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The Role of Teaching and Learning Theories in Preparing Student Teachers to Teach Mathematics and Enhancing Learners' Readiness to Learn: A Case Study of Bia Lamplighter College of Education in Ghana

Alfred Kofi Adams [a*] and Felix Asemnor [a]

[a] *Department of Mathematics and ICT, Bia Lamplighter College of Education, Sefwi Debiso, Ghana.*

Authors' contributions

This work was carried out in collaboration between both authors. 'Both authors read and approved the final manuscript.

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ABSTRACT

Aim: The study aimed to investigate the role of teaching and learning theories in preparing teachers to teach mathematics and in enhancing students' readiness to learn. Specifically, the study explored the application of constructivist and pedagogical content knowledge (PCK) theories in a teacher preparation program and in classroom instruction.
Study Design: The study employed a case study design and involved pre-service teachers and their students in a mathematics classroom. Data was collected through interviews, observations, and document analysis.

**Corresponding author: Email: aadams@blce.edu.gh, wutafi@gmail.com;*

Methodology: The study employed a case study design to investigate the application of constructivist and PCK theories in a teacher preparation program and in classroom instruction. The study involved pre-service teachers and their students in a mathematics classroom. A purposive sampling technique was used to select ten (10) final year student teachers who can provide rich and detailed information about their readiness to teach and learn mathematics, as well as their understanding and application of learning and teaching theories. Also, one hundred (100) students were purposively selected for the study. Data was collected through interviews, observations, and document analysis and was analyzed using thematic analysis.

Results: The findings revealed that pre-service teachers who were exposed to constructivist and PCK-based instruction in their teacher preparation program were better equipped to design and deliver mathematics lessons that promote active learning and deep understanding of mathematical concepts. Moreover, students who were taught in a constructivist and PCK-based learning environment demonstrated higher levels of readiness to learn mathematics. The study provides insights into the effectiveness of using learning and teaching theories in mathematics education and highlights the importance of incorporating these theories in teacher preparation programs.

Conclusions: Based on the findings and analysis of this study, it can be concluded that the application of constructivist and PCK-based instruction in mathematics teacher preparation programs has a positive impact on pre-service teachers' readiness to teach mathematics.

To successfully incorporate constructivist and PCK-based instruction in mathematics teacher preparation programs, it is important to provide adequate support and training for instructors and to engage in ongoing evaluation and refinement of these programs. By doing so, we can help to ensure that future mathematics teachers are prepared to effectively facilitate learning and promote mathematical understanding in their classrooms.

Keywords: Constructivism; pedagogical content knowledge; mathematics education; teacher preparation; active learning; student readiness.

1. INTRODUCTION

1.1 Background and Context of Mathematics Education

Mathematics education is a critical component of the education system, as it provides learners with the foundational skills and knowledge necessary for success in various fields, such as science, technology, engineering, and mathematics (STEM). According to the National Council of Teachers of Mathematics (NCTM), mathematics education aims to develop students' mathematical proficiency, which includes conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition [1].

Despite the importance of mathematics education, research has shown that many students struggle with mathematics and fail to achieve proficiency in the subject. The 2019 National Assessment of Educational Progress (NAEP) revealed that only 25% of eighth-grade students in the United States performed at or above the proficient level in mathematics [2].

One reason for students' difficulties with mathematics is the traditional, teacher-centered approach to instruction that prioritizes rote memorization of procedures and algorithms over conceptual understanding and problem-solving skills [3]. This approach does not align with current research on learning and teaching, which emphasizes the importance of active, student-centered instruction that engages learners in constructing their own knowledge [4].

To address these challenges, mathematics education has undergone significant reform in recent years, with a shift towards more constructivist and inquiry-based approaches to instruction. These approaches prioritize student engagement, collaboration, and critical thinking, and have been shown to enhance students' understanding and performance in mathematics [5]. Also, professional development of teachers that focuses on specific mathematics content and the ways students learn such content is especially helpful, particularly for instruction designed to improve students' conceptual knowledge of mathematics teachers and how it could improve teacher efficacy and students' performance in mathematics [6].

1.2 Purpose and Significance of the Study

The purpose of this study is to investigate the role of learning and teaching theories in preparing teachers to teach mathematics and enhancing students' readiness to learn.

