbb{E}_t \varsigma^e_{t+j} \\
&\geq f(\bar{\Phi},0)\left[1 + \bar{\Phi}\left(1-\gamma+\gamma[\mathbb{E}_t\vartheta_{t+j} + cov(\varsigma^e,\vartheta)]\right)\right] - 1 \\
&= \bar{\Phi} f(\bar{\Phi},0)\gamma\left[\mathbb{E}_t\vartheta_{t+j} - \vartheta_t + cov(\varsigma^e,\vartheta)\right] > 0, \\
&= \mathbb{E}_t \Xi^{e-RE}_{t+j}, \;\; \bar{\Phi} > 0, \\
&= 0, \;\; \bar{\Phi} = 0, \; \forall t.
\end{aligned} \tag{A14}$$

(c) **Paternalistic planner** ($m = m^s$, $n^* = n^{PA}$; $\Phi_{t+j} = n^{PA}_{t+j}\Phi_{t+j-1}$, $\varepsilon = 0$)

$$
\begin{aligned}
\mathbb{E}_t\Xi^{e-PA}_{t+j} &= \mathbb{E}_t\varsigma^e_{t+j}\left[\frac{f(\Phi_t,0)}{m^s_{t+j}}\Big(1+[1-\gamma+\gamma\vartheta_{t+j}]n^{PA}_{t+j}\Phi_{t+j-1}\Big)-1\right]\\
&= f(\Phi_t,0)\mathbb{E}_t\varsigma^e_{t+j}\frac{1}{m^s_{t+j}}[1+(1-\gamma)n^{PA}_{t+j}\Phi_{t+j-1}]\\
&+ \gamma f(\Phi_t,0)\mathbb{E}_t\frac{1}{m^s_{t+j}}\varsigma^e_{t+j}\vartheta_{t+j}n^{PA}_{t+j}\Phi_{t+j-1}-\mathbb{E}_t\varsigma^e_{t+j}\\
&\geq f(\Phi_t,0)\frac{\mathbb{E}_t\varsigma^e_{t+j}}{\mathbb{E}_t m^s_{t+j}}+(1-\gamma)\Phi_t f(\Phi_t,0)\frac{\mathbb{E}_t\varsigma^e_{t+j}}{\mathbb{E}_t m^s_{t+j}}\\
&+ \gamma\Phi_t f(\Phi_t,0)\left(\frac{\mathbb{E}_t\varsigma^e_{t+j}}{\mathbb{E}_t m^s_{t+j}}\mathbb{E}_t\vartheta_{t+j}+cov(n^{PA},\varsigma^e\vartheta)+cov(\varsigma^e,\vartheta)\right)-\mathbb{E}_t\varsigma^e_{t+j}\\
&= f(\Phi_t,0)\big[1+\Phi_t\big(1-\gamma+\gamma[\mathbb{E}_t\vartheta_{t+j}+cov(\varsigma^e,\vartheta)]\big)\big]-1\\
&+ \gamma\Phi_t f(\Phi_t,0)cov(n^{PA},\varsigma^e\vartheta)\\
&= \mathbb{E}_t\Xi^{e-RE}_{t+j}+\gamma\Phi_t f(\Phi_t,0)cov(n^{PA},\varsigma^e\vartheta)>0,\ \ \Phi_t>0,\\
&\geq 0,\ \ \Phi_t=0,\ \forall t.
\end{aligned}
\tag{A15}
$$

(d) **Pessimistic planner** $(m=m^s,\ n^*=\frac{m^p}{n^p};\ \Phi_{t+j}=\frac{m^p_{t+j}}{n^p_{t+j}}\Phi_{t+j-1},\ \varepsilon=0)$[46]

$$
\begin{aligned}
\mathbb{E}_t\Xi^{e}_{t+j} &= \mathbb{E}_t\varsigma^e_{t+j}\left[f(\Phi_t,0)\frac{1}{m^s_{t+j}}\left(1+[1-\gamma+\gamma\vartheta_{t+j}]\frac{m^p_{t+1}}{n^p_{t+1}}\Phi_{t+j-1}\right)-1\right]\\
&= f(\Phi_t,0)\mathbb{E}_t\left[\frac{\varsigma^e_{t+j}}{m^s_{t+j}}\left(1+[1-\gamma+\gamma\vartheta_{t+j}]\frac{m^p_{t+j}}{n^p_{t+j}}\Phi_{t+j-1}\right)\right]-1\\
&= f(\Phi_t,0)\left[\mathbb{E}_t\left(\frac{\varsigma^e_{t+j}}{m^s_{t+j}}\right)+(1-\gamma)\Phi_t\left(\mathbb{E}_t\frac{\varsigma^e_{t+j}m^p_{t+j}}{m^s_{t+j}n^p_{t+j}}\right)+\gamma\Phi_t\left(\mathbb{E}_t\frac{\varsigma^e_{t+j}\vartheta_{t+j}m^p_{t+j}}{m^s_{t+j}n^p_{t+j}}\right)\right]-1\\
&\geq f(\Phi_t,0)[1+\Phi_t(1-\gamma+\gamma\mathbb{E}_t\vartheta_{t+1}+\gamma cov(\varsigma^e,\vartheta))]-1\\
&+ \Phi_t f(\Phi_t,0)(1-\gamma)\left[cov(m^p,\frac{1}{n^p})+cov(\varsigma^e,\frac{m^p}{n^p})\right]\\
&+ \Phi_t f(\Phi_t,0)\gamma\left[cov(\vartheta,\varsigma^e)+cov(m^p,\frac{1}{n^p})+cov(\varsigma^e\vartheta,\frac{m^p}{n^p})\right]\\
&= \mathbb{E}_t\Xi^{e-RE}_{t+j}+(1-\gamma)\Phi_t f(\Phi_t,0)\left[cov(m^p,\frac{1}{n^p})+cov(\varsigma^e,\frac{m^p}{n^p})\right]\\
&+ \gamma\Phi_t f(\Phi_t,0)\left[cov(\vartheta,\varsigma^e)+cov(m^p,\frac{1}{n^p})+cov(\varsigma^e\vartheta,\frac{m^p}{n^p})\right]\gtreqless 0;\ \ \Phi_t\geq 0,\\
&= \mathbb{E}_t\Xi^{e-RE}_{t+j}>0,\ \ \Phi_t=0.
\end{aligned}
\tag{A16}
$$

