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A Mathematical Model for the Control of the Spread of Ebola Virus Disease in West Africa – A Disease-free Equilibrium Approach

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Short Research Article

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Abstract

The current Ebola Virus disease outbreak in West Africa is so far, the worst outbreak of the disease in any part of the world. It began in Guinea in December 2013 and then spread to Liberia, Sierra-Leone, Nigeria, Mali and Senegal. It has already claimed so many thousand lives and threatening those of so many others. In order to help control the spread or even completely eradicate the disease in West Africa in particular, we present a mathematical model based on the standard SEIR model. The disease-free equilibrium point of the model was established and its stability analysis carried out using the Routh-Hurwitz criteria. From the stability analysis it was found out that the necessary and sufficient condition for the control or possibly total eradication of the disease in West Africa is that the product of total break-down of the susceptible and latent classes must be less than the product of the total removal rates from both the latent and the infectious classes. We made recommendations on what should be done in order to meet the established condition.

Keywords: Disease free equilibrium; stability analysis; Jacobian matrix; Routh-Hurwitz criteria.

1 Introduction: Virology

Ebola virus disease is a viral disease caused by four of five virus disease classified in the genus Ebolavirus. Of the four disease-causing viruses, **Ebola virus** (formerly and often still called the Zaire virus) is the most dangerous and is the species responsible for the ongoing epidemic in West Africa. Ebola virus disease has

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been confined to areas in central Africa, where it is endemic since it was discovered in 1976 when outbreaks occurred in Sudan and the Democratic Republic of Congo (then called Zaire). With the current outbreak, it was initially thought that a new species endemic to Guinea might be the cause, rather than being imported from central to West Africa. However, further studies have shown that the current outbreak is likely caused by an Ebola virus lineage that has spread from central Africa via an animal host within the last decade, with the first viral transfer to humans in Guinea [1].

It is not entirely clear how an Ebola outbreak starts. The initial infection is believed to have occurred after an Ebola virus was transmitted to a human by contact with an infected animal's body fluids. Evidence strongly implicates bats as the reservoir hosts for Ebola viruses [2]. Bats drop partially eaten fruits and pulp, then land mammals such as gorillas and duikers feed on these fallen fruits. This chain of events forms a possible indirect means of transmission from the natural host to animal populations [3]. Human-to-human transmission occurs only via direct contact with blood or bodily fluids from an infected person who is showing signs of infection or by contact with objects recently contaminated by an actively ill infected person [4].

Air born transmission has not been documented during Ebola outbreaks, however, in February 2015 a group of researchers published a paper suggesting that it is very likely that at least some degree of Ebola virus transmission currently occurs via infectious aerosols but this has never been demonstrated in humans [5].

The time interval from infection with the virus to onset of symptoms is two to twenty-one days. Dead bodies are still infectious and so must be handled with great caution.

One study suggested that the virus can live up to 7 days in a diseased individual. Semen and possibly other body fluids such as breast milk may be infectious in survivors for months [6].One of the primary reasons for the spread of the virus is the poorly-functioning health systems in the part of Africa where the disease occurs. Other reasons are illiteracy, poverty and lack of adequate information on the mode of spread of the virus. The risk of transmission is increased among those caring for infected people. Recommended measures when caring for those infected include medical isolation through the proper use of gloves, masks, gowns, boots and goggles as well as sterilizing equipment and surfaces [7].

One of the biggest dangers of infection faced by medical staff is learning how to suit up and remove personal protective equipment. Full training for wearing protective body clothing can take 10 to 14 days. Even with proper isolation equipment available working conditions such as lack of running water, climate control, and flooring have made direct care difficult. Difficulties in halting the transmission of the virus also include the multiple disease outbreaks across country bounders [5]. Furthermore, past epidemics have occurred in remote regions but the current outbreak has spread to large urban areas, which has increased the number of contacts an infected person may have and has made transmission faster [8]. Symptoms of Ebola virus disease may begins as early as two days or as long as 21 days after infection. They usually begin with a sudden influenza-like stage characterized by tiredness, fever, and pain in the muscles and joints. Later symptoms may include headache, nausea, sore throat and abdominal pain. This is often followed by severe vomiting and diarrhea [9]. In past outbreaks the symptoms included loss of blood through internal and/or external bleeding but this symptom has been rare in the current outbreak. No proven Ebola virus-specific treatment presently exists, however there are measures that can be taken to improve a patient's chances of survival [10].