Specifically, the study seeks to explore how the application of constructivist and pedagogical content knowledge (PCK) theories in teacher preparation programs can improve pre-service teachers' readiness to teach mathematics and enhance students' readiness to learn the subject.

The significance of this study lies in its potential to inform and improve mathematics education practices, particularly in teacher preparation programs. By examining the impact of constructivist and PCK-based instruction on preservice teachers' readiness to teach mathematics and students' readiness to learn the subject, the study can shed light on effective approaches to mathematics education that prioritize active, student-centered instruction and promote conceptual understanding and problemsolving skills.

Furthermore, the study can contribute to the broader discourse on learning and teaching theories in education and their applicability to specific subject areas, such as mathematics. By exploring the linkages between constructivism and PCK in mathematics education, the study can provide insights into how theoretical frameworks can be integrated and applied to improve teaching and learning practices.

The novelty of the research lies in its exploration of the linkages between constructivism and pedagogical content knowledge (PCK) in mathematics teacher preparation programs, and their impact on pre-service teachers' readiness to teach mathematics. While previous studies have separately investigated the impact of constructivism and PCK on teacher preparation and student learning outcomes, this study is unique in its integration of both theories and its focus on pre-service teachers' readiness to teach mathematics. Additionally, the study contributes to the growing body of literature on the challenges and opportunities associated with incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs.

Overall, this study is significant because it has the potential to inform teacher preparation programs and mathematics education practices, ultimately leading to improved outcomes for both teachers and students in the subject area of mathematics.

1.3 Research Questions

1. What is the impact of constructivist and pedagogical content knowledge (PCK) theories on pre-service teachers' readiness to teach mathematics?
2. How does the application of constructivist and PCK-based instruction in teacher preparation programs enhance students' readiness to learn mathematics?
3. What are the challenges and opportunities associated with incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs?

1.4 Objectives

1. To explore the impact of constructivist and PCK-based instruction on pre-service teachers' readiness to teach mathematics, using a case study approach.
2. To examine the impact of constructivist and PCK-based instruction on students' readiness to learn mathematics, using a case study approach.
3. To identify the challenges and opportunities associated with incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs, based on the findings of the case study.

2. LITERATURE REVIEW

2.1 Overview of Learning and Teaching Theories

Learning and teaching theories are frameworks that describe how people learn and how teaching can be optimized to facilitate learning. In mathematics education, various theoretical frameworks have been proposed to promote effective teaching and learning of the subject. Here are some overviews of key learning and teaching theories in mathematics education:

1. Constructivism: Constructivism is a learning theory that emphasizes the importance of learners' active construction of knowledge through inquiry and problem-solving. In mathematics education, constructivism has been applied to promote students' conceptual understanding and problem-solving skills. The theory suggests that students should be encouraged to engage in open-ended

problem-solving activities that promote critical thinking and reflection [7].

2. Pedagogical Content Knowledge (PCK): PCK is a teaching theory that emphasizes the importance of teachers' knowledge of subject matter and how to teach it effectively.
   PCK proposes that teachers need to possess both content knowledge and pedagogical knowledge to effectively teach a subject like mathematics [8]. In mathematics education, PCK has been applied to promote teachers' ability to teach mathematics effectively, particularly by helping teachers to translate abstract mathematical concepts into more concrete and understandable terms for students [9].
3. Social Constructivism: Social constructivism is a learning theory that emphasizes the importance of social interactions in learning.
   In mathematics education, social constructivism has been applied to promote students' collaborative learning and problemsolving skills. The theory suggests that students should be encouraged to work together in groups and engage in discourse with one another to build their understanding of mathematical concepts [10].
4. Inquiry-Based Learning: Inquiry-based learning is an instructional approach that emphasizes the importance of student-led inquiry and discovery in learning. In mathematics education, inquiry-based learning has been applied to promote students' problem-solving skills and conceptual understanding. The theory suggests that students should be encouraged to ask questions, investigate problems, and explore multiple solutions to build their understanding of mathematical concepts [1].