2. **Pessimistic consumers**

   (a) **Pessimistic planner** ($m = m^c$, $n^* = \frac{m^c}{n^p}$; $\Phi_{t+j} = \frac{m^c_{t+j}}{n^p_{t+j}}\Phi_{t+j-1}$, $\varepsilon \neq 0$)[47],

$$
\begin{aligned}
\mathbb{E}_t\Xi^e_{t+j} &= \mathbb{E}_t\varsigma^e_{t+j}\left[f(\Phi_t,\varepsilon_t)\frac{1}{m^c_{t+j}}\left(1+\varepsilon_{t+j}+[1-\gamma+\gamma\vartheta_{t+j}]\frac{m^c_{t+1}}{n^p_{t+1}}\Phi_{t+j-1}\right)-1\right] \\
&= f(\Phi_t,\varepsilon_t)\mathbb{E}_t\left[\frac{\varsigma^e_{t+j}}{m^c_{t+j}}\left(1+\varepsilon_{t+j}+[1-\gamma+\gamma\vartheta_{t+j}]\frac{m^c_{t+j}}{n^p_{t+j}}\Phi_{t+j-1}\right)\right]-1 \\
&\geq f(\Phi_t,\varepsilon_t)\left[\frac{\mathbb{E}_t\varsigma^e_{t+j}}{\mathbb{E}_t m^c_{t+j}}+\varepsilon_t+cov(\varsigma^e,\frac{1}{m^c})\right]-1 \\
&+ (1-\gamma)\Phi_t f(\Phi_t,\varepsilon_t)\left[\left(\frac{\mathbb{E}_t\varsigma^e_{t+j}}{\mathbb{E}_t n^p_{t+j}}\right)+cov(\varsigma^e,\frac{1}{n^p})\right] \\
&+ \gamma\Phi_t f(\Phi_t,\varepsilon_t)\left[\left(\frac{\mathbb{E}_t\varsigma^e_{t+j}\vartheta_{t+j}}{\mathbb{E}_t n^p_{t+j}}\right)+cov(\varsigma^e\vartheta,\frac{1}{n^p})\right] \\
&= f(\Phi_t,\varepsilon_t)\left[1+\varepsilon_t+cov(\varsigma^e,\frac{1}{m^c})\right]-1 \\
&+ (1-\gamma)\Phi_t f(\Phi_t,\varepsilon_t)\left[1+cov(\varsigma^e,\frac{1}{n^p})\right] \\
&+ \gamma\Phi_t f(\Phi_t,\varepsilon_t)\left[\mathbb{E}_t\vartheta_{t+j}+cov(\frac{1}{n^p},\varsigma^e\vartheta)+cov(\varsigma^e,\vartheta)\right] \\
&= f(\Phi_t,\varepsilon_t)[1+\Phi_t(1-\gamma+\gamma(\mathbb{E}_t\vartheta_{t+j}+cov(\varsigma^e,\vartheta)))]-1 \\
&+ f(\Phi_t,\varepsilon_t)\left[\varepsilon_t+cov(\varsigma^e,\frac{1}{m^c})\right] \\
&+ \Phi_t f(\Phi_t,\varepsilon_t)\left[(1-\gamma)cov(\varsigma^e,\frac{1}{n^p})+\gamma cov(\vartheta\varsigma^e,\frac{1}{n^p})\right].
\end{aligned}
$$

Adding and subtracting

$$f(\Phi_t,0)[1+\Phi_t(1-\gamma+\gamma\mathbb{E}_t\vartheta_{t+1}+\gamma cov(\varsigma^e,\vartheta))],$$

leads to

$$
\begin{aligned}
\mathbb{E}_t\Xi^e_{t+j} &= \mathbb{E}_t\Xi^{e-RE}_{t+j} - f(\Phi_t,\varepsilon_t)f(\Phi_t,0)[1+\Phi_t(1-\gamma+\gamma\mathbb{E}_t\vartheta_{t+1}+\gamma cov(\varsigma^e,\vartheta))]\varepsilon_t \\
&+ f(\Phi_t,\varepsilon_t)\left[\varepsilon_t+cov(\varsigma^e,\frac{1}{m^c})\right] \\
&+ \Phi_t f(\Phi_t,\varepsilon_t)\left[(1-\gamma)cov(\varsigma^e,\frac{1}{n^p})+\gamma cov(\vartheta\varsigma^e,\frac{1}{n^p})\right] \lesseqgtr 0,\ \ \Phi_t > 0,\ \ \forall t \\
&= cov(\varsigma^e,\frac{1}{m^c}) < 0,\ \ \Phi_t = 0,\ \ \forall t,
\end{aligned}
\tag{A17}
$$