1.1 Ebola Virus Epidemics in West Africa

The most widespread epidemic of Ebola virus disease in history is currently ongoing in several West Africa countries. It has caused significant mortality, with reported case fatality rates of up to 70% and specifically 57-59% among hospitalized patients [11]. Ebola virus disease was first described in 1976 in two simultaneous outbreaks in sub-Saharan Africa. This is the 26th outbreaks and the first to occur in West Africa. It began in Guinea in December 2013 and then spread to Liberia and Sierra-Leone. A small outbreak of twenty cases occurred in Nigeria and one case occurred in Senegal. Several cases were also declared in

Mali [1]. Isolated cases have been reported in the United Kingdom, United States and Spain. As at 31st March 2015, the world Health Organization (WHO) and respective governments have reported a total of 25, 263 suspected cases and 10, 477 deaths, (Table 1).

Table 1. World health organization (WHO) report as at 31st March 2015

Country	Cases	Death
Liberia	9,712	4,332
Sierra Leone	12,022	3,810
Guinea	3,494	2,320
Nigeria	20	8
Mali	8	6
United States	4	1
United Kingdom	1	0
Senegal	1	0
Spain	1	0
Total	**25,263**	**10,477**

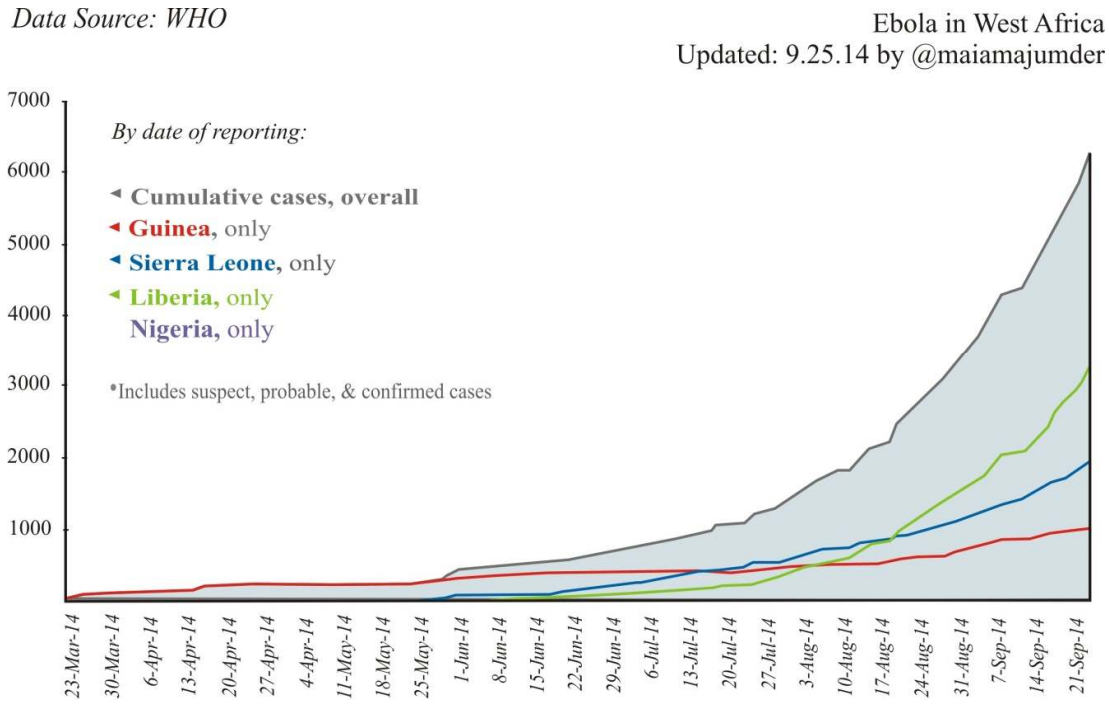

Fig. 1. Ebola virus disease in West Africa in 2014

1.2 Aim or Objective of the Research

The aim or objective of this research work is to contribute in the global effort in controlling the spread of the deadly Ebola virus disease in West Africa in particular and in the whole world in general, using a mathematical model.

2 Materials and Methods

2.1 The Model Description

The SLIR model, in which the population is partitioned into four (4) compartments or classes based on the epidemiological state of individuals in the population, is used to describe the Ebola disease dynamics within the popular. The compartments are: the Susceptible, the Latent, the Infectious and the Recovered compartments. The susceptible compartment increases due to the coming in of newborn babies and those treated and recovered from the disease. The class reduces due to infection of some people in the susceptible class who become latently infected in the sense that they have the virus but have not started exhibiting the symptoms of the disease. The class also reduces as a result of death from natural causes.

The population of the Latent class increases as a result of the infection of some people in the susceptible class with the virus. The compartment decreases due to the progression of some people from this class to the infectious class, the recovery of latently infected individuals though treatment and care, as well as death from natural causes.

The infectious compartment increases due to the progression of latently infected individual into this class. The compartment decreases due to the recovery of people who have received treatment and are cured of the disease, as well as due to death from Ebola-related causes and natural death.