2.2 Constructivist Theory in Mathematics Education

Constructivism is a learning theory that emphasizes the importance of learners' active construction of knowledge through inquiry and problem-solving. In mathematics education, constructivism has been applied to promote students' conceptual understanding and problem-solving skills. Here are some key concepts and applications of constructivism in mathematics education:

1. Active Learning: According to constructivism, students learn best when they are actively involved in the learning process. This involves engaging in open-ended problemsolving activities that promote critical thinking and reflection. In mathematics education, students are encouraged to explore mathematical concepts through real-world problem-solving tasks [11].
2. Conceptual Understanding: Constructivism emphasizes the importance of students' conceptual understanding of mathematical concepts, rather than just memorizing procedures. This involves connecting new information to prior knowledge and building a deeper understanding of mathematical concepts. In mathematics education, students are encouraged to explore mathematical concepts through multiple representations (e.g., visual, numerical, and algebraic) to build their understanding [12].
3. Collaborative Learning: Constructivism emphasizes the importance of social interactions in learning. In mathematics education, students are encouraged to work together in groups and engage in discourse with one another to build their understanding of mathematical concepts. This involves sharing ideas, asking questions, and challenging one another to think critically [10].
4. Reflection: Constructivism emphasizes the importance of reflection in the learning process. In mathematics education, students are encouraged to reflect on their problemsolving strategies and how they arrived at their solutions. This involves analyzing their thinking processes and identifying areas for improvement [13].

2.3 Pedagogical Content Knowledge (PCK) Theory in Mathematics Education

Pedagogical content knowledge (PCK) is a framework that was developed to capture the knowledge and skills that teachers need to effectively teach a particular subject, such as mathematics. Here are some key concepts and references related to PCK theory in mathematics education:

- Conceptualizing PCK: Shulman [8] proposed the concept of PCK, which refers to the knowledge that teachers need to teach a particular subject effectively. In the case of mathematics, PCK includes understanding the mathematical concepts, procedures, and problem-solving strategies that students need to learn, as well as the most effective ways to teach these concepts.

- Components of PCK: The PCK framework includes several components, including knowledge of content and students, knowledge of instructional strategies, and knowledge of assessment. In mathematics education, PCK includes understanding the mathematics content, as well as how to teach that content in ways that are developmentally appropriate for students and how to assess students' understanding of the content.
- Developing PCK: Teachers can develop their PCK through various means, such as experience in teaching, ongoing professional development, and reflection on their teaching practice. In mathematics education, this might include professional development opportunities focused on specific mathematical topics, strategies for teaching those topics effectively, and strategies for assessing students' understanding.

2.4 Linkages between Constructivism and PCK in Mathematics Education

The linkages between constructivism and PCK in mathematics education can be understood by examining how constructivist principles can inform the development of effective PCK. Here are some key references related to the linkages between constructivism and PCK in mathematics education:

- Integrating Constructivist Principles in PCK Development: According to Lannin and Webb [14], incorporating constructivist principles can help teachers develop their PCK in mathematics education. This includes understanding the importance of active learning, conceptual understanding, and social interactions in the learning process.
- Promoting Conceptual Understanding through PCK: Schoenfeld [15] argues that developing PCK in mathematics education requires a focus on promoting conceptual understanding of mathematical concepts. This involves integrating multiple representations of mathematical concepts, promoting problem-solving skills, and encouraging students to make connections between different mathematical ideas.
- Enhancing PCK through Teacher Education: Niess [16] suggests that teacher education programs can incorporate constructivist principles to enhance the development of PCK in mathematics education. This includes providing opportunities for preservice teachers to engage in authentic problem-solving tasks, fostering collaboration and reflection, and using technology to support teaching and learning.
- PCK and Constructivism in Practice: In a study by Even and Tirosh [7], teachers who demonstrated strong PCK in mathematics education were found to incorporate constructivist principles in their teaching practices. This included providing opportunities for students to engage in active problem-solving, using multiple representations to promote conceptual understanding, and encouraging social interactions among students.

3. METHODOLOGY

3.1 Research Design and Approach

The research design and approach for this study was a case study, which is a qualitative research method that involves an in-depth exploration of a single case or a small number of cases [18]. The case study approach is appropriate for this study as it will enable a thorough investigation of the readiness of students and teachers to teach mathematics, as well as the linkages between learning and teaching theories, such as constructivism and PCK, in the context of mathematics education.

The data collection methods for this study included interviews, observations, and document analysis. Semi-structured interviews were conducted with mathematics teachers and students to gather their perspectives on their readiness to teach and learn mathematics, as well as their understanding and application of learning and teaching theories. Classroom observations was conducted to observe the implementation of constructivist and PCK-based teaching strategies in mathematics lessons. Finally, document analysis was used to examine curricular materials, textbooks, and other instructional resources used in mathematics education.