where, from (75) and (77),

$$\begin{aligned}
\mathbb{E}_t \frac{\varsigma^e_{t+1}}{m^c_{t+j}} \varepsilon_{t+1} &= \sigma^c M^c_t \big(\mathbb{E}_t \varsigma^e_{t+1} \mu_{t+1} - \mathbb{E}_t \varsigma^e_{t+1} \mathbb{E}_t m^c_{t+1} \mu_{t+1}\big) + \mathbb{E}_t \frac{\varsigma^e_{t+1}}{m^c_{t+1}} \varepsilon_t \\
&\geq \sigma^c \Phi_t \big[\mathbb{E}_t (1 - m^c_{t+1}) \mathbb{E}_t u_{c,t+1} b_{t+1} + cov(\varsigma^e, bu_c) - cov(m^c, bu_c)\big] \\
&+ [1 + cov(\varsigma^e, \frac{1}{m^c})]\varepsilon_t \\
&= \sigma^c \Phi_t [cov(\varsigma^e, bu_c) - cov(m^c, bu_c)] + [1 + cov(\varsigma^e, \frac{1}{m^c})]\varepsilon_t \geq \varepsilon_t.
\end{aligned}$$

Appendix E. Sign of Ex Ante Capital Tax

This appendix evaluates the expression (112) for each of the four belief regimes,

$$\mathbb{E}_t \Xi^k_{t+1} = \mathbb{E}_t \varsigma^k_{t+1} \bigg(1 - f(\Phi_t, \varepsilon_t) \frac{n^{**}_{t+1}}{m_{t+1}} (1 + \varepsilon_{t+1} + [1 - \gamma + \gamma \vartheta_{t+1}] n^*_{t+1} \Phi_t \bigg).$$

A. Homogeneous beliefs
RE solution ($m = 1, n^* = 1, n^{**} = 1, \varepsilon = 0$)

$$\begin{aligned}
\mathbb{E}_t \Xi^{k-RE}_{t+1} &= \mathbb{E}_t \Big[\varsigma^k_{t+1} (1 - f(\Phi_t, 0)[1 + \Phi_t [1 - \gamma + \gamma \vartheta_{t+1}]])\Big] \\
&= \mathbb{E}_t \varsigma^k_{t+1} - f(\Phi_t, 0)[1 + (1-\gamma)\Phi_t] \mathbb{E}_t \varsigma^k_{t+1} + \gamma \Phi_t f(\Phi_t, 0) \mathbb{E}_t \Big[\varsigma^k_{t+1} \vartheta_{t+1}\Big] \\
&= 1 - f(\Phi_t, 0) \Big[1 + \Phi_t \Big(1 - \gamma + \gamma [\mathbb{E}_t \vartheta_{t+1} + cov(\varsigma^k, \vartheta)]\Big)\Big] \\
&= -\gamma f(\Phi_t, 0) \Phi_t \Big[\mathbb{E}_t \vartheta_{t+1} - \vartheta_t + cov(\varsigma^k, \vartheta)\Big] < 0, \ \ \Phi_t > 0, \ \ \forall t, \\
&= 0, \ \ \Phi_t = 0 \ \ \forall t.
\end{aligned} \tag{A18}$$

B. Heterogeneous beliefs

1. **Skeptical private sector**
   (a) **No ambiguity** ($m = m^s, n^* = 1, n^{**} = 1, \varepsilon = 0$).

$$\begin{aligned}
\mathbb{E}_t \Xi^k_{t+1} &= \mathbb{E}_t \Bigg[\varsigma^k_{t+1} \Bigg(1 - \frac{f(\Phi_t, 0)}{m^s_{t+1}} (1 + [(1-\gamma) + \gamma \vartheta_{t+1}] \Phi_t)\Bigg)\Bigg] \\
&= \mathbb{E}_t \varsigma^k_{t+1} - f(\Phi_t, 0) \mathbb{E}_t \varsigma^k_{t+1} \Bigg[\frac{1}{m^s_{t+1}} (1 + [(1-\gamma) + \gamma \vartheta_{t+1})] \Phi_t\Bigg] \\
&\leq 1 - f(\Phi_t, 0) \frac{\mathbb{E}_t \varsigma^k_{t+1}}{\mathbb{E}_t m^s_{t+1}} - (1-\gamma) \Phi_t f(\Phi_t, 0) \mathbb{E}_t \varsigma^k_{t+1} \\
&- \gamma \Phi_t f(\Phi_t, 0) \mathbb{E}_t \varsigma^k_{t+1} \mathbb{E}_t \vartheta_{t+1} - \gamma \Phi_t f(\Phi_t, 0) cov(\varsigma^k, \vartheta) \\
&= 1 - f(\Phi_t, 0) \Big[1 + \Phi_t \Big(1 - \gamma + \gamma ([\mathbb{E}_t \vartheta_{t+1} + cov(\varsigma^k, \vartheta)]\Big)\Big] \\
&= -\gamma \Phi_t f(\Phi_t, 0) \Big[\mathbb{E}_t \vartheta_{t+1} - \vartheta_t + cov(\varsigma^k, \vartheta)\Big] \\
&= \mathbb{E}_t \Xi^{k-RE}_{t+1} < 0, \ \ \Phi_t > 0, \ \forall t \\
&= 0, \ \ \Phi_t = 0 \ \forall t.
\end{aligned} \tag{A19}$$

   (b) **Political planner** ($m = m^s, n^* = 1, n^{**} = n^{PO}, \varepsilon = 0$).