The recovered compartment increases due to the coming in of latently and actively infected people who have successfully been treated and so are recovered and decreases due to the fact that recovered individual do not have any form of immunity against the virus and so once again become susceptible to the infection. The compartment also decreases due to death from natural causes. This dynamics is described in Fig. 2.

2.2 Assumptions of the Model

The model is based on the following assumptions;

1) That the individuals that make up the population can be grouped into different compartments or classes based on their epidemiological state. In other words, the population is assumed to be heterogeneous.
2) That the population size in each compartment is differentiable with respect to time and that the epidemic process is deterministic. In other words, that the changes in population of a compartment can be calculated using only history to develop the model.
3) That the population mixes homogeneously. That is, all the susceptible individuals are equally likely to be infected by infectious individuals in case of contact.
4) That people in each compartment have equal natural death rate of μ.
5) That the only way of entry into the population is through newborn babies and the only way of exit is through death from natural causes or death from Ebola virus diseases- related causes. In other words, that there are no immigration or emigration.
6) That the infection does not confer any immunity to the cured and recovered individuals and so they go back to the susceptible compartment.
7) That all newborns are previously uninfected by Ebola virus and therefore join only the susceptible class.
8) That there is presently no vaccination that provides immunity against the Ebola virus disease.
9) That infected individuals could be treated and cured of the virus.

2.3 Variables and Parameters

The variables and parameters used in this model are:

S(t): The population of susceptible individuals at time t

L(t): The population of latently infected individuals at time t
I(t): The population of infected individuals at time t
R(t): The population of individuals who have been treated and have recovered from the infection at time t.
α: The rate at which susceptible individuals become latently infected with the virus.
β: The rate at which latently infected individual become actively infected with the disease.
π: The rate at which actively infected individuals recover from the disease.
r: The rate at which latently infected individuals recover from the disease.
k: The rate at which recovered individuals return to the susceptible compartment.
μ: The natural mortality or death rate.
d: The Ebola virus disease-induced mortality or death rate
Λ: The population of newborn babies entering the population.
N: The total population size.