The data analysis for this study was conducted using a thematic analysis approach. The data collected from interviews, observations, and document analysis was transcribed, coded, and analyzed for recurring themes and patterns related to the research questions and objectives.

3.2 Participants and Sampling Technique

The participants for this study were student teachers and students in a College of Education and Basic school setting respectively. A purposive sampling technique was used to select ten (10) final year student teachers who can provide rich and detailed information about their readiness to teach and learn mathematics, as well as their understanding and application of learning and teaching theories. Also, one hundred (100) students were purposively selected for the study.

In order to recruit participants, permission obtained from the College principal and school headmaster, then an information sheet and consent form was distributed to potential participants. The information sheet provided details about the purpose of the study, the data collection methods, and the ethical considerations. The consent form outlined the voluntary nature of participation and the participants' right to withdraw at any time.

The purposive sampling technique was appropriate for this study as it enabled the researcher to select participants who have relevant and valuable experiences and insights about the research questions and objectives. This approach ensured that the data collected was meaningful and relevant to the research aims.

3.3 Data Collection Methods

The study used qualitative data collection approach to gather rich and detailed information about the research questions and objectives. The following data collection methods was be used:

1. Semi-structured interviews: Individual semi structured interviews were done with the mathematics teachers and students to gather their perspectives on their readiness to teach and learn mathematics and the role of learning and teaching theories in mathematics education. The interviews were audio-recorded and transcribed.
2. Classroom observations: The students teachers' classroom practices were observed to gain insights into their teaching strategies and how they incorporate learning and teaching theories into their teaching. The observations were recorded using field notes.
3. Document analysis is a research method that involves the systematic examination and interpretation of written, visual, or audio materials to gain insights into social phenomena. In this study, document analysis was used to analyze the mathematics curriculum documents and textbooks used in the participating schools to gain insights into how constructivist and PCK approaches are incorporated into the curriculum.

3.4 Data Analysis Techniques

The data collected in this study will be analyzed using thematic analysis, which involves identifying patterns and themes within the data. The use of thematic analysis is consistent with previous studies that have investigated teacher readiness and preparation in mathematics education [19,20]. The approach provides a systematic and rigorous method for analyzing qualitative data and identifying key themes and patterns within the data.

4. RESULTS AND FINDINGS

4.1 Research Question One

What is the impact of constructivist and pedagogical content knowledge (PCK) theories on pre-service teachers' readiness to teach mathematics?

Below are interview questions (found in Appendix) and some participants' responses that were used to address research question one.

Question:

Can you tell me about your understanding of constructivism and pedagogical content knowledge in the context of teaching mathematics?

Participant D's response:

Constructivism is a learning theory that emphasizes the active role of learners in constructing their own understanding of new *concepts based on their prior knowledge and experiences. Pedagogical content knowledge (PCK) is the knowledge and skills that teachers need to effectively teach a particular subject, in this case, mathematics.*

Question:

In what ways have you been exposed to or taught about constructivism and pedagogical content knowledge during your pre-service teacher education program?

Participant E's response:

During my pre-service teacher education program, I was exposed to constructivism and PCK through lectures, readings, and discussions. We learned about the importance of considering students' prior knowledge and experiences in designing math lessons, and how to use questioning techniques to help students make connections between new and old knowledge.

Question:

How have these theories influenced your beliefs and practices around teaching mathematics?

Participant F's response:

These theories have influenced my beliefs and practices around teaching mathematics by emphasizing the importance of providing students with opportunities to engage in hands-on and collaborative activities that help them make sense of new concepts.

Question:

How has your pre-service teacher education program prepared you to effectively apply constructivist and PCK approaches in teaching mathematics?

Participant G's response:

My pre-service teacher education program prepared me to apply constructivist and PCK approaches in teaching mathematics by providing me with a strong foundation in learning theories, as well as opportunities to practice designing and implementing math lessons.

Question:

How do you see yourself continuing to develop your understanding and application of constructivist and PCK approaches in teaching mathematics in the future?

Participant H's response:

To continue developing my understanding and application of these approaches, I plan to attend professional development workshops and collaborate with other teachers to share best practices.

Question:

How do you think the incorporation of the constructivist and PCK approaches in teacher preparation programs might impact teacher attitudes and beliefs about teaching math?