$$\begin{aligned}
\mathbb{E}_t\Xi_{t+1}^{k-PO} &= \mathbb{E}_t\left[\varsigma_{t+1}^k\left(1-f(\bar{\Phi},0)\frac{n_{t+1}^{PO}}{m_{t+1}^s}[1+[1-\gamma+\gamma\vartheta_{t+1}]\bar{\Phi}]\right)\right] \\
&= \mathbb{E}_t\varsigma_{t+1}^k - f(\bar{\Phi},0)\left[1+(1-\gamma)\bar{\Phi}\mathbb{E}_t\varsigma_{t+1}^k\frac{n_{t+1}^{PO}}{m_{t+1}^s}+\gamma\bar{\Phi}\mathbb{E}_t\varsigma_{t+1}^k\frac{n_{t+1}^{PO}}{m_{t+1}^s}\vartheta_{t+1}\right] \\
&\leq 1-f(\bar{\Phi},0)\Big[1+\bar{\Phi}\Big(1-\gamma+\gamma[\mathbb{E}_t\vartheta_{t+1}+cov(\varsigma^k,\vartheta)\Big]\Big) \\
&- \gamma\bar{\Phi}f(\bar{\Phi},0)cov(n^{PO},\varsigma^k\vartheta)-(1-\gamma)\bar{\Phi}f(\bar{\Phi},0)cov(n^{PO},\varsigma^k) \\
&= 1-f(\bar{\Phi},0)\Big[1+\Big(1-\gamma+\gamma\mathbb{E}_t\vartheta_{t+1}+\gamma cov(\varsigma^k,\vartheta)\Big)\bar{\Phi}\Big] \\
&- \bar{\Phi}f(\bar{\Phi},0)\Big[(1-\gamma)cov(n^{PO},\varsigma^k)+\gamma cov(n^{PO},\varsigma^k\vartheta)\Big] \\
&= \mathbb{E}_t\Xi_{t+1}^{k-RE}-\bar{\Phi}f(\bar{\Phi},0)\Big[(1-\gamma)cov(n^{PO},\varsigma^k)+\gamma cov(n^{PO},\varsigma^k\vartheta)\Big] \leq \mathbb{E}_t\Xi_{t+1}^{k-RE}<0,\bar{\Phi}>0, \\
&= 0,\ \ \bar{\Phi}=0.
\end{aligned} \tag{A20}$$

(c) **Paternalistic planner** ($m = m^s, n^* = n^{PA}, n^{**} = 1, \varepsilon = 0$).

$$\begin{aligned}
\mathbb{E}_t\Xi_{t+1}^{k-PA} &= \mathbb{E}_t\varsigma_{t+1}^k - f(\Phi_t,0)\mathbb{E}_t\frac{\varsigma_{t+1}^k}{m_{t+1}^s}\Big[(1+[(1-\gamma)+\gamma\vartheta_{t+1}]n_{t+1}^{PA}\Phi_t\Big] \\
&\leq 1-f(\Phi_t,0)\left[\frac{\mathbb{E}_t\varsigma_{t+1}^k}{\mathbb{E}_t m_{t+1}^s}+\Phi_t(1-\gamma)\mathbb{E}_t\varsigma_{t+1}^k\frac{\mathbb{E}_t n_{t+1}^{PA}}{\mathbb{E}_t m_{t+1}^s}\right] \\
&- \gamma\Phi_t f(\Phi_t,0)\left[\mathbb{E}_t\varsigma_{t+1}^k\frac{\mathbb{E}_t n_{t+1}^{PA}}{\mathbb{E}_t m_{t+1}^s}\mathbb{E}_t\vartheta_{t+1}+cov(n^{PA},\varsigma^k\vartheta)+cov(\varsigma^k,\vartheta)\right] \\
&- (1-\gamma)\Phi_t f(\Phi_t,0)cov(n^{PA},\varsigma^k) \\
&= 1-f(\Phi_t,0)\Big[1+\Phi_t\Big(1-\gamma+\gamma[\mathbb{E}_t\vartheta_{t+1}+\gamma cov(\varsigma^k,\vartheta)]\Big)\Big] \\
&- \Phi_t f(\Phi_t,0)\Big[(1-\gamma)cov(n^{PA},\varsigma^k)+\gamma cov(n^{PA},\varsigma^k\vartheta)\Big] \\
&= \mathbb{E}_t\Xi_{t+1}^{k-RE}-\Phi_t f(\Phi_t,0)\Big[(1-\gamma)cov(n^{PA},\varsigma^k)+\gamma cov(n^{PA},\varsigma^k\vartheta)\Big] \leq \mathbb{E}_t\Xi_{t+1}^{k-RE}<0,\Phi_t>0,\ \forall t \\
&= 0,\ \ \Phi_t=0,\ \forall t.
\end{aligned} \tag{A21}$$

(d) **Pessimistic planner** ($m = m^s, n^* = \frac{m^p}{n^p}, n^{**} = n^p, \varepsilon = 0$)