Based on the assumptions and the inter-relationships between the variables and the parameters as described in the compartmental model in Fig. 2, the Ebola virus disease dynamics can be described by the following differential equations:

$$\frac{dS}{dt} = \Lambda - \alpha SI - \mu S + \kappa R \tag{1}$$

$$\frac{dL}{dt} = \alpha SI - (\mu + \beta)L - rL \tag{2}$$

$$\frac{dI}{dt} = \beta L - (\pi + \mu + d)I \tag{3}$$

$$\frac{dR}{dt} = \pi I - (\kappa + \mu)R + rL \tag{4}$$

$$N = S + L + I + R$$

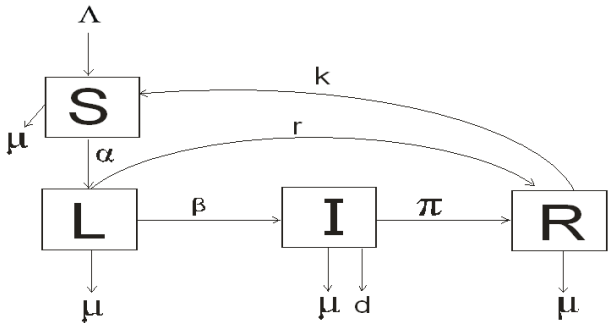

Fig. 2. Schematic presentation of the model

3 Results and Discussion

3.1 Equilibrium Point

At the equilibrium point we have

$$\frac{dS}{dt} = \frac{dL}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$$

That is,

$$\Lambda - \alpha SI - \mu S + \kappa R = 0 \tag{5}$$

$$\alpha SI - (\mu + \beta)L - rL = 0 \tag{6}$$

$$\beta L - (\pi + \mu + d)I = 0 \tag{7}$$

$$\pi I - (\kappa + \mu)R + rL = 0 \tag{8}$$

3.2 Disease-Free Equilibrium State

The disease-free equilibrium state is the state of total eradication of the disease. Let the disease-free equilibrium point be$E^* = (S^*, L^*, I^*, R^*)$. At the disease-free equilibrium state.

$$L^* = I^* = 0 \tag{9}$$

Substituting eqn (9) into egn. (5) – (8) and solving we have

$$\text{From (5), } \Lambda - \mu S + \kappa R = 0 \tag{10}$$

$$\text{From (8) } -(\kappa + \mu)R = 0$$
$$\Longrightarrow R^* = 0 \tag{11}$$

Substitute $R = 0$ into (10) to have

$$\Lambda - \mu S = 0$$

$$\Longrightarrow S^* = \frac{\Lambda}{\mu} \tag{12}$$

Therefore, $E^* = (\frac{\Lambda}{\mu}, 0,0,0)$

3.3 Stability Analysis of the Disease-free Equilibrium Point

We need to find the Jacobian matrix of the system to analyse the stability of the disease-free equilibrium. The Jacobian matrix, J, of the system of equations is:

$$J = \begin{pmatrix} -(\alpha I + \mu) & 0 & -\alpha S & \kappa \\ \alpha I & -(\mu + \beta + r) & \alpha S & 0 \\ 0 & \beta & -(\pi + \mu + d) & 0 \\ 0 & r & \pi & -(\kappa + \mu) \end{pmatrix}$$

At the disease-free equilibrium point the Jacobian matrix becomes:

$$J^* = \begin{pmatrix} -\mu & 0 & -\alpha\frac{\Lambda}{\mu} & \kappa \\ 0 & -(\mu + \beta + r) & \alpha\frac{\Lambda}{\mu} & 0 \\ 0 & \beta & -(\pi + \mu + d) & 0 \\ 0 & r & \pi & -(\kappa + \mu) \end{pmatrix}$$

If λi are the eigen values of the system, then the characteristic equation is $|J^* - \lambda I| = 0$

$$\begin{vmatrix} -\mu & 0 & -\alpha\frac{\Lambda}{\mu} & \kappa \\ 0 & -(\mu+\beta+r) & \alpha\frac{\Lambda}{\mu} & 0 \\ 0 & \beta & -(\pi+\mu+d) & 0 \\ 0 & r & \pi & -(\kappa+\mu) \end{vmatrix} = 0$$

$$-(\mu+\lambda)\begin{vmatrix} -(\mu+\beta+r)-\lambda & \alpha\frac{\Lambda}{\mu} & 0 \\ \beta & -(\pi+\mu+d)-\lambda & 0 \\ r & \pi & -(\kappa+\mu)-\lambda \end{vmatrix} = 0$$

$$(\mu+\lambda)(\kappa+\mu+\lambda)\begin{vmatrix} -(\mu+\beta+r+\lambda) & \alpha\frac{\Lambda}{\mu} \\ \beta & -(\pi+\mu+d+\lambda) \end{vmatrix} = 0$$