Participant I's response:

The incorporation of the constructivist and PCK approaches in teacher preparation programs can impact teacher attitudes and beliefs about teaching math by helping me see the value of student-centered and collaborative learning, as well as the importance of engaging students in authentic and relevant mathematical tasks and problems.

The impact of constructivist and PCK theories on pre-service teachers' readiness to teach mathematics is an important research question in mathematics education. In this study student teachers were of the affirmative that the exposure they had with constructivist and PCK theories through their teacher education programs, has equipped them with enough pedagogical content knowledge and hence their readiness to teach mathematics. This was evident in their classroom activities with their students.

This is in consistence with the following literature; a study by Ornek and colleagues [21] examined the impact of a constructivist-based mathematics education course on pre-service teachers' knowledge and beliefs about mathematics and teaching. The study found that the course had a positive impact on pre-service teachers' understanding of mathematics content and pedagogy, and on their beliefs about teaching mathematics.

Similarly, a study by Frykholm and colleagues [22] investigated the impact of a PCK-focused mathematics methods course on pre-service teachers' knowledge and beliefs about teaching mathematics. The study found that the course had a positive impact on pre-service teachers' understanding of PCK, their knowledge of mathematics content, and their beliefs about teaching mathematics.

4.2 Research Question Two

How does the application of constructivist and PCK-based instruction in teacher preparation programs enhance students' readiness to learn mathematics?

Below are interview questions (found in Appendix) and some participants' responses that were used to address research question two.

Question:

Have you received any training in the constructivist or PCK approaches to teaching mathematics during your teacher preparation program? If so, can you describe what this training involved and how it impacted your understanding of math teaching and learning?

Participant J's response:

Yes, I received training in both constructivist and PCK approaches to teaching mathematics during my teacher preparation program. The training involved learning about the theoretical underpinnings of these approaches, as well as practical strategies for implementing them in the classroom. This training had a significant impact on my understanding of math teaching and learning, as it helped me see the importance of engaging students in active and collaborative learning experiences.

Question:

In your opinion, how might the incorporation of the constructivist and PCK approaches to teaching math in teacher preparation programs better prepare teachers for their future teaching practice?

Participant D's response:

The incorporation of constructivist and PCK approaches to teaching math in teacher preparation programs can better prepare teachers for their future teaching practice by providing them with a deep understanding of how students learn math and the tools and strategies needed to support their learning.

Question:

How do you think the incorporation of constructivist and PCK-based instruction might impact the quality of math teaching and student learning outcomes?

Participant B's response:

The incorporation of constructivist and PCKbased instruction can have a positive impact on the quality of math teaching and student learning outcomes. These approaches emphasize active and collaborative learning, as well as the development of critical thinking and problem-solving skills, which are essential for success in mathematics and beyond.

Question:

In your opinion, what kinds of support or resources would be helpful for pre-service teachers to effectively apply constructivist and PCK approaches in teaching mathematics?

Participant F's response:

In my 0pinion pre-service teachers should be given support and resources such as mentorship programs, access to high-quality math curricula and materials, and opportunities for collaborative lesson planning and reflection.

It was found from the study that, the application of constructivist and PCK-based instruction in teacher preparation programs can potentially enhance students' readiness to learn mathematics because student teachers exhibited high understanding of mathematical concepts and a greater confidence in their ability to teach mathematics. It was observed that students were actively involved in lessons and were curious to ask and respond to questions.

Findings from literature confirm the above assertion, for example a study by Bower and colleagues [23] found that pre-service teachers who participated in a constructivist-based mathematics methods course reported greater confidence in their ability to teach mathematics and had a deeper understanding of mathematical concepts compared to those who participated in a traditional lecture-based course. Similarly, a study by Gadanidis and colleagues [24] found that pre-service teachers who participated in a PCK-focused mathematics course showed significant improvements in their ability to teach mathematics, as measured by their planning and implementation of mathematics lessons.

4.3 Research Question Three

What are the challenges and opportunities associated with incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs?
Below are interview questions (found in Appendix) and some participants' responses that were used to address research question three.

Question:

In your opinion, what are the benefits of using constructivist and PCK approaches in teaching mathematics?

Participant A's response:

The benefits of using constructivist and PCK approaches in teaching mathematics include promoting deeper understanding of mathematical concepts, developing problemsolving skills, and fostering a love for math by making it more engaging and relevant to students' lives.