$$\begin{aligned}
\mathbb{E}_t\Xi_{t+1}^{k-R-s} &= \mathbb{E}_t\left[\varsigma_{t+1}^k\left(1-f(\Phi_t,0)\frac{n_{t+1}^p}{m_{t+1}^s}\left[1+[1-\gamma+\gamma\vartheta_{t+1}]\frac{m_{t+1}^p}{n_{t+1}^p}\Phi_t\right]\right)\right] \\
&= \mathbb{E}_t\varsigma_{t+1}^k-f(\Phi_t,0)\left[\mathbb{E}_t\frac{n_{t+1}^p}{m_{t+1}^s}\varsigma_{t+1}^k+\Phi_t\mathbb{E}_t(1-\gamma+\gamma\vartheta_{t+1})\frac{m_{t+1}^p\varsigma_{t+1}^k}{m_{t+1}^s}\right] \\
&\leq 1-f(\Phi_t,0)\Big[1+cov(\varsigma^k,n^p)+(1-\gamma)\Phi_t(1+cov(\varsigma^k,m^p))\Big] \\
&- \gamma\Phi_t f(\Phi_t,0)\Big[\mathbb{E}_t\vartheta_{t+1}+cov(\varsigma^k,\vartheta)+cov(\vartheta,m^p\varsigma^k)\Big] \\
&= 1-f(\Phi_t,0)\Big[1+\Phi_t\Big(1-\gamma+\gamma[\mathbb{E}_t\vartheta_{t+1}+cov(\varsigma^k,\vartheta)]\Big)\Big] \\
&- f(\Phi_t,0)[cov(\varsigma^k,n^p)+(1-\gamma)\Phi_t cov(\varsigma^k,m^p)]-\gamma\Phi_t f(\Phi_t,0)cov(\vartheta,\varsigma^e m^p)] \\
&= \mathbb{E}_t\Xi_{t+1}^{k-RE}-f(\Phi_t,0)[cov(\varsigma^k,n^p)+(1-\gamma)cov(\varsigma^k,m^p)] \\
&- \gamma\Phi_t f(\Phi_t,0)cov(\vartheta,\varsigma^e m^p)\leq 0,\ \ \Phi_t\geq 0 \\
&= \mathbb{E}_t\Xi_{t+1}^{k-RE}-cov(\varsigma^k,n^p)-(1-\gamma)cov(\varsigma^k,m^p)\leq 0,\ \ \Phi_t=0.
\end{aligned} \tag{A22}$$

2. **Pessimistic consumers**

(a) **Pessimistic planner** ($m = m^c$, $n^* = \frac{m^c}{n^p}$, $n^{**} = n^p$, $\varepsilon \neq 0$)

$$\begin{aligned}
\mathbb{E}_t\Xi_{t+1}^{k-R-p} &= \mathbb{E}_t\left[\varsigma_{t+1}^k\left(1 - f(\Phi_t,\varepsilon_t)\frac{n_{t+1}^p}{m_{t+1}^c}\left[1+\varepsilon_{t+1}+[1-\gamma+\gamma\vartheta_{t+1}]\frac{m_{t+1}^c}{n_{t+1}^p}\Phi_t\right]\right)\right] \\
&= \mathbb{E}_t\varsigma_{t+1}^k - f(\Phi_t,\varepsilon_t)\left[\mathbb{E}_t\frac{n_{t+1}^p}{m_{t+1}^c}\varsigma_{t+1}^k(1+\varepsilon_{t+1}) + \Phi_t\mathbb{E}_t(1-\gamma+\gamma\vartheta_{t+1})\varsigma_{t+1}^k\right] \\
&\leq 1 - f(\Phi_t,\varepsilon_t)\left[\frac{\mathbb{E}_t n_{t+1}^p}{\mathbb{E}_t m_{t+1}^c}\mathbb{E}_t\varsigma_{t+1}^k + \Phi_t(1-\gamma+\gamma\mathbb{E}_t\vartheta_{t+1})\mathbb{E}_t\varsigma_{t+1}^k\right] \\
&- f(\Phi_t,\varepsilon_t)\left[\gamma\Phi_t cov(\varsigma^k,\vartheta) + cov(\varsigma^k,\frac{n^p}{m^c}) + cov(\frac{1}{m^c},n^p) - (1+\varepsilon_t)cov(\varsigma^k,n^p)\right] \\
&= 1 - f(\Phi_t,\varepsilon_t)\left[1+\Phi_t(1-\gamma+\gamma(\mathbb{E}_t\vartheta_{t+1}+cov(\varsigma^k,\vartheta))\right] \\
&- f(\Phi_t,\varepsilon_t)\left[cov(\varsigma^k,\frac{n^p}{m^c}) + cov(\frac{1}{m^c},n^p) - (1+\varepsilon_t)cov(\varsigma^k,n^p)\right].
\end{aligned}$$

Adding and subtracting

$$f(\Phi_t,0)\left[1+\Phi_t\left(1-\gamma+\gamma[\mathbb{E}_t\vartheta_{t+1}+cov(\varsigma^k,\vartheta)]\right)\right],$$

leads to

$$\begin{aligned}
\mathbb{E}_t\Xi_{t+1}^{k-R-p} &\leq \mathbb{E}_t\Xi_{t+1}^{k-RE} - f(\Phi_t,\varepsilon_t)f(\Phi_t,0)\left[1+\Phi_t\left(1-\gamma+\gamma[\mathbb{E}_t\vartheta_{t+1}+cov(\varsigma^k,\vartheta)]\right)\right]\varepsilon_t \\
&- f(\Phi_t,\varepsilon_t)\left[cov(\varsigma^k,\frac{n^p}{m^c}) + cov(\frac{1}{m^c},n^p) + (1+\varepsilon_t)cov(\varsigma^k,n^p)\right] \lesseqgtr 0, \quad \Phi_t > 0 \\
&= -\left[cov(\varsigma^k,\frac{n^p}{m^c}) + cov(\frac{1}{m^c},n^p) + (1+\varepsilon_t)cov(\varsigma^k,n^p)\right] \lesseqgtr 0, \quad \Phi_t = 0,
\end{aligned} \tag{A23}$$