Hence, either $[(\mu+\lambda)(\kappa+\mu+\lambda)] = 0$ (13)

Or $\begin{vmatrix} -(\mu+\beta+r+\lambda) & \alpha\frac{\Lambda}{\mu} \\ \beta & -(\pi+\mu+d+\lambda) \end{vmatrix} = 0$ (14)

Solving eqn. (13) gives: $\lambda_1 = -\mu \quad and \quad \lambda_2 = -(\mu+\kappa)$

$$Let\ A = \begin{pmatrix} -(\mu+\beta+r) & \alpha\frac{\Lambda}{\mu} \\ \beta & -(\pi+\mu+d) \end{pmatrix}$$

The Routh-Hurwitz necessary and sufficient conditions (criteria) for the remaining two eigen values (roots) of the characteristic equation to have negative real part, implying asymptotic stability, is that trace A < O and det A > O.

$$Trace\ A = -(\mu+\beta+r) - (\pi+\mu+d)$$

Clearly, trace A < 0 since all the parameters are positive. For det A to be positive,(ie det A> 0) we must have

$$\begin{vmatrix} -(\mu+\beta+r) & \alpha\frac{\Lambda}{\mu} \\ \beta & -(\pi+\mu+d) \end{vmatrix} > 0$$

or $(\mu+\beta+r)(\pi+\mu+d) - \alpha\beta\frac{\Lambda}{\mu} > 0$

$$\Rightarrow \alpha\beta\frac{\Lambda}{\mu} < (\mu+\beta+r)(\pi+\mu+d) \quad (15)$$

Equation (15) establishes the necessary and sufficient condition for the disease-free equilibrium to be asymptotically stable. This means that Ebola virus disease can effectively be controlled if effort is made to ensure that the total breakdown of the susceptible and latent classes is always less than the total removal rate from both the latent and the infectious classes.

4 Conclusion

A mathematical model for the control of the spread of Ebola virus disease in West Africa was developed based on some assumptions and on the standard SEIR model, where the population is partitioned into compartments or classes based on the epidemiological state of individuals within the population. Using differential equations the disease infection dynamics within the population was described and the disease-free equilibrium was established. The stability analysis of the disease–free equilibrium state shows that Ebola virus disease can effectively be controlled in West Africa if effort is made to ensure that the total breakdown of the susceptible and latent classes is always less than the total removal rate from both the latent and the infectious classes.

5 Recommendations

The condition for the control or total eradication of the Ebola virus disease can be met if the following recommendations are considered:

1. People should be educated on the mode of transmission and on the symptoms of the diseases.
2. The conditions that promote rapid spread of the virus such as illiteracy, lack of adequate medical facilities, overcrowded accommodation etc. should be taken care of.
3. People who are infected should be encouraged to voluntarily report to designated health centers for immediate attention.
4. There should be more training of medical staff to specially handle Ebola virus disease.
5. There should be insurance policy for all medical staff handling Ebola virus disease to encourage them to be very serious and committed on handling infected individuals.
6. There should not be stigmatization of people infected by the disease or people fully cured of the disease.
7. People should freely submit themselves for Ebola tests and those found to be infected with the virus should co-operate with medical personnel.
8. Infected individuals should promptly be quarantined.
9. There should be more international co-operation to prevent cross-border transmission of the disease.
10. There should be more effort at producing vaccines and drugs for the virus by national and international organizations.

Competing Interests

Authors have declared that no competing interests exist.

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