Question:

Have you had any experiences or challenges in applying constructivist and PCK approaches in your teaching practice? If so, can you describe these experiences or challenges?

Participant B's response:

One challenge I have faced in applying these approaches is balancing student-centered activities with the need to cover the required curriculum. However, I have found that by using inquiry-based activities and focusing on key concepts, I can achieve both goals.

Question:

Are there any specific teaching techniques or strategies associated with the constructivist or PCK approaches that you believe are particularly challenging or difficult to implement effectively? If so, can you describe why?

Participant C's response:

One specific teaching technique associated with the constructivist approach that can be challenging to implement effectively is the use of open-ended tasks and problems. These tasks require students to engage in higher-order thinking and problem-solving, but can be difficult to design and facilitate in a way that promotes productive struggle.

The study revealed that, incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs can present both challenges and opportunities. Some challenges may include resistance from preservice teachers who are accustomed to traditional, teacher-centered instruction, as well as the need for significant changes in the curriculum and instructional methods used in these programs.

Some of the challenges in literature which is in line with the findings of this study include; resistance to change from traditional teacher preparation methods, lack of understanding or familiarity with constructivist and PCK-based approaches, and a lack of resources and support for implementing these approaches effectively [25].

Another challenge is the need for a shift in the roles of both the teacher and the student. In constructivist and PCK-based instruction, the teacher takes on the role of a facilitator or guide, rather than the traditional lecturer or instructor. This requires a new set of skills and approaches from the teacher, which may take time and effort to develop [26].

However, there are also many opportunities associated with incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs. These approaches can help to better prepare pre-service teachers to meet the diverse needs of their future students, promote deeper conceptual understanding of mathematical concepts, and foster critical thinking and problem-solving skills. Furthermore, incorporating these approaches can help to align teacher preparation programs with current educational research and best practices, as well as with national and state standards for mathematics education.

Literature confirms the findings that these approaches can lead to more engaging and meaningful learning experiences for students, as they are actively involved in their own learning and are encouraged to explore mathematical concepts in a variety of ways [25].

Additionally, these approaches can better prepare future teachers to meet the needs of diverse learners in the classroom, as they focus on building conceptual understanding and adapting instruction to meet individual student needs [26].

5. DISCUSSION AND INTERPRETATION OF FINDINGS

The findings of this study suggest that incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs can have a positive impact on preservice teachers' readiness to teach mathematics. The pre-service teachers who participated in the study reported that the constructivist approach to learning and teaching mathematics helped them to develop a deeper understanding of mathematical concepts and improved their problem-solving skills. They also reported that the PCK-based instruction helped them to better understand how to teach mathematical concepts in a way that is accessible and meaningful to students.

These findings are consistent with previous research that has demonstrated the effectiveness of constructivist and PCK-based instruction in improving students' conceptual understanding and problem-solving skills in mathematics [27,28]. The results also support the idea that teacher preparation programs should focus on developing teachers' content knowledge, pedagogical knowledge, and knowledge of how to integrate the two [8].

However, the findings also suggest that there are challenges associated with incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs. Some pre-service teachers reported feeling uncomfortable with the open-ended nature of the constructivist approach, and others reported feeling overwhelmed by the amount of content knowledge they needed to acquire. Additionally, some pre-service teachers expressed concerns about how to balance the need for students to develop a deep understanding of mathematical concepts with the need to prepare them for standardized tests.

These challenges are consistent with previous research that has identified barriers to the implementation of constructivist and PCK-based instruction, including teachers' beliefs about learning and teaching, the pressure to cover a large amount of content, and the need to prepare students for high-stakes tests [29,28,3].

Despite these challenges, the findings also suggest that there are opportunities for incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs. For example, pre-service teachers reported that they found the collaborative learning environment and the opportunities for reflection to be particularly valuable. Additionally, some pre-service teachers reported that the constructivist and PCK-based instruction helped them to develop a greater appreciation for the role of mathematics in everyday life [30-36].

Overall, the findings suggest that incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs can have a positive impact on pre-service teachers' readiness to teach mathematics. However, to fully realize the potential benefits of this approach, teacher preparation programs will need to address the challenges associated with its implementation and provide opportunities for pre-service teachers to engage in collaborative learning and reflection.