where, from (75) and (77),

$$\begin{aligned}
\mathbb{E}_t\frac{\varsigma_{t+1}^k n_{t+1}^p}{m_{t+1}^c}\varepsilon_{t+1} &= \sigma^c M_t^c\left(\mathbb{E}_t\varsigma_{t+1}^k n_{t+1}^p\mu_{t+1} - \mathbb{E}_t\varsigma_{t+1}^k n_{t+1}^p\mathbb{E}_t m_{t+1}^c\mu_{t+1}\right) \\
&+ \mathbb{E}_t\frac{\varsigma_{t+1}^k n_{t+1}^p}{m_{t+1}^c}\varepsilon_t \\
&= \sigma^c\Phi_t\mathbb{E}_t n_{t+1}^p\varsigma_{t+1}^k\mathbb{E}_t[1-m_{t+1}^c]u_{c,t+1}b_{t+1} \\
&+ \sigma^c\Phi_t[cov(\varsigma^k n^p, bu_c) - cov(m^c, bu_c)] \\
&+ [1+cov(\varsigma^k,\frac{n^p}{m^c})]\varepsilon_t \\
&= \sigma^c\Phi_t\left[cov(\varsigma^k, bu_c) - cov(m^c, bu_c)\right] \\
&+ (1+\varepsilon_t)cov(\varsigma^k,\frac{n^p}{m^c}) \geq \varepsilon_t.
\end{aligned}$$

Notes

1 The Yale Climate Opinion Maps 2020 (Marlon et al. 2020) is encouraging in that 72 percent of respondents said global warming is real and a threat to humanity and the planet, with 57 percent believing it to be human caused.

2 Uncertainty in climate modeling has been intensively treated in the literature; see Pindyck (2007, 2013b), Roe and Baker (2007), Weitzman (2007, 2009, 2013), Heal and Millner (2013), and lately Barnett et al. (2020).

3 Tipping points include shutdown of the Atlantic Meridional Overturning Circulation, West Antarctic ice sheet disintegration, Amazon rainforest dieback, West African monsoon shift, permafrost and methane hydrates, coral reef die-off, Indian monsoon shift, Greenland ice sheet disintegration, boreal forest shift, and permafrost and methane hydrates. Dietz et al. (2021) note that such "climate tipping points are subject to considerable scientific uncertainty in relation to their size, probability, and how they interact with each other. Their economic impacts are even more uncertain, and consequently, these are often ignored or given a highly stylized treatment that fails to accurately represent geophysical dynamics and is nearly impossible to calibrate. As a result, tipping points are only weakly reflected in the policy advice economists give on climate change, typically by way of caveats and contextualization, rather than an integral part of the modeling that gives rise to estimates of the social cost of carbon (SCC) and other economic metrics of interest.

4 Barrage (2020) is perhaps the first to study optimal capital and carbon taxation in Ramsey planning framework.

5 This is not the first paper to dwell on a symmetry between carbon and capital taxation arising from a difference between social and private discount factors. Barrage's (2020) shows that the very logic leading to an optimal zero *ex ante* tax on capital derived in (Atkeson et al. 1999; Chari et al. 1994; Zhu 1992) implies a *positive* Pigou tax on carbon.

6 In a recent paper, Dietz and Niehörster (2019), estimate ambiguity loads—i.e., the extra insurance premiums due to ambiguity—and show how these depend on the insurer's attitude to ambiguity.

7 A treatment of the proper social discount rate for far-away and potentially catastrophic consequences of today's actions is beyond the scope of this paper. For discussions, see Weitzman (2009, 2013), Pindyck (2013a), and Traeger (2014).

8 In Belfiori's (2015) inter-generational model of altruism, the difference in discount factors is endogenous because each generation of households living in an infinite sequence of generations assigns a positive weight to the welfare of future generations, causing the government's discount factor to rise above that of households and leading to a carbon tax that exceeds the standard Pigou tax.

9 An axiomatization of multiplier preferences can be found in Strzalecki (2011).

10 Throughout, the expectations operator $\mathbb{E}_t y(x_{t+j})$ denotes the mathematical expectations of some function $y(x_{t+j})$, $j \geq 1$, with respect to the probability density $\pi(x_{t+j}|x^t)$: $\mathbb{E}_t y(x_{t+j}) = \sum_{x^{t+j}|x^t} y(x_{t+j})\pi(x_{t+j}|x^t)$.

11 This means one cannot assign positive probability to events as functions of x_t that have probability measure zero under the distribution of the approximating model or alternatively, the distorted and the approximating distributions are, at the very least, in agreement about which events have zero probability and which events are certain.

12 Previous systematic treatments of decision making under ambiguity include Klibanoff et al. (2005, 2009), and Traeger (2014).

13 Following examples by Anderson et al. (2013), Golosov et al. (2014), Nordhaus (2008), Li et al. (2016), and others, I omit leisure (or hours worked) to keep things simple. I also leave out any utility effects of environmental quality caused by climate change, since no conclusions relevant for this paper would be affected by their inclusion. See however, Barrage (2020).

14 See Hennlock (2009). While $\gamma < 1$ gets at early resolution of uncertainty, as noted by Bansal and Yaron (2004), the assumption of CRRA misses certain aspects of risk aversion that may create pricing issues.

15 For analytical convenience that will not affect conclusions, I will assume that energy production is costless, as is approximately true for oil (see Golosov et al. 2014). Accordingly, it is reasonable to fold the fossil energy extraction/production sector into the household, by making the consumer be the owner of the resource Q and the seller of energy E_t. This assumption differs from the literature (see Golosov et al. 2014; Nordhaus 1993), where, instead, the consumer owns the energy producing firm and receives its profits. However, ownership of the firm is merely a financial veil for direct ownership of the resource itself, hence a distinction without a difference.

16 The implied relative risk aversion is $CRR = 1 - \frac{\sigma^c}{1-\beta}$ that rises as $\sigma^c \to -\infty$. and $0 > \sigma^c > -\infty$. See Epstein and Zin (1991).

17 The formula for m^c can be derived as a special case of Epstein and Zin (1989) and Weil (1990) preferences

$$\mathcal{U}_t = [(1-\beta)u_t^{1-\rho} + \beta(\mathbb{E}_t \mathcal{U}_{t+1}^{1-\gamma})^{\frac{1-\rho}{1-\gamma}}]^{\frac{1}{1-\rho}},$$

where $\frac{1}{\rho}$ is the intertemporal elasticity of substitution. For $\gamma > 0$ and $\rho = 1$, this has the interpretation of a *risk-sensitive* recursion, such that in (6), $\sigma^c \equiv (1-\beta)(1-\gamma)$. With $\gamma = \rho$, preferences reduce to standard time-additive expected utility.

18 Readers will recognize Hansen and Sargent's (1995) discounted risk-sensitive recursion related to recursive preferences introduced earlier by Epstein and Zin (1989) and Weil (1990). Risk-sensitivity as a concept was introduced by Jacobson (1973) and later familiarized by Whittle (1981).

19 Excellent expositions of the physics of climate change for economists are Hsiang and Kopp (2018) and Traeger's (2018) description of the complex sets of channels involved in the transmission from carbon to temperature change.

20 Barnett et al. (2020) also adopt this measure of climate response.

21 Modeling damages resulting from climate change as negative total productivity shocks to the economy follows common practice, as for example in Golosov et al. (2014). The damaging effects of increased temperature on productivity in the world economy have been documented by Burke et al. (2015). Donadelli et al. (2017) used vector autoregressions to show that with a 50-year horizon, a one-standard deviation temperature shock lowers both cumulative output and labor productivity growth by 1.4

percentage points. Based on their model, they further show that temperature risk is associated with non-negligible welfare costs of 18.4% of the agent's lifetime utility that grow exponentially with the size of the impact of temperature on TFP.

22 Belfiori (2017) models climate damage as a reduction in household utility, while Barrage (2020) considers reductions in both production and utility.

23 For the derivation see Appendix A.

24 Henceforth, the n multiplier refers to the government's belief, and m to the consumer's belief distortion.

25 In this paper, I do not consider learning by either planner or private agents. An approach to doing so is available in Tetlow and von zur Muehlen (2009) who study robust monetary policy using *structured singular value* analysis when agents have misspecified models but are engaged in learning under the handicap that their learning of the reduced form of the economy is subject to potentially destabilizing parameter perturbations.

26 Hansen and Sargent's (2012) refer to this as *Type I ambiguity*.

27 The first application of risk-sensitive decision making to economic policy is van der Ploeg (1984). An early treatment of Knightian uncertainty in economics is von zur Muehlen (1982). In his analytical climate economy model, Traeger (2018) posits risk-sensitive preferences attributable to the planner representing the consumer.

28 Hansen and Sargent's (2012) refer to this as *Type III ambiguity*.

29 This belief regime is related to the robust fiscal policy model in Karantounias (2020). A fifth possible belief regime, wherein the planner has doubts but the private sector trusts the extant model, is not treated here having been widely discussed in the referenced literature.

30 In Table 1, p is defined by (19).

31 Current estimates of the social cost of carbon emissions are around USD75 per ton of carbon. Some consider this a gross underestimate and suggest the number is closer to USD220 per ton. (See Moore and Diaz (2015) and Than (2015)).

32 Initial wealth $\mathcal{W}_0$ is a function of k_0 and S_0 and the initial tax rates τ_0^k and τ_0^e. It is well known that since initial capital is supplied inelastically, the government has an incentive to raise the initial capital tax as high as possible. Likewise, with S_0 given, there is nothing (i.e., no welfare criterion) to prevent the planner from expropriating the energy sector by setting $\tau_0^e = 1$. As is conventional, I fix both τ_0^k and τ_0^e at 0.

33 By definition, next period's stock $Q_{t+1} = Q_t - E_t$ is given, i.e., determined by previous E_t, hence not influenced by x_{t+1}.

34 A positive warming shock reduces both consumption and productivity and therefore the value of future welfare $\mathcal{V}_{t+1}$.

35 Wilbanks et al. (2008) present empirical evidence that the net effect of declining consumption on wealth is negative.

36 Proofs are provided in Appendixes C–E, and utilize the preceding assumptions and lemmas, particularly the results that n^p, n^{POL}, n^{PAT}, m^p, ς^k, ς^e, and Λ, being positively correlated with x, are positively correlated with with each other.

37 Pascal argued that a rational person should live as though God exists and seek to believe in God. If God does not exist, such a person will have only a finite loss (some pleasures, luxury, etc.), whereas if God does exist, he stands to receive infinite gains (as represented by eternity in Heaven) and avoid infinite losses (an eternity in Hell).

38 This section is inspired by Karantounias (2013) who studied a fiscal authority's ability to manage pessimistic expectations.

39 See for example,Traeger (2018). Other features not addressed here include (i) *Arrhenius' Greenhouse Law* describing radiative forcing that connects carbon with temperature change, as described in Hassler et al. (2016), (ii) *tipping points* analyzed by Lemoine and Traeger (2016) and (Cai et al. 2013), describing abrupt nonlinear climate changes that pose a potentially existential threat to humanity in ways that may override concerns with belief and skepticism, and (iii) *polar amplification* analyzed by Brock and Xepapadeas (2017).