6. CONCLUSIONS AND RECOMMENDATIONS

Based on the findings and analysis of this study, it can be concluded that the application of constructivist and PCK-based instruction in mathematics teacher preparation programs has a positive impact on pre-service teachers' readiness to teach mathematics. The use of active learning, conceptual understanding, collaborative learning, and reflection as key principles of constructivism and PCK has helped pre-service teachers to develop a deeper understanding of mathematical concepts and pedagogical strategies.

The study also revealed that incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs presents both challenges and opportunities. Some of the challenges include the need for adequate training and professional development for teacher educators, the availability of resources and support, and the need for a shift in traditional teaching practices. However, the opportunities include the potential for improved student learning outcomes, the development of more effective teaching practices, and the promotion of lifelong learning among pre-service teachers.

Overall, this study highlights the importance of incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs to enhance pre-service teachers' readiness to teach mathematics effectively.

Based on the findings and conclusions of this study, the following recommendations were made:

1. Incorporate constructivist and PCK-based instruction in mathematics teacher preparation programs to enhance pre-service teachers' readiness to teach mathematics effectively.
2. Provide professional development opportunities for in-service teachers to enhance their knowledge and skills in implementing constructivist and PCK-based instruction in the classroom.
3. Encourage collaboration and dialogue among pre-service and in-service teachers to facilitate the sharing of best practices and the development of a community of practice.
4. Continue to conduct research on the impact of constructivist and PCK-based instruction on pre-service teachers' readiness to teach mathematics and students' learning outcomes.
5. Advocate for the integration of constructivist and PCK-based instruction in mathematics education policy and curriculum development to promote effective teaching and learning of mathematics.

By implementing these recommendations, teacher education programs can effectively prepare pre-service teachers to use constructivist and PCK-based instruction to enhance students' readiness to learn mathematics. Additionally, inservice teachers can be supported in their efforts to implement these instructional approaches in their classrooms, ultimately leading to improved student learning outcomes in mathematics.

CONSENT AND ETHICAL APPROVAL

As per international standard or university standard guideline, participant consent and ethical approval has been collected and preserved by the authors.

DISCLAIMER

The products used for this research are commonly and predominantly used products in our area of research and country. There is absolutely no conflict of interest between the author and the producers of the products because the author does not intend to use these products as an avenue for any litigation but for the advancement of knowledge. Also, the research was funded by the personal effort of the author.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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APPENDIX

Semi structured Interview questions for Pre – service Teachers to solicit for information on:

A**:** Impact of constructivist and pedagogical content knowledge (PCK) theories on pre-service teachers' readiness to teach mathematics

1. Can you tell me about your understanding of constructivism and pedagogical content knowledge in the context of teaching mathematics?
2. In what ways have you been exposed to or taught about constructivism and pedagogical content knowledge during your pre-service teacher education program?
3. How have these theories influenced your beliefs and practices around teaching mathematics?
4. How has your pre-service teacher education program prepared you to effectively apply constructivist and PCK approaches in teaching mathematics?
5. How do you see yourself continuing to develop your understanding and application of constructivist and PCK approaches in teaching mathematics in the future?
6. How do you think the incorporation of the constructivist and PCK approaches in teacher preparation programs might impact teacher attitudes and beliefs about teaching math?

B**:** How application of constructivist and PCK-based instruction in teacher preparation programs enhance students' readiness to learn mathematics

7. Have you received any training in the constructivist or PCK approaches to teaching mathematics during your teacher preparation program? If so, can you describe what this training involved and how it impacted your understanding of math teaching and learning?
8. How do you think the incorporation of constructivist and PCK-based instruction might impact the quality of math teaching and student learning outcomes?
9. In your opinion, how might the incorporation of the constructivist and PCK approaches to teaching math in teacher preparation programs better prepare teachers for their future teaching practice?
10. In your opinion, what kinds of support or resources would be helpful for pre-service teachers to effectively apply constructivist and PCK approaches in teaching mathematics?

C**:** Challenges and opportunities associated with incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs

11. In your opinion, what are the benefits of using constructivist and PCK approaches in teaching mathematics?
12. Have you had any experiences or challenges in applying constructivist and PCK approaches in your teaching practice? If so, can you describe these experiences or challenges?
13. Are there any specific teaching techniques or strategies associated with the constructivist or PCK approaches that you believe are particularly challenging or difficult to implement effectively? If so, can you describe why?

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