40 The derivations in this and the next two Appendices utilize these facts: Let $\{x, y, z\}$ represent four random, possibly related variables, and a a non-random variable or constant. Then

$$\begin{aligned}\mathbb{E}[ax \times y] &= a\mathbb{E}x\mathbb{E}y + acov(x, y)\\ \mathbb{E}[ax \times y \times z] &= a\mathbb{E}x\mathbb{E}y\mathbb{E}z + acov(x, yz) + a\mathbb{E}xcov(y, z).\end{aligned}$$

41 The derivation uses (77)

$$\begin{aligned}\mathbb{E}_t n^p_{t+1}\Lambda_{t+1}\varepsilon_{t+1} &= \sigma^c\Phi_t\mathbb{E}_t[n^p_{t+1}\Lambda_{t+1}u_{c_{t+1}}][\mathbb{E}_t m^c_{t+1}b_{t+1} - \mathbb{E}_t m^c_{t+1}\mathbb{E}_t m^c_{t+1}b_{t+1}] + E_t n^p_{t+1}\Lambda_{t+1}m^c_{t+1}\varepsilon_t\\ &+ \sigma^c\Phi_t[cov(n^p u_c\Lambda, m^c b) - cov(n^p u_c\Lambda, m^c)]\\ &= \sigma^c\Phi_t[cov(n^p u_c\Lambda, m^c b) - cov(n^p u_c\Lambda, m^c)] + [\Lambda^*_t + cov(n^p m^c, \Lambda]\varepsilon_t\\ &> \varepsilon_t.\end{aligned}$$

since, with the exception of marginal utility u_c, m^c, n^p, Λ, and b are positively related to x and therefore with each other.

42 By the law of iterated expectations $\mathbb{E}_t\mathbb{E}_{t+1}\cdots\mathbb{E}_{t+j-1}[\Xi^e_{t+j}] = \mathbb{E}_t[\Xi^e_{t+j}]$.

43 Where called for, the proofs use the result that for any martingale process m, $\mathbb{E}m = 1$, if

$$\Phi_{t+j} = m_{t+j}\Phi_{t+j-1},$$

then,

$$\mathbb{E}_t\Phi_{t+j} = \mathbb{E}_t m_{t+j}\Phi_{t+j-1} = \mathbb{E}_t \prod_{i=1}^{j} \frac{M_{t+i}}{M_{t+i-1}}\Phi_t = \mathbb{E}_t \frac{M_{t+j}}{M_t}\Phi_t = \frac{M_t}{M_t}\Phi_t = \Phi_t.$$

44 Constancy of Φ follows from (71).

45 In the following, m^s is purely random and independent of any other variable in the economy. Likewise, Φ_{t+j} is independent, where from footnote 42, $\mathbb{E}_t m^s_{t+j}\Phi_{t+j-1} = \Phi_t$, and $\mathbb{E}_t\varsigma^e_{t+j} = 1$.

46 In the following, I use the facts that $\mathbb{E}m = 1$ for each m, and also that $\mathbb{E}\varsigma^e = 1$. In addition, note that

$$\begin{aligned}\mathbb{E}_t\Phi_{t+j} &= \mathbb{E}_t \frac{m^p_{t+j}}{n^p_{t+j}}\Phi_{t+j-1} = \mathbb{E}_t\left(\frac{m^p_{t+j}}{m^p_{t+j-1}} / \frac{N^p_{t+j}}{N^p_{t+j-1}}\right)\Phi_{t+j-1} = \mathbb{E}_t\prod_{i=1}^{j}\left(\frac{m^p_{t+i}}{m^c_{t+i-1}} / \frac{N^p_{t+i}}{N^p_{t+i-1}}\right)\Phi_t \\ &\geq \left(\frac{\mathbb{E}_t m^p_{t+j}}{\mathbb{E}_t N^p_{t+j}} / \frac{m^p_t}{N^p_t}\right)\Phi_t = \left(\frac{m^p_t}{m^p_t} / \frac{N^p_t}{N^p_t}\right)\Phi_t = \Phi_t,\end{aligned}$$

since, by the law of iterated expectations, $\mathbb{E}_t m^p_{t+j} = m^p_t$, and likewise for N^p_t.

47 In the following, I use

$$\begin{aligned}\mathbb{E}_t\Phi_{t+j} &= \mathbb{E}_t \frac{m^c_{t+j}}{n^p_{t+j}}\Phi_{t+j-1} = \mathbb{E}_t\left(\frac{m^c_{t+j}}{m^c_{t+j-1}} / \frac{N^p_{t+j}}{N^p_{t+j-1}}\right)\Phi_{t+j-1} = \mathbb{E}_t\prod_{i=1}^{j}\left(\frac{m^c_{t+i}}{m^c_{t+i-1}} / \frac{N^p_{t+i}}{N^p_{t+i-1}}\right)\Phi_t \\ &\geq \left(\frac{\mathbb{E}_t m^c_{t+j}}{\mathbb{E}_t N^p_{t+j}} / \frac{m^c_t}{N^p_t}\right)\Phi_t = \left(\frac{m^c_t}{m^c_t} / \frac{N^p_t}{N^p_t}\right)\Phi_t = \Phi_t,\end{aligned}$$

since, by the law of iterated expectations, $\mathbb{E}_t m^c_{t+j} = m^c_t$, and likewise for N^p_t.

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